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ABSTRACT

The Las Colinas landslide that occurred at Santa Tecla (El Salvador, Central America) due to the earthquake of 13 January 2001 is considered as one of the most destructive landslides ever known. This paper studies the ability of Hill's sufficient condition of stability (1958), which is based on the sign of second-order work, for predicting and describing this catastrophic massive landslide. The general expressions of both local and global second-order work criteria and its implementation into finite element codes are given. By using the non-associated elasto-plastic Hardening Soil Model and the local second-order work criterion, it is demonstrated that potentially unstable stress-strain states can occur strictly inside the Mohr-Coulomb failure surface in axisymmetric conditions. The Las Colinas landslide under earthquake shaking is simulated in plane strain conditions, using the pseudo-static method as loading variable with the non-associated Hardening Soil Model available in the finite element code PLAXIS. The location of the zone of negative values of the local second-order work makes it possible to successfully exhibit the landslides mechanism observed on the Las Colinas slope. Moreover, the comparison with the safety factors calculated by using the methods of slices and the shear strength reduction technique confirms that the global second-order work is a more pertinent indicator for predicting the global stability of the Las Colinas slope.

INTRODUCTION

In seismically active regions, earthquakes are major trigger for instability of natural and man-made slopes. Therefore, seismic effects are essential design considerations for slope stability analysis. Currently, the majority of the methods used by practitioners for the evaluation the failure mechanism of slopes during earthquakes are based on the pseudo-static approach. In this approach, the effect of earthquake on a potential sliding soil mass is represented in an approximate manner by a static force acting in the horizontal and/or vertical directions. The stability of the soil under this force is expressed by a safety factor that is usually defined as the ratio of the resisting force to the destabilising force. Failure occurs when the safety factor drop below one. Although the pseudo-static approach has certain limitations (Kramer 1996), this methodology is considered to be generally conservative, and is the one most often used in current practice.

The most popular technique used for design is limit equilibrium analysis based on methods of slices. Some of these methods have been advocated by Bishop (1955), Janbu (1968), Morgenstern and Price (1965), Spencer (1967). A difficulty with most of the limit equilibrium methods is that they are based on some assumptions made in advance about the shape or location of the failure surfaces and the shear strength at the interface along these failure surfaces. Moreover, it is well known that the behaviour of soil is often non-linear and path dependent. An accurate analysis of a slope stability problem and the related failure mechanism needs to consider the evolving strains and path dependency by means of a constitutive model.

In recent years, it has become more common to use more numerical methods, especially the finite element method, for stability analyses of slope (Duncan 1996; Griffiths and Lane 1999; Zienkiewicz and Taylor 1994). The finite element method allows to model non-linear material behaviour and, complex boundary and loading conditions. These more sophisticated approaches, generally based on elasto-plasticity, usually analyze the landslide mechanism by considering a failure criterion which is the so-called plastic limit condition (e.g. the Mohr-Coulomb's criterion) and/or the localized plastic strain condition (Rice 1976). The modelling of slope stability problems with these modern tools has led to large improvements compared to the conventional limit equilibrium methods. This paper focuses on the use of Hill's sufficient condition of stability, which is based on the sign of second-order work, in finite element modeling to analyze failure in geomaterials.

The formulations of both local and global second-order work criteria are firstly presented. The local second-order work allows to determine a potentially unstable material points or zones. Recently some authors (Darve *et al.* 2004; Darve and Vardoulakis 2004; Khoa *et al.* 2006; Nicot *et al.* 2007; Sibille *et al.* 2007; etc.) have demonstrated that for non-associated materials (i.e. the plastic flow direction is given by a potential surface that is different from the yield surface) such as soils, this local criterion allows to describe some failure modes which can occur strictly inside the Mohr-Coulomb limit condition. The global second-order work obtained by integrating the local second-order work over the considered volume allows to describe the stability of the overall domain (Khoa 2007; Nicot *et al.* 2007).

Then the next part is dedicated to numerical modeling of a real case. It concerns the catastrophic massive landslide occurring at the Las Colinas (El Salvador) under earthquake shaking in 2001. This landslide is analyzed by using the pseudo-static method as loading variable with the non-associated Hardening Soil Model available in the commercial finite element code PLAXIS. The sign of the local and global second-order works are checked in the calculated non-homogeneous stress and strain fields in order to analyze the potentially unstable zones and the global stability of the model, respectively.

During the pseudo-static simulation the safety factors are also calculated by using both the methods of slices (Bishop's simplified method, Janbu's generalized procedure of slices) and the shear strength reduction technique available in the code PLAXIS. The pertinence of the safety factors and the global second-order work in predicting the global stability of the Las Colinas is then discussed.

OUTLINE OF THE LAS COLINAS LANDSLIDE

El Salvador, one of the smallest and most crowded nations in Central America, is located on the Pacific coast of isthmus. This country has experienced, on average, one destructive earthquake every decade during the last hundred years. This paper focuses on the devastating earthquake of 13 January 2001 that struck El Salvador and caused damage to thousands of traditionally built houses and trigged hundreds of landslides, which were the main cause of fatalities. Among those landslides, the Las Colinas landslide (neighbourhood of Santa Tecla) was the most tragic. A huge amount off the rim of a mountain ridge rising south behind Las Colinas area caused the greatest loss of life in a single location from the earthquake. The exposed failure surface can be roughly divided into three zones as shown in Fig. 1.

Fig. 1. A bird's eyes view of the Las Colinas landslide obtained from a helicopter (USGS 2001).

Figure 2 presents the longitudinal cross-section of the Las Colinas slope. The relative thin layer of paleo-soil might have played an important role in initiating the movement. The main failure surface and the finite element model (FE-model) area are also shown in this figure.

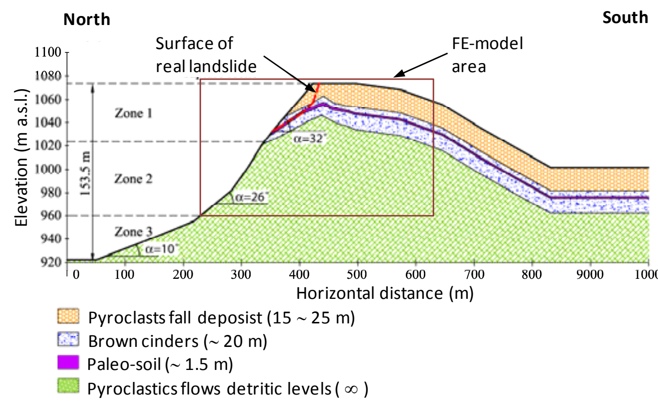


Fig. 2. Longitudinal cross section of the Las Colinas landslide area (Lotti & Associati-Enel.Hydro 2001).

HILL'S SUFFICIENT CONDITION OF STABILITY

This part presents the general expressions of both local and global second-order work criteria and its implementation into finite element codes.

Second-order work criterion at a material point level

Within the framework of continuum mechanics, Hill's sufficient condition of stability (Hill 1958) states that a mechanical stress-strain state is considered as stable if, for any couple ($\mathrm{d}\underline{\sigma}$, $\mathrm{d}\underline{\varepsilon}$) linked by incremental constitutive relation (constitutive matrix: $\underline{\underline{M}}$), the second-order work d^2W defined below is strictly positive:

$$\begin{aligned} &\forall\left(\mathrm{d}\underline{\sigma},\mathrm{d}\underline{\varepsilon}\right)\in \mathbb{R}^n\setminus\{0\} \quad \text{and} \quad \mathrm{d}\underline{\varepsilon}=\underline{\underline{M}}\cdot\mathrm{d}\underline{\sigma} \\ &\mathrm{d}^2W=\mathrm{d}\underline{\sigma}\cdot\mathrm{d}\underline{\varepsilon}>0 \end{aligned} \quad (1)$$

where n is the stress space dimension. Note that the second-order work is a directional quantity in the stress space.

Although this criterion of stability is not established on a thermodynamic basis, its physical meaning is clear. From a mechanical standpoint, if a given "material point" is not stable in Hill's sense, the deformation will continue in an infinitesimal manner at least in one direction without any additional energy contribution from external agencies (all loading conditions being maintained constant furthermore). The instability thus corresponds to the violation of Eq. 1 and that could be considered as an indicator of local (material) instability and of a possible global failure triggering.

The general expression of the second-order work d^2W which depends only on the symmetric part of the constitutive matrix is given by Eq. 2. In other terms, the instability argument coincides with the loss of positive definitenesss of the symmetric part of the constitutive matrix:

$$d^2W = d\underline{\sigma} \cdot d\underline{\varepsilon} = d\underline{\sigma} \cdot \underline{\underline{M}} \cdot d\underline{\sigma} = d\underline{\sigma} \cdot \underline{\underline{M}}^s \cdot d\underline{\sigma} \qquad (2)$$

where $\underline{M}^s$ is the symmetric part of the constitutive matrix $\underline{M}$.

For associated materials (the potential surface coincides with the yield surface), the constitutive matrix $\underline{M}$ is symmetric. Thus the unstable states will coincide with the failure surface. On the contrary, for non-associated materials like geomaterials, $\underline{M}$ is no longer symmetric and consequently the determinant of the symmetric part of $\underline{M}$ will vanish (following a given parameter) before the determinant of the matrix itself: unstable states according to Hill's criterion appear before the failure surface (see more details in Darve *et al.* (2004), Khoa *et al.* (2006), Nicot *et al.* (2007)).

Several experimentalists have observed the sudden collapse of soil-type materials for effective stress states strictly inside the Mohr-Coulomb failure surface (see for example: Chu and Leong (2003), Eckersley (1990), Gajo *et al*. (2000), Lade (2002), Sasitharan *et al.* (1993)). The well-known liquefaction phenomenon of loose sands under undrained (isochoric) triaxial conditions is a typical example where collapse occurs before reaching the failure surface. In this test, the deviatoric stress q passes through a maximum value (q–peak) which is located strictly inside the Mohr–Coulomb failure line (see Fig. 3). The experiments show that, if an infinitesimal additional axial load (axial stress control) is applied at q-peak, the specimen abruptly collapses without any localization pattern (Han and Vardoulakis 1991; Khoa 2006; Lade 1992). Re-writing the second-order work for the undrained triaxial loading path in axisymmetric conditions gives:

$$d^2W = d\sigma_1 d\varepsilon_1 + 2 d\sigma_3 d\varepsilon_3 \qquad (3)$$

Taking into account the isochoric condition ($d\varepsilon_1+2d\varepsilon_3=0$) implies that:

$$d^2W = dq d\varepsilon_1 \quad \text{with} \quad dq = d\sigma_1 - d\sigma_3 \qquad (4)$$

Thus, the second-order work vanishes at q–peak. According to Hill's condition, q–peak has to be considered as an unstable state.

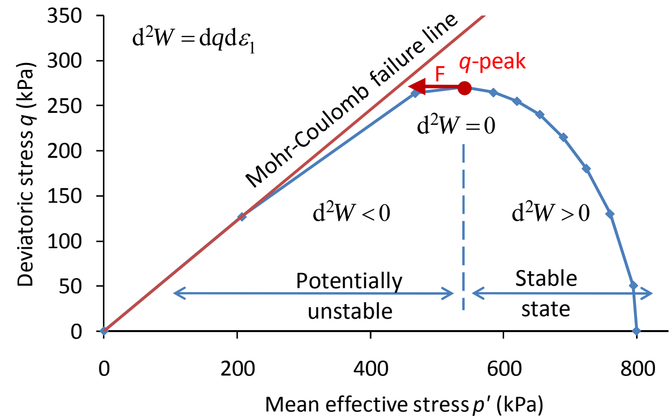


Fig. 3. Typical undrained triaxial behavior of Hostun loose sand S_{28} (test-CUSPR-02). At the q-peak, following the triaxial control parameters, the specimen can liquefy (strain control) or can give rise to a sudden failure (stress control). After Khoa et al. (2006).

In finite element codes, the second-order work is checked at all integration points *pi* (Gauss points) of the discretised domain between two successive and sufficiently close equilibrium steps of the incremental process. To make the plots of d^2W more legible, d^2W is normalized in relation to the norm of the stress increment $||d\underline{\sigma}||$ and strain increment $||d\underline{\varepsilon}||$ as follows:

$$d^2W_{pi,norm} = \frac{d\underline{\sigma}_{pi} \cdot d\underline{\varepsilon}_{pi}}{\left\|d\underline{\sigma}_{pi}\right\| \left\|d\underline{\varepsilon}_{pi}\right\|} \qquad (5)$$

The special domain of nil or negative values of $d^2W_{pi,norm}$ determines the potentially unstable area in the system.

To numerically determine the potentially unstable domain, Darve and Laouafa (2000) have proposed a procedure consisting of two principal steps:

- for a given confining pressure, we simulate a defined monotonic loading path (triaxial, plane strain,…);
- for each incremental stress along the loading path, we perform a stress-probing as presented in Fig. 4 to check for the first stress state that exhibits one stress direction giving a nil second-order work. This gives one point in the stress space of the boundary of the

instability domain and a corresponding unstable direction. Then the full instability domain is obtained by repeated the same procedure for different confining pressures.

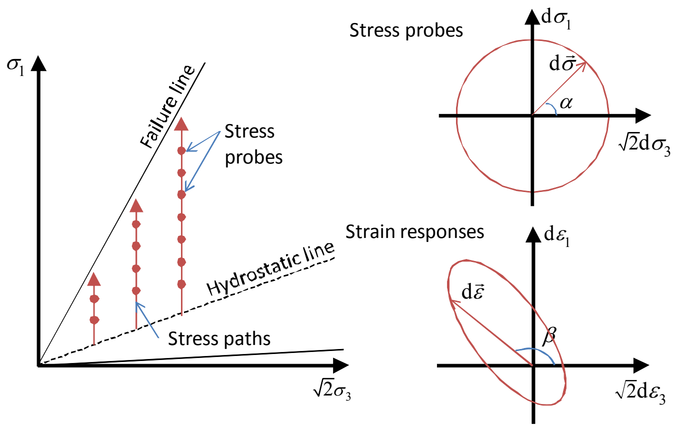


Fig. 4. Stress paths (left) and stress probe with corresponding strain response (right) in Rendulic plane in axisymmetric conditions.

Note that if this stress-probing procedure is conducted for a stress state inside the instability domain, a set of unstable directions is obtained and can be grouped in a cone (Darve *et al.* 2004; Khoa *et al.* 2006).

Global second order work criterion

Hill (1958) proposed a sufficient condition of stability for boundary problems where the geometry, the equilibrium equations, the initial condition and the boundary conditions are taken into account. Hill's criterion for a domain Ω is reviewed below:

$$\begin{aligned} &\forall\left(d\underline{\sigma}, d\underline{\varepsilon}\right) \in \mathbb{R}^n \setminus \{0\} \quad \text{and} \quad d\underline{\varepsilon} = \underline{\underline{M}} \cdot d\underline{\sigma} \\ &D^2W = \int_\Omega d\underline{\sigma} \cdot d\underline{\varepsilon}\, d\Omega > 0 \end{aligned} \tag{6}$$

In finite element analysis, the value of the global second-order work D^2W is the sum of the weighted values of d^2W_{pi} for all integration points (N_{pi}) of the medium:

$$\begin{aligned} D^2W &= \sum_{pi=1}^{N_{pi}} d^2W_{pi} \cdot \omega_{pi} \cdot \det \underline{\underline{J}}_{pi} \\ &= \sum_{pi=1}^{N_{pi}} d\underline{\sigma}_{pi} \cdot d\underline{\varepsilon}_{pi} \cdot \omega_{pi} \cdot \det \underline{\underline{J}}_{pi} \end{aligned} \tag{7}$$

where N_{pi} is the total number of integration points, $\det\underline{\underline{J}}_{pi}$ is the determinant of Jacobian transformation matrix at the integration point pi and ω_{pi} is the integration weight factor for the same point.

In order to eliminate the influence of the mesh and of the loading increment size, Khoa (2005) proposed a normalized form of the global second-order work $D^2W_{normalized}$ as follows:

$$D^2W_{normalized} = \frac{1}{\sum_{pi=1}^{N_{pi}} \omega_{pi} \cdot \det \underline{\underline{J}}_{pi}} \frac{\sum_{pi=1}^{N_{pi}} d\underline{\sigma}_{pi} \cdot d\underline{\varepsilon}_{pi} \cdot \omega_{pi} \cdot \det \underline{\underline{J}}_{pi}}{\sum_{pi=1}^{N_{pi}} \left\| d\underline{\sigma}_{pi} \right\| \cdot \left\| d\underline{\varepsilon}_{pi} \right\|} \tag{8}$$

The vanishing value of $D^2W_{normalized}$ along a given loading path determines the critical value of the loading parameter where the instability may occur due to a perturbation.

In the following, we will apply this second-order work criterion will be used to investigate the Las Colinas landslide.

NUMERICAL MODELING OF THE LANDSLIDE AT LAS COLINAS (EL SALVADOR)

Figure 5 depicts the FE-model of the Las Colinas slope. The stability analysis is carried out using the commercial FE code PLAXIS. Displacements along the bottom boundary are fully fixed while the right side is fixed only in the vertical direction. The mesh discretisation is composed of 1191 15-node triangular elements with 12 Gauss points.

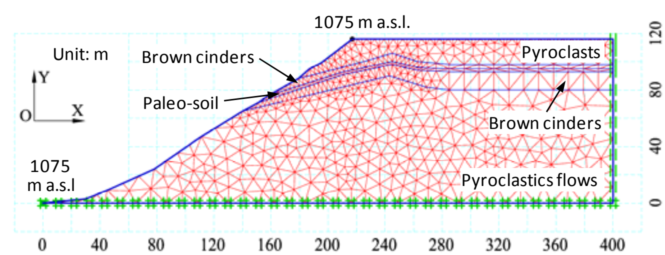


Fig. 5. Dimensions and discretisation of the FE-model of the Las Colinas slope.

The soil behaviors are modeled by using the non-associated Hardening Soil Model available in the PLAXIS code (Schanz 1999). The soil properties used in this study are based on several in situ and laboratory tests performed by Lotti & Associati-Enel.Hydro (2001). Table 1 summaries four sets of mechanical parameters of the soils at the site. It can be seen from the table that the paleo-soil is the weakest layer which probably facilitated the slope instability.

Table 1. Hardening-Soil parameters used in the FE-model of the Las Colinas slope.

Parameter	Unit	Pyroclasts	Brown cinders	Paleo-soil	Pyroclastics flows
Unit weight γ	kN/m^3	11	11	11	18
E_{50}^{ref} (p^{ref} = 100 kPa)	MPa	60	360	360	3780
E_{oed}^{ref} (p^{ref} = 100 kPa)	MPa	60	360	360	3780
E_{ur}^{ref} (p^{ref} = 100 kPa)	MPa	180	1080	1080	11340
Cohesion c	kPa	60	30	20	200
Friction angle φ	degree	30	30	25	38
Dilatancy angle ψ	degree	0	0	0	0
Poisson's ratio ν_{ur}	-	0.42	0.33	0.33	0.2
Power m	-	1	1	1	1

Figure 6 presents the cones of potentially unstable stress directions and the boundary of the unstable domain for the paleo-soil under triaxial axisymmetric conditions. It is apparent that a large domain of potentially material instability in Hill's sense (i.e. nil or negative value of the second-order work) is exhibited strictly inside the Mohr-Coulomb failure line. This results can be explained by the fact that since the non-associated character of the plastic strain is considered, the constitutive matrix is non symmetric and, therefore, loss of material stability in the form $d^2W = 0$ (i.e. $\det(\underline{M}^s) = 0$) precedes loss of uniqueness $\det(\underline{M}) = 0$. It is also interesting to remark that some directions will never give rise to failure by material instabilities, such as drained triaxial compressions and the axisymmetric compression or extension at constant mean pressures. The reason for performing the stress-probing procedure only in triaxial conditions is that the code used for calculating the unstable domain was limited to the triaxial stress conditions.

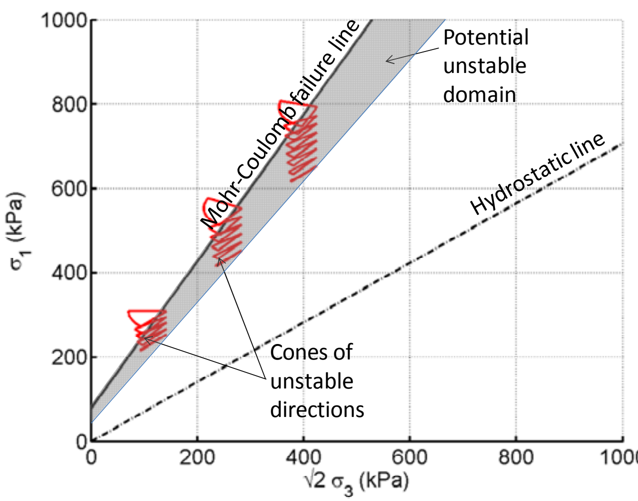


Fig. 6. Cones of unstable stress directions and boundary of the unstable domain in Rendulic plane for the paleo-soil.

To analyze the stability of the Las Colinas slope under earthquake shaking, the pseudo-static method is used as loading variable. In the pseudo-static analysis, the cyclic earthquake motion is replaced with a constant acceleration equal to $k_c(g)$, where k_c is the seismic coefficient, and g is the acceleration of gravity. A pseudo-static force applied to the soil mass is equal to the product of the acceleration and the weight of the soil mass. Figure 7 presents the seismic coefficients increased automatically by the PLAXIS code. Four loading steps are chosen to analyze the stability and determine the safety factor of the Las Colinas slope under the earthquake loading.

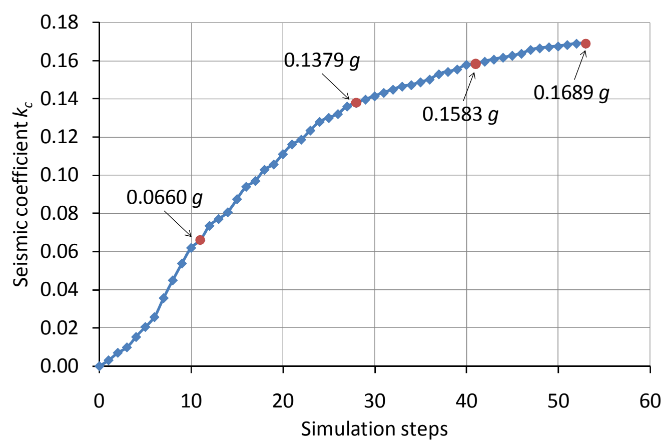


Fig. 7. Seismic coefficient k_c versus simulation steps.

When the simulation reaches step 53, i.e. $k_c = 0.1689$, the loss of convergence appears. The reason is that in the PLAXIS code, one cannot apply the horizontal gravity force independent from the vertical gravity force when using the pseudo-static method. If the horizontal gravity force could be applied as one load component it would have been possible to continue beyond the unstable point and continue into the unstable regime in order to analyze the development of the full failure model. Figure 8 plots the total displacement vectors for the last convergence step of the pseudo-static simulation. It can be seen that the largest displacement occurs in the paleo-soil layer which contains the weakest mechanical properties. It is worth noting that the critical constant horizontal acceleration is below the 0.5g peak ground acceleration recorded by a nearby Santa Tecla seismic station during the earthquake.

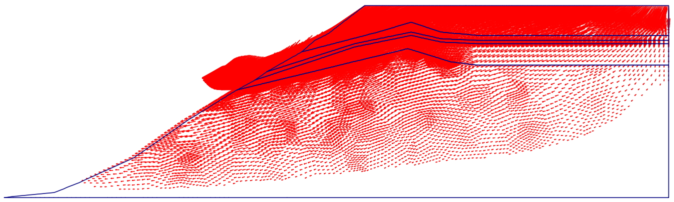


Fig. 8. Total displacement vectors at the last convergence step of the pseudo-static simulation (k_c = 0.1689).

Figure 9 presents a plot of the stress points in a plastic state (plastic points) which is often used in geotechnical engineering to determine the unstable material points and unstable zones. The hardening points represent points on the mobilized friction envelope, the Mohr-Coulomb points correspond to stress states lying on the surface of the Mohr-Coulomb failure envelope and the tension cut-off points indicates stress states fulfilling the tension cut-off criterion (see more details in Schanz (1999)). It is clear from the figure that at the last convergence step of the pseudo-static simulation, no stress state in the FE-model violates the Mohr-Coulomb's criterion. Therefore, it is necessary to use more robust stability criterion to analysis the Las Colinas landslide. Next parts devote to apply the Hill's sufficient condition of stability to predict and describe the failure mechanisms in the Las Colinas slope.

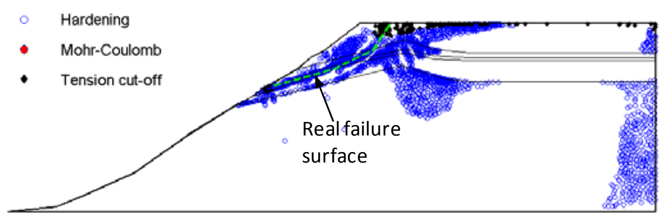
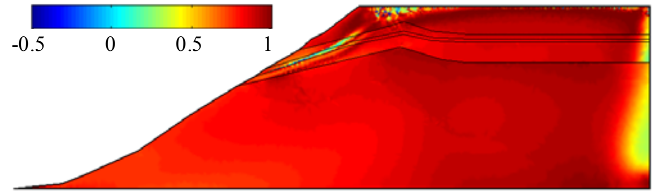


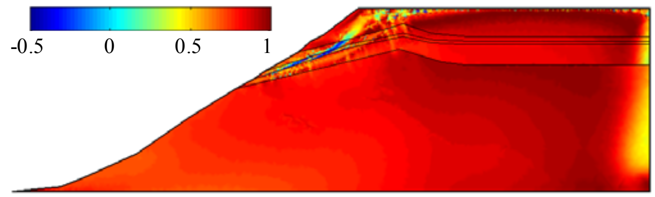
Fig. 9. Plastic points (hardening, Mohr-Coulomb and tension cut-off) at the last convergence step of the pseudo-static simulation (k_c = 0.1689).

Material instability analysis

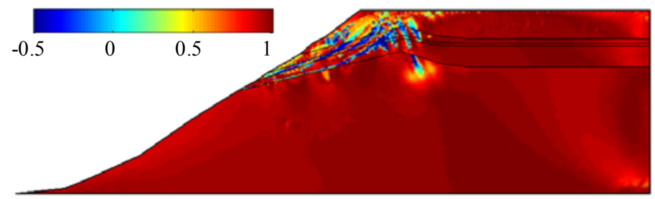
Using Eq. 5, the second-order work is calculated at all Gauss points in the FE-model, between two successive simulation steps, during the application of the pseudo-static force. The location of the zones, where the stability in Hill's sense is not fulfilled, is given in Fig. 10 for three values of the seismic coefficient. It can be seen from the figure that the more the seismic coefficient (acceleration) increase the larger zones of nil and negative values of d^2W are observed. At the last convergence step of the pseudo-static simulation, the potentially unstable zones in Hill's sense coincide in a remarkable way with the real landslide zones on the Las Colinas slope.



(a) k_c = 0.1379



(b) k_c = 0.1583



(c) k_c = 0.1689

Fig. 10. Isovalues of the normalized second order work in the Las Colinas slope for three values of the seismic coefficient.

The results confirm that the sufficient condition of stability proposed by Hill (1958), based on the sign of the second-order work, allow a successful prediction and location of the real landslide occurring on the Las Colinas slope. In the next part of this paper, the global stability is analyzed by considering the normalized global second-order work given by Eq. 8.

Global stability analysis

Figure 11 presents the evolution of the normalized global second-order work versus the increase of the seismic coefficient. It can be seen that $D^2W_{normalized}$ decreases as the acceleration increases. This reduction of $D^2W_{normalized}$ can well be explained by the fact that the increase of the pseudo-static forces due to the acceleration destabilized the Las Colinas slope. When the acceleration exceeds about 0.16 *g*, a sudden chute of $D^2W_{normalized}$ value appears. The loss of positive definiteness of the global second-order work at the last step of simulation is thus a good indicator to forecast the loss of stability of the whole Las Colinas slope.

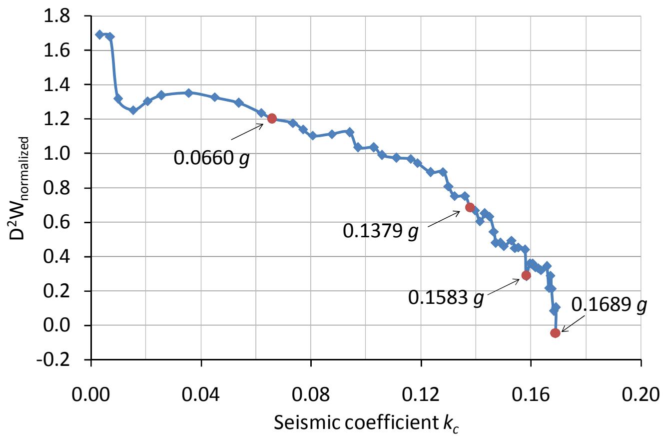


Fig. 11. Evolution of the normalized global second-order work during the pseudo-static simulation.

CACULATION OF THE SAFETY FACTOR

For slope, the traditional factor of safety is usually determined using the shear strength reduction technique and can be defined as (Zienkiewicz *et al.* 1975; Brinkgreve and Bakker, 1991):

$$\text{Safety factor} = \frac{c + \sigma' \tan\varphi}{c_c + \sigma' \tan\varphi_c} \tag{9}$$

where c is the cohesion, φ is the friction angle and σ' is the effective normal stress. c and φ are the peak or ultimate strength parameters and the subscript 'c' indicates the mobilized strength parameter. In the PLAXIS code, an automatic reduction of strength parameters is performed until equilibrium cannot longer be satisfied giving the global safety factor.

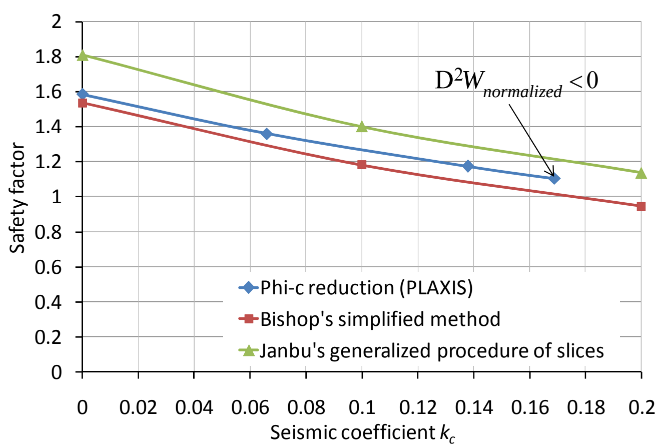


Fig. 12. Computed safety factors during the pseudo-static simulation.

Figure 12 presents the safety factors calculated by the PLAXIS code (called the so-called phi-c reduction method), and the methods of slices such as the Bishop's simplified method and Janbu's generalized procedure of slices obtained by Luo *et al.* (2004). The Bishop's method gives a lowest safety factor, however, none of those methods predicts the global failure (i.e. safety factor ≤ 1) in the Las Colinas slope before k_c = 0.17. Using $D^2W_{normalized}$, it can be concluded that the Las Colinas is unstable when k_c = 0.1689 (see Fig. 11). Based on those results, it can be concluded that the global second-order work is a pertinent indicator which allows to predict the global stability of the Las Colinas slope.

CONCLUSION

The ability of Hill's sufficient stability condition, which is based on the second-order work, for analyzing the stability of geomaterials has been discussed within the context of the elasto-plastic theory. The general expressions of both local and global second-order work criteria and its formulations introduced into finite element codes have been proposed.

The material instability phenomena have been numerically studied by considering the non-associated Hardening Soil Model and the local second-order work criterion. By analyzing the cones of potentially unstable stress directions and the boundary of the unstable domain of the specific case of the paleo-soil material, it was shown that the large domains of potentially material instabilities described by the local second-order work criterion can appear strictly inside the Mohr-Coulomb failure surface.

As a practical application, the Las Colinas landslide has been considered. The stability analysis at both local and global levels employing the second-order work criterion made it possible to successfully exhibit the main landslide zones in-situ. The increased interest of these pertinent criteria is that problems of greater complexity in terms of geometry, boundary conditions, loading parameters as well as material behavior, etc. can be considered. Moreover, the comparison with the safety factors calculated by using the methods of slices and the shear strength reduction technique confirms that the global second-order work is a more pertinent indicator for predicting the global stability of the Las Colinas slope.

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