

Missouri University of Science and Technology

[Scholars' Mine](https://scholarsmine.mst.edu/)

[International Conferences on Recent Advances](https://scholarsmine.mst.edu/icrageesd) [in Geotechnical Earthquake Engineering and](https://scholarsmine.mst.edu/icrageesd) [Soil Dynamics](https://scholarsmine.mst.edu/icrageesd)

[2010 - Fifth International Conference on Recent](https://scholarsmine.mst.edu/icrageesd/05icrageesd) [Advances in Geotechnical Earthquake](https://scholarsmine.mst.edu/icrageesd/05icrageesd) [Engineering and Soil Dynamics](https://scholarsmine.mst.edu/icrageesd/05icrageesd)

27 May 2010, 4:30 pm - 6:20 pm

Accuracy of Empirical Equations Predicting Sliding-Block Displacement

Tryfon Thomaidis Hellenic Open University, Greece

Constantine A. Stamatopoulos Hellenic Open University, Greece

Follow this and additional works at: [https://scholarsmine.mst.edu/icrageesd](https://scholarsmine.mst.edu/icrageesd?utm_source=scholarsmine.mst.edu%2Ficrageesd%2F05icrageesd%2Fsession04b%2F4&utm_medium=PDF&utm_campaign=PDFCoverPages)

Part of the Geotechnical Engineering Commons

Recommended Citation

Thomaidis, Tryfon and Stamatopoulos, Constantine A., "Accuracy of Empirical Equations Predicting Sliding-Block Displacement" (2010). International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics. 4.

[https://scholarsmine.mst.edu/icrageesd/05icrageesd/session04b/4](https://scholarsmine.mst.edu/icrageesd/05icrageesd/session04b/4?utm_source=scholarsmine.mst.edu%2Ficrageesd%2F05icrageesd%2Fsession04b%2F4&utm_medium=PDF&utm_campaign=PDFCoverPages)

This work is licensed under a [Creative Commons Attribution-Noncommercial-No Derivative Works 4.0 License.](https://creativecommons.org/licenses/by-nc-nd/4.0/)

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor I.M. Idriss May 24-29, 2010 · San Diego, California

ACCURACY OF EMPRICAL EQUATIONS PREDICTING SLIDING-BLOCK DISPLACEMENT

Tryfon Thomaidis
 Constantine A. Stamatopoulos
 Constantine A. Stamatopoulos
 Constantine A. Stamatopoulos
 Constantine A. Stamatopoulos
 Constantine A. Stamatopoulos 1 Anixeos Avenue, Veroia, GREECE

Instructor, Hellenic Open University
5 Isavron, 114 71 Athens, GREECE

ABSTRACT

The sliding-block model forms the basis of simple models predicting permanent co-seismic shear displacements of soils. For excitations consisting of actual accelerograms, different parameters of the applied motion have been used and different expressions have been proposed by researchers. Recently, many accelerograms have been recorded and these accelerograms are available in internet sites. These accelerograms allow the investigation of the accuracy of the expressions described above, some of which were based in a small number of accelerograms. In the present work the accuracy of empirical equations predicting sliding-block displacement is studied thru the application of 101 accelerograms covering a wide range of magnitudes, maximum accelerations, maximum velocities and dominant periods. The analysis illustrated that the accuracy of the methods vary. The Whitman and Liao (1984) method was found to produce the best predictions.

INTRODUCTION

Newmark's sliding-block model (Newmark, 1965) forms the basis of simple models predicting permanent co-seismic shear displacements of soils (Kramer, 1996). In this model, as shown in Fig. 1, a block rests in an inclined plane. In the general case, frictional and cohesional components of resistance exist in the interface between the block and the inclined plane. Critical acceleration is defined as the horizontal acceleration that causes the shear stress to become equal to the shear strength at the base of the block. When the applied acceleration is larger than the critical acceleration, the block slides. The total displacement is obtained by the addition of the slips where relative motion develops.

This model has been used for the estimation of permanent seismic deformations of natural slopes without considerable earthquake-induced loss of strength, of earth dams, of rockfill dams, and of gravity walls retaining dry soil (Makdisi and Seed, 1978, Richard, and Elms, 1979, Gazetas and Dakoulas, 1992, Whitman, 1993, Sarma, 1999). The solutions giving the distance moved by the sliding-block are used for the prediction of permanent seismic movement of these problems by replacing the maximum applied acceleration and critical acceleration of the block with those of the potential sliding mass under consideration.

For excitations of given analytical functions, analytical expressions giving the seismic displacement of the slidingblock model can be obtained and have been proposed

(Kramer, 1996). For excitations consisting of actual accelerograms, different parameters of the applied motion have been used and different expressions have been proposed by researchers (Ambraseys and Menu, 1988, Richard and Elms, 1979, Whitman and Liao, 1984, Sarma, 1999, Ambraseys and Srbulov, 1993, Yegian et al, 1991).

Recently, many accelerograms have been recorded and these accelerograms are available in internet sites. These accelerograms allow the investigation of the accuracy of the expressions described above, some of which were based in a small number of accelerograms. In the present work the accuracy of empirical equations predicting sliding-block displacement is studied thru the application of more than 100 accelerograms covering a wide range of magnitudes, maximum accelerations, maximum velocities, dominant periods and time durations.

The equation that describes the motion of the sliding-block model described above and shown in Fig. 1 (e.g. Stamatopoulos, 2003) is:

$$
d^2u/dt^2 = A^*g^*[k(t) \cdot k_c] \text{ for } du/dt > 0
$$

where

$$
A = \cos(\varphi \cdot \theta)^* \cos\theta / \cos\varphi
$$

$$
k_c = [m^*g^* \sin(\varphi \cdot \theta) + c^*L^* \cos\varphi]/[m^*g^* \cos(\varphi \cdot \theta)]
$$
 (1)

In the above equations, u is the relative downward displacement of the block, g is the acceleration of gravity, $k(t)$ is the applied acceleration history normalized by the acceleration of gravity, k_c is the critical acceleration defined above normalized by the acceleration of gravity, m is the mass of the block (per unit length), θ is the angle of the block with the horizontal, φ and c is the friction angle and the cohesion at the interface between the block and the inclined plane and L is the length of the slip surface. The inequality du/dt >0 indicates that displacement accumulates only when the applied horizontal acceleration exceeds the critical acceleration at one direction (the downward).

Equation (1) is usually solved in the bibliography with the assumption A=1.0, or

$$
d^{2}u/dt^{2} = g^{*}[k(t) - k_{c}] \text{ for } du/dt > 0
$$
 (2)

Two consecutive intergrations of equation (2) give the velocity du/dt and the displacement u in terms of time.

In real slopes the typical values of the angles θ and φ are such that the factor A does not differ considerably from unity. Thus, equation (2) is a reasonable approximation of equation (1).

SOLUTIONS THAT HAVE BEEN PROPOSED

For excitations of given analytical functions, analytical expressions giving the seismic displacement of equation (2) can be obtained and have been proposed (Kramer, 1996). Usually, the final seismic displacement predicted by the sliding-block model (uf) is expressed as the dimensionless factor (uf/[g $k_{max}T^2N$]), where u is the displacement, k_{max} is the maximum acceleration of the applied accelerogram normalized by the acceleration of gravity and T and N are the period and number of cycles of the excitation respectively, in terms of the ratio of the critical by the maximum acceleration (k_c/k_{max}) .

For excitations consisting of actual accelerograms, different parameters of the applied motion have been used and different expressions have been proposed by researchers. In particular, all expressions that have been proposed depend on the ratio (k_c/k_{max}) . Ambraseys and Menu (1988) propose a relationship that gives uf in terms of only the ratio k_c/k_{max} :

$$
\log (uf) = 0.90 + \log (1 - k_c/k_{\text{max}})^{2.53} (k_c/k_{\text{max}})^{1.09}) \quad (3a)
$$

where the displacement uf is in cm.

Richard and Elms (1979) give an upper limit on the displacement for earthquakes of magnitude M=7.5, in a relationship that in addition to the ratio k_c/k_{max} includes the value of the maximum acceleration $(k_{max} g)$ and the maximum velocity V_{max} of the applied excitation:

$$
uf < 0.09 \, \left[V_{\text{max}}^2 / (g \, k_{\text{max}}) \right] \left(k_c / k_{\text{max}} \right)^4 \tag{3b}.
$$

Whitman and Liao (1984), similarly, using the 14 ground motions, propose a relationship that in addition to the ratio k_c / k_{max} includes the value of the maximum acceleration and the maximum velocity of the applied excitation:

$$
uf = 37 * [V_{max}^2 / (g k_{max})] * e^{(.9.4 k c / k max)}
$$
 (3c)

Expressions (3b) and (3c) were derived for the design of gravity walls, but also apply for any structure designed by the predictions of the sliding-block model.

As described by Sarma (1999), Sarma in 1988 proposes a relationship that predicts uf in terms of the ratio $k_c/k_{\rm max}$, the critical period T_k , and the maximum value of the acceleration of the applied exchitation:

$$
\log (4uf/(g k_{\text{max}} T_{\kappa}^{2})) = 1.07 - 3.83 k_{c}/k_{\text{max}}
$$
 (3d)

Ambraseys and Srbulov (1993) give the seismic displacement uf in terms of the ratio k_c/k_{max} and of seismological parameters of (a) the magnitude M_s and (b) the distance r of the fault of the earthquake :

$$
\log \left(\text{uf} \right) = -2.41 + 0.47 \text{ M}_s - 0.010 \text{ r} + \log \left[\left(1 - \text{k}_c/\text{k}_{\text{max}} \right)^{2.64} \left(\text{k}_c/\text{k}_{\text{max}} \right)^{-1.02} \right]
$$
(3e)

where the displacement uf is in cm and the distance r is in Km The relationship has the advantage of using directly seismological parameters of a given location, that can be estimated from seismological studies.

Finally, Yegian et al. (1991), propose a relationship that in addition to the ratio k_c/k_{max} includes the value of the maximum acceleration, the significant number of cycles Neq and the dominant period:

$$
\log (\text{ uf/(g k}_{\text{max}} N_{\text{ eq}} T_{\kappa}^{2})) = 0.22 - 10.12 \text{ k}_{c} / \text{k}_{\text{max}} + 16.38 (\text{k}_{c} / \text{k}_{\text{max}})^{2} - 11.48 \text{ (k}_{c} / \text{k}_{\text{max}})^{3}
$$
 (3f)

At this point it should be noted that, as described by Seed and Idriss (1982), Neq can be related to the Earthquake Magnitude (M). This is illustrated graphically in Fig. 2. Fig. 2 also gives an empirical expression predicting the data:

$$
Neq = 0.07 \exp(0.70 M) \tag{4}
$$

DATA BASE OF ACCELEROGRAMS

During the present study, one hundered and one (101) accelerograms were collected from earthquakes with magnitudes 5 to 7.9 from the internet. The accelerograms were retrieved primarily from the sites [http://peer.berkeley.edu/nga/ earthquakes.html], [\http://peer.berkeley.edu/smcat/search.html], [http://www .isesd.cv.ic.ac.uk/ESD/frameset.htm] , [http://web.ics.purdue.edu/ ~braile/edumod/seisres/] , [http://www.pnsn.org/ seismosurfing.html] , [http://nsmp.wr.usgs.gov/].

The accelerograms were convered to fomat readable by the program seismograph, used to evaluate the dominant period of the accelerogram, and the program multi-block (Stamatopoulos and Mavromihalis, 2009), that was used to apply the sliding-block model.

The accelerograms that are included in the data base are given in the Appendix. For each accelerogram, the following parameters were recorded or estimated: maximum acceleration, maximum velocity, critical period, average period, earthquake magnitude, earthquake distance, earthquake energy, Arias earthquake intensity. In Fig. 3 characteristics of the data base are given. In particular, the maximum acceleration (a_{max}), the maximum velocity (V_{max}), the dominant period (T_k) , the average period (T_{mean}) and the earthquake magnitude of the earthquake records is presented. It can be observed that the accelerograms cover a wide range of magnitudes, maximum accelerations, maximum velocities and dominant periods

The possible relationship between the maximum horizontal acceleration and the earthquake magnitude and distance is investigated for the accelerograms of the data base collected in Fig. 4. It is observed that as theory predicts (Kramer, 1996), the maximum horizontal acceleration decreases as the earthquake distance increases and increases as the magnitude of the earthquake increases. The coefficient of correlation is very small for the earthquake distance and reasonable for the earthquake magnitude.

Fig. 3. The maximum acceleration, maximum velocity, dominant period and earthquake magnitudre of the data base of the earthquake records

ESTIMATION OF THE SEISMIC DISPLACEMENT

The seismic displacement of the sliding-block model was estimated by the multi-block program that has been developed by Stamatopoulos and described by Stamatopoulos and Mavromihalis (2009). A block having an inclination of appoximately 10° was considered. By varying the frictional strength at the interface between the block and the inclined plane, the cases of critical acceleration coefficient of kc=0.03, 0.13 and 0.18 were considered. All accelerograms of the data base described above were applied for each case of kc.

CHARACTERISTICS OF THE APPLIED ACCELEROGRAM THAT AFFECT THE SEISMIC DISPLACEMENT

The correlation of the seismic displacement of the slidingblock model (uf) with parameters of the applied excitation is first studied. The maximum acceleration, the maximum velocity, the dominant period and the average period of the applied accelerogram are considered. In addition, the effect of seismological parameters of the earthquake that caused the accelerogram such as the Earthquake magnitude, the distance, the Energy, the Arias Intensity (T_{Arias}) is also considered.

where Z is a quantity of the applied excitation correlated to the displacement (uf) and A1 and A2 are best-fit parameters. Figs 5, 6, 7 give the obtained correlations for the cases of kc=0.03, 0.13, 0.18. Table 1 gives the best-fit parameters $A1$ and A2 of equation (5). Table 2 summarizes the coefficient of correlation $[\overline{R}^2]$ for all cases.

It is observed that, as theory (Kramer, 1996) predicts, as the maximum acceleration, the maximum velocity, the dominant period and the average period of the applied accelerogram increase, the seismic displacement increases. In addition, as the earthquake magnitude, energy and Arias intensity increase, the seismic displacement increases. Table 1 illustrates that the parameters of the linear expression A1 and A2 change with kc in a consistent manner. Furthermore, from table 2 it is observed that the best correlation exists between the seismic displacement and the maximum velocity and the maximum applied acceleration. On the other hand, T_k and T_{mean} of the applied accelerogram and the Arias Intensity, the magnitude, the distance and the energy of the earthquake that caused the excitation do not correlate well the computed displacement.

Table 1. Best-fit parameters A1 and A2 of equation (5) of the relationship between the seismic displacement and parameters of the applied excitation

| PARAMETER | A1 | | | A2 | | |
|---------------------|------------|------------|------------|------|------|------|
| kc | 0.03 | 0.13 | 0.18 | 0.03 | 0.13 | 0.18 |
| Maximum | | | | | | |
| acceleration | | | | | | |
| (m/s2) | 0.29 | 0.09 | 0.04 | 0.22 | 0.16 | 0.10 |
| Maximum | | | | | | |
| velocity (m/s) | 3.27 | 0.61 | 0.32 | 0.45 | 0.10 | 0.06 |
| Critical Period (s) | 0.14 | 0.22 | 0.09 | 0.36 | 0.06 | 0.04 |
| Average Period | | | | | | |
| (s) | 1.64 | 0.21 | 0.07 | 0.06 | 0.02 | 0.03 |
| Earthquake | | | | | | |
| magnitude (Ms) | 0.61 | 0.11 | 0.06 | 3.30 | 0.62 | 0.33 |
| Earthquake | | | | | | |
| distance (km) | 0.001 | 0.002 | 0.002 | 0.77 | 0.23 | 0.08 |
| Earthquake | $2*$ | $2*$ | $8*$ | | | |
| Energy (erg) | 10^{-28} | 10^{-29} | 10^{-30} | 0.71 | 0.13 | 0.06 |
| Arias Intensity (s) | 0.04 | 0.005 | 0.002 | 0.35 | 0.07 | 0.04 |

Table 2. Comparison of the correlation between the seismic displacement and parameters of the applied excitation

INVESTIGATION OF THE ACCURACY OF EMPIRICAL RELATIONSHIPS OF THE BIBLIOGRAPHY

The data base of accelerograms that was created was used to investigate the accuracy of the empirical equations that have been proposed in the bibliography and are given in the section "Solutions that have been proposed" predicting the seismic displacement of the sliding-block model (uf). In the method of Yegian et al, the equivalent number of cycles was obtained from the earthquake magnitude using equation (4).

The predicted by the various methods versus the computed seismic displacement is given in Figs 8 and 9. In addition, the predicted (P) versus actual (X) values of seismic displacement are compared assuming linear relationship:

$$
P = A X \tag{5}
$$

where A is a factor. The factor A, as well as the coefficient of correlation (R^2) must be close to unity for good prediction. Table 3 gives the factor A, as well as the coefficient of correlation (R^2) for all methods for kc=0.03, 0.13 and 0.18. It can by observed that the accuracy of the predictions depends on the method and the critical acceleration value.

Referring to equation (5), satisfactory prediction is defined in the present work as the cases where (a) $1.4 > A > 0.7$ and (b) R^2 >0.4. Table 4 gives the satisfactory predictions for each case. It can be observed that the Whitman and Liao method is satisfactory for all values of kc. The Ambraseys and Menu and Yegian methods give satisfactory predictions for kc=0.03, but not for kc=0.13 and 0.18. The Ambraseys and Srbulov and Sarma methods do not give satisfactory predictions for kc=0.03, 0.13 and 0.18. It can be noted that the above are consistent with the observations of table 1 indicating that the seismic displacement is related better with the maximum velocity and maximum acceleration than to other factors of the applied accelerogram such as the dominant period, the earthquake magnitude and the earthquake distance.

As the Richard and Elms method provides an upper limit, and not the average displacement, the above comparison is not relevant. Yet, Fig. 8 illustrates that the upper limit predicted by the Richard and Elms method is a reasonable approximation for $uf<0.1m$, but not for $uf>0.1m$, where it is over-conservative.

| | $k = 0.03$ | | | $k = 0.13$ | | $k = 0.18$ |
|---------------|------------|-------|------|------------|------|------------|
| METHOD | A | R^2 | A | R^2 | A | R^2 |
| Ambraseys | | | | | | |
| and Menu | 0.84 | 0.41 | 0.45 | 0.46 | 0.46 | 0.36 |
| Richard and | | | | | | |
| Elms | 930 | 0.60 | 14.0 | 0.69 | 6.72 | 0.72 |
| Whitman | | | | | | |
| and Liao | 0.94 | 0.44 | 0.88 | 0.68 | 0.82 | 0.69 |
| Ambraseys | | | | | | |
| and Srbulov | 0.43 | 0.54 | 0.26 | 0.56 | 0.28 | 0.69 |
| Sarma | 1.13 | 0.25 | 2.00 | 0.43 | 2.48 | 0.57 |
| Yegian et al | 0.87 | 0.42 | 0.54 | 0.55 | 0.52 | 0.71 |

Table 3. Comparison of the accuracy of the methods

Table 4. Satisfactory prediction $(1.4 > A > 0.7$ and $R^2 > 0.4$ of equation (5)), in terms of the method and the kc value

| | \sim \prime \prime \sim | | | | | | |
|------------------|---------------------------------|------------|------------|--|--|--|--|
| METHOD | $k = 0.03$ | $k = 0.13$ | $k = 0.18$ | | | | |
| Ambraseys and | yes | no | no | | | | |
| Menu | | | | | | | |
| Whitman and Liao | yes | yes | yes | | | | |
| | | | | | | | |
| Ambraseys and | no | no | no | | | | |
| Srbulov | | | | | | | |
| Sarma | no | no | no | | | | |
| | | | | | | | |
| Yegian et al | yes | no | no | | | | |
| | | | | | | | |

CONCLUSIONS

One hundered and one (101) accelerograms of earthquakes of magnitudes 5 to 7.9 were collected from the internet. This data base of accelerograms was first used to investigate the correlation of the sliding-block seismic displacement with characteristics of the applied accelerogram and the earthquake that produced it. It was observed that best correlation exists between the seismic displacement and the maximum velocity and the maximum acceleration. On the other hand, the dominant and average periods of the applied accelerogram and the Arias Intensity, the magnitude, the distance and the energy of the earthquake that caused the excitation do not correlate well the computed displacement.

Then, the data base was used to investigate the accuracy of the empirical equations that have been proposed in the bibliography and predict the seismic displacement of the sliding-block model. It was observed that the accuracy of the predictions depends on the critical acceleration value (kc). Referring to equation (5), satisfactory prediction was defined as the cases that both (a) $1.4 > A > 0.7$ and (b) $R^2 > 0.4$. As illustrated in tables 3 and 4, the Whitman and Liao method is satisfactory for all values of kc. The Ambraseys and Menu and Yegian methods give satisfactory predictions for kc=0.03, but not for kc=0.13 and 0.18. The Ambraseys and Srbulov and Sarma methods do not give satisfactory predictions for either kc=0.03, 0.13 or 0.18. It can be noted that the above are consistent with the observations of table 1 indicating that the seismic displacement is related better with the maximum velocity and maximum acceleration than to other factors of the applied accelerogram such as the dominant period, the earthquake magnitude and the earthquake distance.

REFERENCES

Ambraseys N., Srbulov M. (1995) "Earthquake induced displacements of slopes," Soil Dynamics and Earthquake Engineering; 14, pp. 59-71

Ambaseys N. and Menu J. (1988) "Earhquake induced ground displacements, Earthquake engineering and structural dynamics, 16, 7, 985-1006.

Gazetas G., Dakoulas P. (1992). Seismic analysis and design of rockfill dams: State-of-the-Art", Soil Dynamics and Eathquake Enginering Journal, 11, pp. 27-61 (1992).

Kramer, S.L. "Geotechnical Earthquake Engineering", Prentice Hall, New Jersey, 1996.

Makdisi F. I. and Seed H. B. (1978), "Simplified procedure for estimating dam and embankment earthquake-induced deformations", ASCE Proceedings, Journal Geotechnical Engineering Division, Vol. 104, GT 7, pp. 849-867.

Newmark, N. M. (1965). "Effect of earthquakes on dams and embankments", Geotechnique, Vol. 15, No. 2, London, England, June, pp. 139-160.

Richard, R. and Elms, D.(1979), "Seismic Behavior of Gravity Retaining Walls", Journal of the Geotechnical Engrg Division, ASCE, Vol. 105 No. 4.

Paper No. 4.03b 8

Sarma, S. K. (1999). "Seismic slope stability - The critical acceleration", Proceedings of the Second International Conference on Earthquake Geotechnical Engineering, Balkema, Lisbon, 1999, pp. 1077-1082.

Seed H. B. and Idriss I. M. (1982). "Ground motions and soil liquefaction during Earthquakes, monograph series, Earthquake Engineering Research Institute, Berkeley, California.

Stamatopoulos C. and Mavromihalis C.(2009) The Effect of the Geometry Changes on Sliding-Block Predictions. Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in honor of professor I. M. Idriss

Stamatopoulos C. (2003). Dynamics of soils and foundations. Program: earthquake engineering and earthquake resistant structures. Hellenic Open University , Patras, Greece (in Greek)..

Whitman R. V. (1993). Predicting earthquake-caused permanent deformations of earth structures, article on "Predictive Soil Mechanics", Thomas Telford, London, pp. 729-741.

Whitman, R. V. and Liao S. (1985). Seismic design of retaining walls. Miscellaneous paper GL-85-1, US Army Engineer Waterways Experimental Station, Vicksburg, Mississipi.

Yegian M. D., Marciano E. and Ghahraman V. G. (1991). Earthquake-inced permanent deformations: probabilistic approach. Journal of Earthquake Engineering, ASCE, vol. 117, no 1, pp 35-50.

APPENDIX. DATA BASE OF ACCELEROGRAMS CONSIDERED

