

29 May 2010, 8:00 am - 9:30 am

## Evaluation of Frequency Dependent Equivalent Linear analysis

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### Recommended Citation

Jeong, Chang-Gyun; Kwak, Dong-Yeop; Park, Duhee; and Kim, Kwangkyun, "Evaluation of Frequency Dependent Equivalent Linear analysis" (2010). *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. 5.

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Fifth International Conference on

## Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor I.M. Idriss

May 24-29, 2010 • San Diego, California

### EVALUATION OF FREQUENCY DEPENDENT EQUIVALENT LINEAR ANALYSIS

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#### ABSTRACT

One dimensional equivalent linear site response analysis is widely used in practice due to the simplicity and ease of use. In the method, the dynamic soil properties are assumed to be independent of the loading frequency. To better simulate the nonlinear behavior, equivalent analysis methods that models the loading frequency dependence of the shear modulus and damping were developed. The backbone of the methods is the frequency-dependent shear strain curves. Various forms of the frequency-dependent shear strain curves were developed. However, the effect of the frequency - shear strain curves are not well known and documented. In this study, a series of frequency – strain curves were used to evaluate the accuracy of the frequency dependent equivalent linear analysis. Results show that the effect of the curves is significant and that the frequency dependent analysis does not always provide an improved estimate and can highly overestimate the amplification of the high frequency components of the ground motion. The degree of overestimation is dependent on the characteristics of the input ground motion and the soil profile. It is therefore concluded that the frequency dependent equivalent linear analysis should be used with caution and that standard equivalent linear analysis can be a more reliable option.

#### INTRODUCTION

One-dimensional (1D) seismic site response analysis is routinely performed to characterize the site amplification effects under an earthquake ground motion. 1D analysis, which simulates the vertical propagation of horizontal shear waves through horizontally layered soil profile, has been known to give reasonable estimates of ground vibration under a seismic event (Idriss, 1990).

1D site response analysis is either performed in frequency or time domain. In a nonlinear analysis, which is performed in time domain, the dynamic equation of motion is integrated at each time step and the nonlinear soil behavior is accurately modeled. However, the non-linear site response analysis is not widely used due to difficulty in performing the analysis and high computational cost. Instead, the equivalent linear analysis (EL), which approximates the nonlinear soil behavior within the linear framework, is widely used in engineering practice (Schnabel et al., 1972). The main limitation of the procedure is that a constant linear shear modulus and damping at a representative level of strain is used throughout the analysis.

Frequency-dependent equivalent linear algorithms (here after FDEL) have been proposed to overcome the limitation and better simulate the nonlinear hysteretic soil response under the

seismic loading (Sugito et al., 1994, Yoshida et al., 2002, Kausel and Assimaki, 2002). It has been shown that the frequency-dependent equivalent linear methods result in improved estimates of the ground motion propagation. However, the degree of improvement of respective methods have not been compared. This study compares the computed responses using the schemes by Yoshida et al. (2002) and Kausel and Assimaki (2002), along with three additional schemes developed in this paper, with those from equivalent linear and nonlinear analyses.

#### FDEL ALGORITHM

Fig. 1 shows a schematic plot of hysteretic loops under large (A) and small (B) shear strain amplitudes, respectively. Assuming that the strain increment at each time step is similar for both loops, the frequency of vibration of loop A will be lower than for loop B. This comparison explains that the frequency of vibration is associated with the amplitude of shear strain.

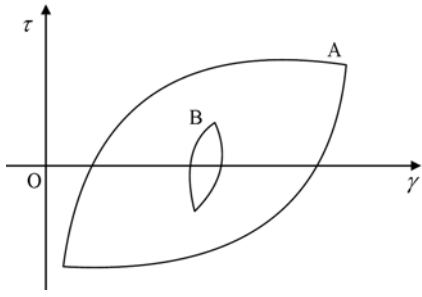


Fig. 1. Comparison of hysteretic loops of large(A) and small(B) strain amplitudes (modified after Yoshida et al., 2002)

Fig. 2, which shows the Fourier spectra of two shear strain time histories, demonstrates the strong dependence of the shear strain amplitude on the frequency. The amplitude of the shear strain decays quickly with increase in frequency. Standard equivalent linear procedure ignores such dependence and applies constant shear modulus and damping throughout the entire frequency range.

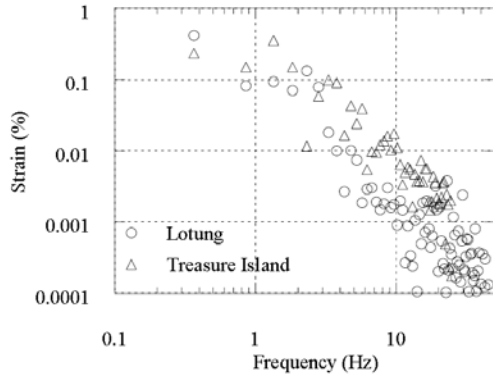


Fig. 2. Frequency-dependence of shear strain Fourier spectra

The FDEL propose use of the smoothed shear strain Fourier spectrum, such as shown in Fig. 3a, in quantifying the frequency dependence of the shear strain in a site response analysis. The normalized shear strain represents the shear strain amplitude normalized to the maximum value. The FDEL applies shear modulus and damping representative of the maximum shear strain at frequencies between 0 and  $f_i$  (Fig. 3a). At higher frequencies, the normalized shear strain ratios obtained from the frequency – shear strain curve is used to select corresponding shear modulus and damping ratio. In this process, different values of shear modulus and damping ratios are applied at frequency range between  $f_i$  and  $f_u$ . At frequencies higher than  $f_u$ , minimum shear strain is applied. All FDEL methods use this procedure. The difference lies in how the shape of the smoothed Fourier spectrum is defined.

Yoshida et al. (2002) proposed the following equation to characterize the smoothed frequency-dependent shear strain:

$$\begin{cases} \gamma_{eff} = \gamma_{max} & f_i > f \\ \gamma_{eff} = \gamma_{max} \left\{ 1 - \left( \frac{\log f - \log f_i}{\log f_u - \log f_i} \right)^m \right\} & f_i \leq f \leq f_u \\ \gamma_{eff} = 0 & f < f_u \end{cases} \quad (1)$$

where  $f_i$  is the frequency at which the amplitude of the Fourier spectrum is maximum,  $f_u = 15$  Hz, and  $m$  is a material constant. Yoshida et al. (2002) used 2 for  $m$ . This model will be termed “YKSM” in the following.

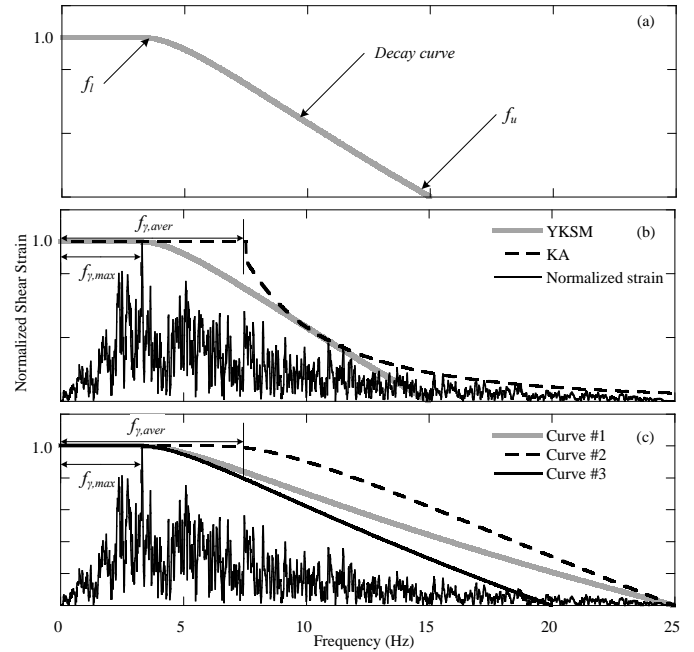


Fig. 3. Smoothed strain curve proposed by Yoshida et al. (2002), Kausel and Assimaki (2002), and this study

Kausel and Assimaki (2002) proposed the following equation:

$$\begin{cases} \gamma_{eff} = \gamma_{max} & f \leq f_0 \\ \gamma_{eff} = \gamma_{max} \times \frac{\exp\left(-\alpha \frac{\omega}{2\pi f_0}\right)}{\left(\frac{\omega}{2\pi f}\right)^\beta} & f > f_0 \end{cases} \quad (2)$$

where  $f_0$  and  $\gamma_0$  are defined as follows

$$f_0 = \frac{\int_0^\infty \omega r(\omega) d\omega}{2\pi \int_0^\infty r(\omega) d\omega} \quad (3)$$

$$r_0 = \frac{1}{\omega_0} \int_0^{\omega_0} r(\omega) d\omega \quad (4)$$

$\alpha$  and  $\beta$  are constants those are determined by least square method.  $f_0 = f_i$ , and  $f_u =$  maximum frequency of the input

ground motion, as shown in Fig. 3b. This model will be termed “KA” in the ensuing. It has been postulated that the form is very convenient, since the curve fitting parameters appear linearly when its logarithm is taken and allows simple estimation of  $\alpha$  and  $\beta$ .

Although both curves are distinctively different in form, both equations are alleged to significantly enhance the accuracy of the site response analysis (Yoshida et al., 2002, Kausel and Assimaki, 2002).

This paper used additional curves to investigate the influence of the shape of the curves. Three additional curves are named curve #1, curve #2 and curve #3, respectively. All curves use the decay function of YKSM, given in Eq. 1. The curves differ in values of  $f_i$  and  $f_u$ . Curve #1 extends  $f_u$  from 15 Hz to 25 Hz. Curve #2 applies  $f_i = f_0$  and  $f_u = 25$  Hz. Curve #3 applies  $f_u = 20$  Hz.

#### VALIDATION OF FDEL VIA RECORDED DATA

The validity of the FDEL was evaluated through measured recordings at two sites. Standard equivalent linear analysis (EL), FDEL, and nonlinear analyses (NL) were performed at selected sites. A total of five frequency-dependent shear strain curves were used in FDEL, shown in Fig. 3. EL and NL were performed using DEEPSOIL v3.7 (Hashash, 2009), while GEOSHAKE (Lee et al., 2008) was used to perform FDEL.

calculated responses for E-W and N-S components, respectively. Figures a & b show the 5% damped surface acceleration response spectra, c & d show the ratio of output and input response spectra, and e & f display the ratio of Fourier spectra, termed as amplification factors. Fig. 6 show that the E-W motion is predicted accurately by all methods. The YKSM and curves #1 - #3 predict the peak ground acceleration (PGA) better than KA, EL, and NL. The KA curves estimation of the PGA is the lowest. For the N-S component, YKSM and KA overestimate the response, while the EL and NL match well with the recordings. The curves #1 - 3 all compare well with the recordings. This comparison demonstrates that the FDEL does not always improve the solution. In this case, the EL provided better match compared to YKSM and KA.

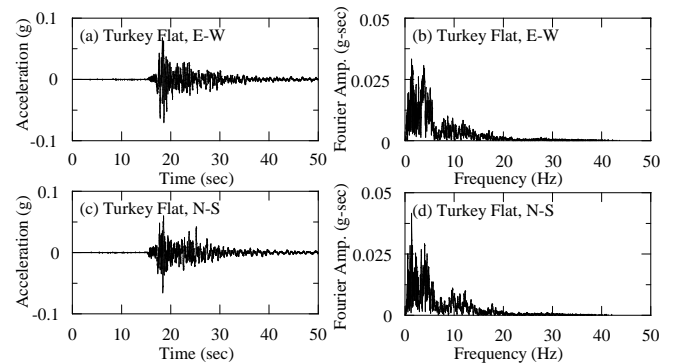


Fig. 5. Input motion acceleration time histories and Fourier spectra (Case 1, Turkey flat, CA, U.S.A.)

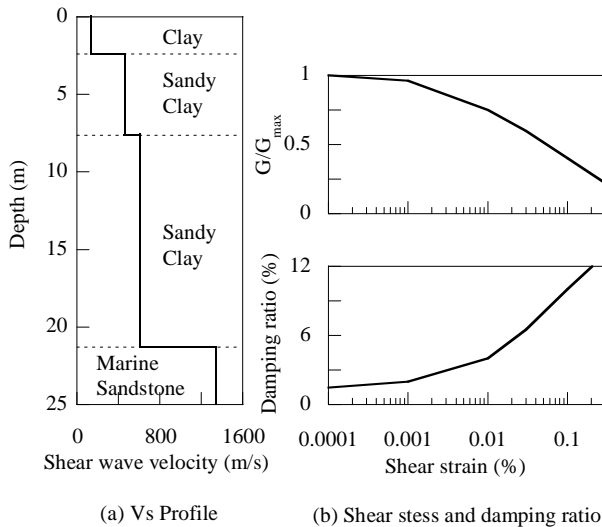


Fig. 4. Shear wave velocity profile and dynamic curves used for Turkey flat site (Real, 1988)

#### Case 1: Turkey flat, California, U.S.A.

The first site used in this study is Turkey Flat, California. Figure 4 shows the shear wave velocity profile and dynamic curves of the site. Input earthquake motions used were recorded motions during Parkfield earthquake (magnitude = 6.0, 2004), as shown in Fig. 5. Fig. 6 and 7 show the

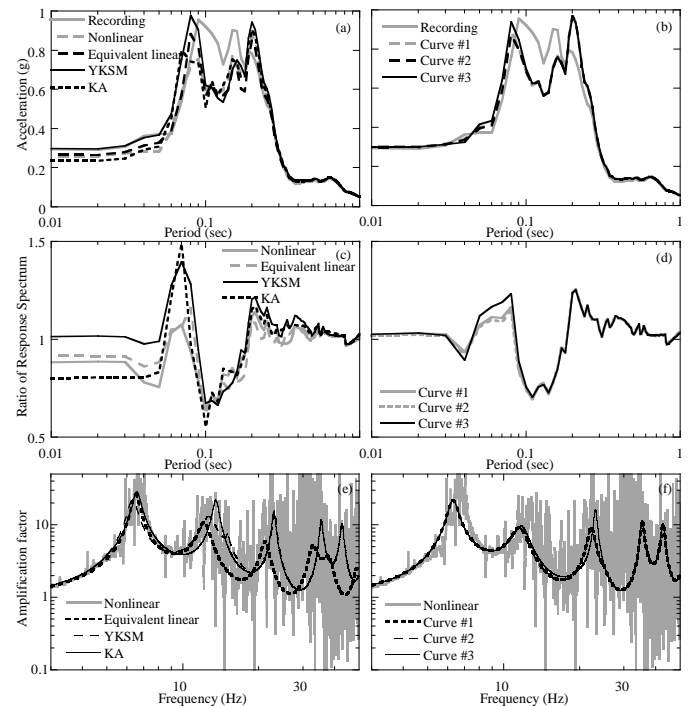


Fig. 6. 5% damped surface acceleration response spectrum, ratio of response spectrum and amplification factor (Case 1, Turkey flat, CA, U.S.A., E-W component)

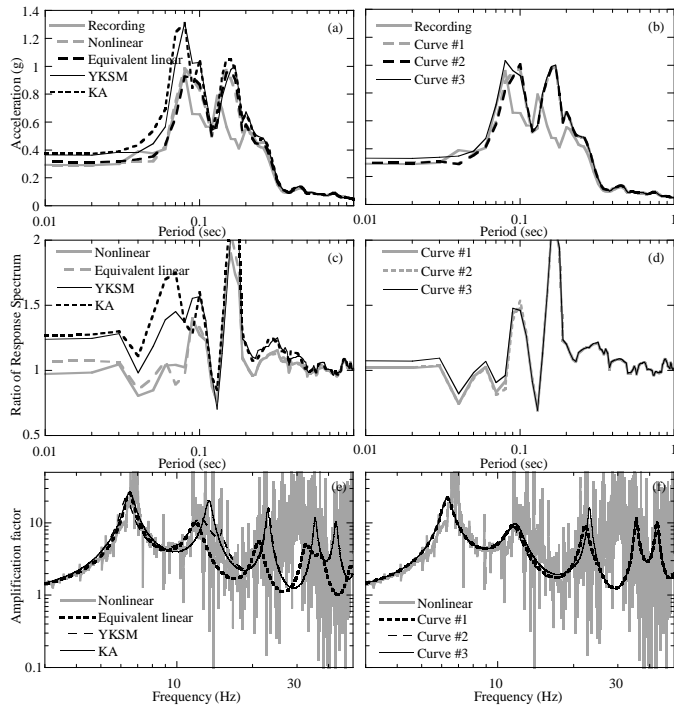


Fig. 7. 5% damped surface acceleration response spectrum, ratio of response spectrum and amplification factor (Case 1, Turkey flat, CA, U.S.A., N-S component)

This case demonstrates that even for an identical event, the accuracy of the FDEL can vary for different components of the ground motion. It means that the FDEL is very sensitive to the frequency characteristics of the input ground motion.

### Case 2: Lotung, Taiwan

The LSST (Lotung Large-Scale Seismic Test) project started in the early 1980s to investigate the seismic site effects and soil-structure interaction. Dense downhole arrays were deployed at the test site. The shear wave velocity profile and dynamic properties of the site are shown in Fig. 11 (Elgamal et al., 1995, Zeghal et al., 1995). The site consists of silty sand and sandy silt with gravel layers.

In this paper, motions recorded at a depth of during the LSST event #7 (1981, magnitude = 5.8) were used as input motions (Fig. 12) in performing the site response analysis. The motions were imposed as “within soil profile” motions.

The results of the site response analyses are shown in Fig. 13 – 14. For the E-W component, Fig. 13, the predictions of all analyses were not accurate. The KA curve resulted in the highest response. The calculated PGA was more than twice the measured value. The EL also resulted in overestimation of the response, while YKSM highly underestimated the response. The NL resulted in the best match with the recording. Curves #1 – 3 were similar to the YKSM.

For the N-W components (Fig. 14), the KA curves resulted in unacceptably high overestimation of the response. EL and

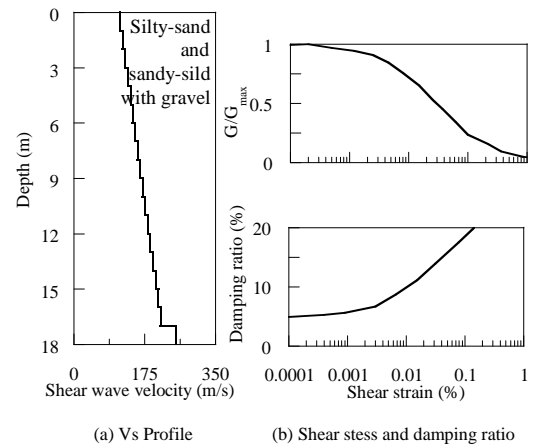


Fig. 8. Shear wave velocity profile and dynamic properties of the Lotung, Taiwan (Elgamal et al., 1995, Zeghal et al., 1995)

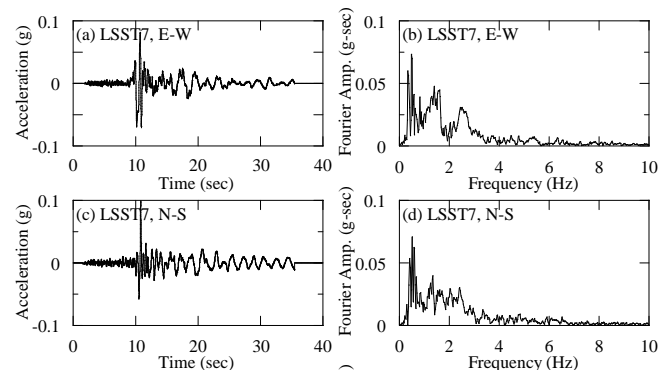


Fig. 9. Input motion acceleration time histories and Fourier spectrum (Case 2, Lotung, Taiwan)

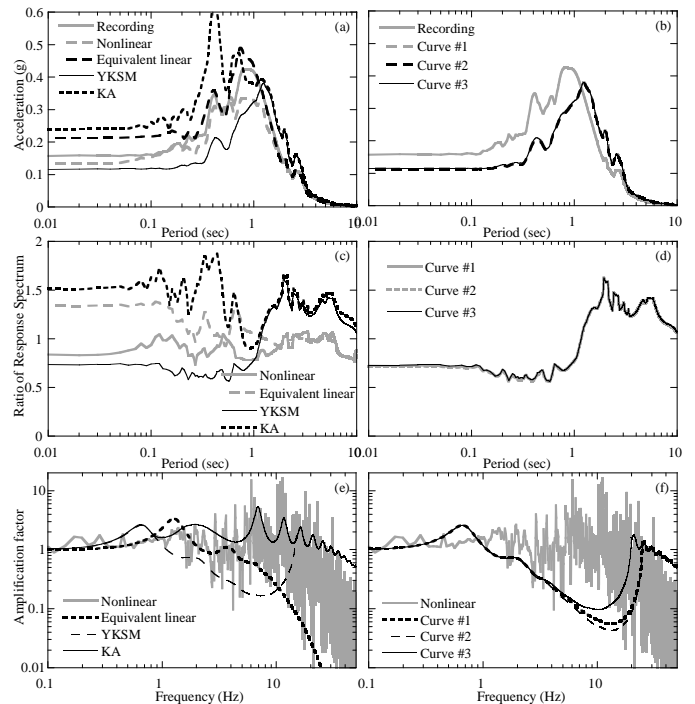


Fig. 10. Comparison of calculated and measured responses (Case 2, Lotung, Taiwan, LSST7 E-W component)

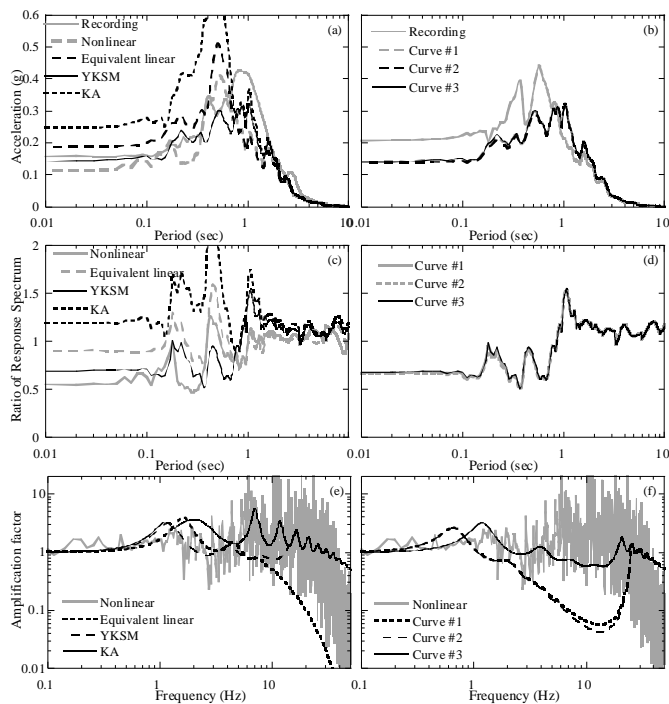


Fig. 11. Comparison of calculated and measured responses (Case 2, Lotung, Taiwan, LSST7 N-S component)

## CONCLUSION

In this study, the accuracy of the frequency-dependent equivalent linear method (FDEL) were evaluated through two case studies. A total of five curves that relate the frequency with the shear strain were implemented in a FDEL program and the results of the analyses were compared to nonlinear (NL) and standard equivalent linear analyses (EL).

The comparisons showed that the predicted responses of the FDEL are highly different. The analysis type (NL – EL – FDEL) that best predicted the measured data varied for all cases. Comparing the FDEL with EL and NL, it is shown that FDEL does not always improve the prediction. This is in direct contradiction with the conclusions of relevant literature which alleged that the FDEL always enhances the prediction. Among the FDEL, pronounced degree of variations were found among the estimated responses. It was not possible to select a single curve that best matches the measurement for all cases. Among various components of the smoothed frequency – shear strain curve, the rate of decay of the shear strain amplitude with frequency had the most critical influence on the calculated response.

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