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DISPLACEMENT DESIGN SPECTRUM MODEL ACCOUNTING FOR NON-LINEAR SITE EFFECTS

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ABSTRACT

Displacement design response spectrum is an essential component for the currently-developing displacement-based seismic design and assessment procedures. This paper proposes a new and simple method for constructing displacement design response spectra on soft soil sites. The method takes into account modifications of the seismic waves by the soil layers, giving due considerations to factors such as the level of bedrock shaking, material non-linearity, seismic impedance contrast at the interface between soil and bedrock, and plasticity of the soil layers. The model is particularly suited to applications in regions with a paucity of recorded strong ground motion data, from which empirical models cannot be reliably developed.

INTRODUCTION

Ground motion characteristics of a soil site can be highly dependent on conditions of the overlying Quaternary sediments. Engineering design spectra stipulated by contemporary codes of practices specify site factors for different site classes and hence enable site effects to be predicted without calculations, or with simple manual calculations. Site classification schemes adopted by major codes of practices typically parameterize soil dynamic properties on the basis of the shear wave velocity (*SWV*) averaged over a certain depth in the sediment (20m in the Chinese Code, and 30m in the International Building Code, IBC). With this approach, which is based on the statistical analyses of abundant empirical data, parameters representing details of the soil layers have been averaged. Consequently, factors controlling the timing of multiple reflections at the boundary between soil and bedrock and those within the soil medium (resulting in conditions pertaining to resonance behavior) have not been parameterized.

The significance of soil resonance phenomenon depends on soil conditions, level of seismic hazard, and so forth. The resonance phenomenon deserves special attention with flexible soil sediments with high impedance contrast at the interface with bedrock, and more so in regions of low and moderate seismicity which are typified by infrastructure with limited ductility which accentuates the effects of resonance. The effects of resonance results in high displacement (drift) demand on structures and are best represented by the displacement response spectrum. It is important to note that displacement response spectrum is the key to the development of reliable displacement-based seismic design and assessment procedures (e.g. Tsang *et al.*, 2009).

The objective of the proposed calculation procedure is to estimate the spectral response ratio (*SR*) which is defined herein as the ratio of the maximum response spectral displacement on the surface of the soil (RSD_{max}) and the corresponding response spectral displacement on the adjacent rock outcrop (RSD_{Tg}) at the fundamental natural period of the site (T_g). The value of T_g can be estimated by equation (1).

$$T_g = \frac{4H}{V_S} \tag{1}$$

where H is the depth of the soil and the V_s is the weighted average shear wave velocity.

Structures found on a soil site and possessing this natural period will experience resonance behavior and hence this period is most critical in terms of the seismic displacement demand. Refer Fig. 1 for a schematic illustration. The amplification from RSD_{Tg} to RSD_{max} is modeled in two parts: (i) amplification of the peak displacement demand at the bedrock surface to that at the soil surface as represented by the peak displacement ratio (*PDR*) and (ii) response amplification of an elastic single-degree-of-freedom system when subject to periodic motion at the soil surface and is represented by the resonance factor (f). The relationship between *SR* and the amplification factors is defined by equation (2).

$$SR = \frac{RSD_{\max}}{RSD_{Tg}} = PDR \cdot f \tag{2}$$

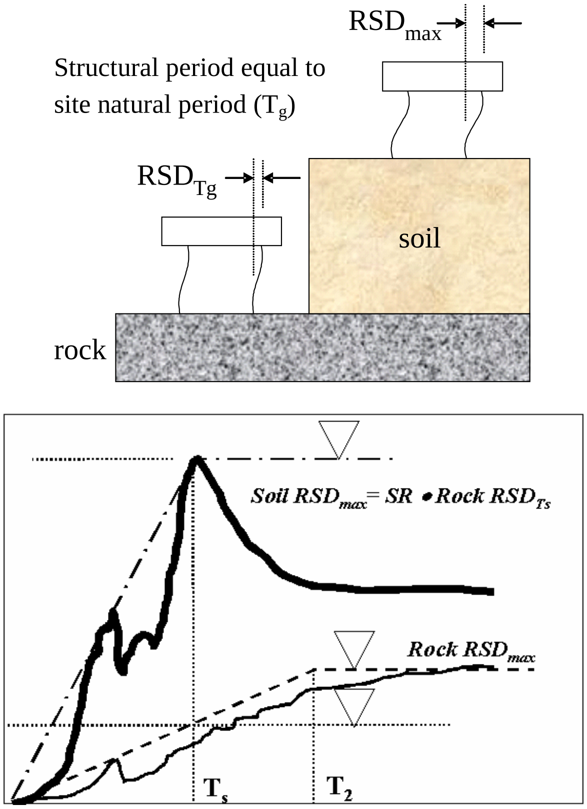


Fig. 1. Schematic representation of displacement amplification on a flexible soil site.

A range of analytical software has been developed to model site effects at varying levels of sophistication. Whilst one-dimensional (1-D) equivalent linear shear wave analysis is a well known and well established analytical tool for site response analysis (e.g. program *SHAKE*), it is yet to be widely used by practicing engineering professionals, and particularly in low and moderate seismic regions. Another issue with the use of time-history analysis programs such as *SHAKE* is the lack of information over details of ground motion and hence there are uncertainties as to what accelerogram data is suitable for input into the analysis.

This paper presents the development of a simple (hand-calculation) model for predicting site effects which are characterized by soil resonance behavior as described above. Importantly, the impedance ratio between the bedrock and the overlying soil has been introduced as a key parameter in the calculation (along with the damping parameters). It is noted that expressions used in developing the proposed formulae are based on well-established wave theories. The predicted amplification has been shown to be very consistent with results obtained from analyses using *SHAKE*. The proposed calculation procedure which is in its early stage of development is based on modeling the soil sediments as homogenous materials overlying bedrock. Intuitively, non-homogenous soil layers may also be analyzed using this method by weighted averaging the soil *SWV* and density. Further study is now underway to improve the capability of the method to take into account complex layering conditions within the soil sediments.

The microtremor array method with the spatial auto-correlation (SPAC) processing technique has been discussed in latter part of the paper. The method appears to be extremely well suited to applications in urban areas due to its non-invasiveness and inexpensive (and speedy) data acquisition processes. It is recommended that SPAC be used as a common tool for obtaining *SWV* information of the site.

THEORETICAL DEVELOPMENT

Non-linear Peak Displacement Ratio (PDR)

Modeling of the non-linear peak displacement ratio (*PDR*) is based on three principal mechanisms: (i) transmission of seismic waves across the interface between two media (bedrock and soil), (ii) reflection of seismic waves at the two boundaries of the soil medium (i.e. boundary with rock and that with air), and (iii) hysteretic energy dissipation during wave transmission within the soil medium.

As upwardly propagating seismic waves reach the interface between the bedrock and the soil, as shown in Fig. 2, only part of the wave energy is transmitted into the soil whilst the rest is reflected back into the half-space of the bedrock. The displacement amplitude of the transmitted wave (A_T) and the reflected wave (A_R) can be calculated using equations (3a) and (3b) for zero angle of incidence (approach the interface at 90° angle).

$$A_R = \frac{\alpha - 1}{\alpha + 1} A_i \tag{3a}$$

and

$$A_T = \frac{2\alpha}{1 + \alpha} A_i \tag{3b}$$

where A_i is the amplitude of the incident wave and α the impedance ratio as defined by equation (4).

$$\alpha = \frac{\rho_R V_R}{\rho_S V_S} \tag{4}$$

where ρ and V are the weighted-average of the density and the *SWV* (the subscripts *R* and *S* represent the rock and soil layers respectively).

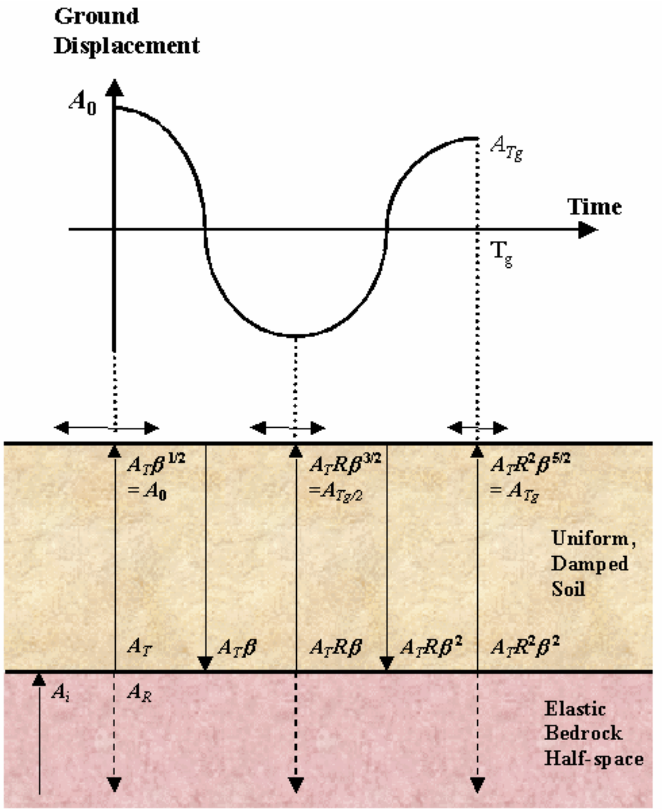


Fig. 2. Illustration of the concept of the site fundamental natural period, multiple wave reflections, material and radiation damping.

Equation (3b) can also be used to model the amplification of seismic waves reaching the soil surface, based on considering the soil and air as two media separated by the interface (with a very high value of α). Amplification factor at the soil-air interface is equal to 2. Meanwhile, there are waves reflecting back down into the soil medium. The amplitude of the downward propagating reflected waves is accordingly equal in amplitude and sign to the incident wave based on equation (3a).

The reflected seismic waves will then reach bedrock for the second time when reflection will again occur. Equation (3a) may, yet again, be used for modeling seismic waves reflecting from the bedrock-soil interface back up into the soil medium, but the value of α is reciprocal to that defined by equation (4) due to the change in direction of the wave transmission. The ratio of the amplitude of the reflected and incident waves, which is defined as the wave reflection coefficient (R), can be calculated using equation (5).

$$R = \frac{1/\alpha - 1}{1/\alpha + 1} = \frac{1-\alpha}{1+\alpha} \tag{5}$$

From equation (5), R varies between 0 and 1 and with a change in sign which means that the polarity of the waves will also change. The de-amplification of the seismic waves ($R < 1$) reflected back up from the bedrock surface can be described as "radiation damping".

Unlike boundary mechanisms, hysteretic damping occurred within the soil medium modifies wave amplitude continuously. The de-amplification of the wave amplitude can be expressed as an exponential function of the number of wave cycles experienced during the wave transmission. The de-amplification factor β for half wave-cycle is given by equation (6).

$$\beta = \exp(-\pi\zeta) \tag{6}$$

where ζ is the damping ratio (as a proportion of critical damping). The dependence of the shaking level (non-linearity) in site response is accounted for by this soil damping ratio. A model for estimating intensity dependent damping in soil has been developed in (Tsang *et al.*, 2006a), and illustrated in Section 4 of the paper. From equation (1), seismic wave components possessing the site natural period (T_g) will experience quarter-of-a-cycle periodic motion during the transmission of the waves through the thickness of the soil medium. The reduction in the wave amplitude is accordingly represented by:

$$A_0 = 2\beta^{\frac{1}{2}} A_T \tag{7}$$

where A_0 is the wave amplitude reaching the soil surface.

The upwardly propagating S-waves after reflecting from the soil-bedrock interface will reach the soil surface to complete half a cycle of wave motion. The displacement amplitude is defined by:

$$A_{\frac{Tg}{2}} = R\beta A_0 \tag{8}$$

The same modifications will be experienced by the reflected waves when undergoing yet another half a cycle of motion (with yet another change in the wave polarity). On completion of the two half-cycles, the displacement amplitude of the wave reaching the soil surface is defined by:

$$A_{Tg} = R\beta A_{\frac{Tg}{2}} = R^2\beta^2 A_0 \tag{9}$$

Equations (7) and (9) represent the displacement amplitudes of the wave when reaching the soil surface at time $T = 0$ and $T = T_g$ (i.e. $n = 0$ and 1), respectively. The polarity of the wavefront at both instances has the same polarity.

Wavefronts with time-lag will superpose as they are reflected onto the soil surface repetitively. The amplitude of two wave components, as defined by equations (7) and (9), corresponding to $n = 0$ and 1 respectively, can be aggregated as shown by equation (10) which satisfies the principle of the conservation of energy.

$$\tilde{A}_{Tg} = \sqrt{A_0^{\ 2} + A_{Tg}^{\ \ 2}} = A_0\sqrt{1 + R^4\beta^4} \tag{10}$$

The superposition of infinite number of wave components (i.e. n = infinity) can also be represented by the algebraic relationship of equation (11) which features the summation of a geometric series with infinite number of terms.

$$A_{soil-surface} = \sqrt{\sum_{n=0}^{\infty} A_{nTg}^{\ \ 2}} = A_o \sqrt{\sum_{n=0}^{\infty} \left(R^{2n}\beta^{2n}\right)^2} \tag{11}$$

where n is the number of wave cycles (of period T_g). Given that the value of $R^{2n}\beta^{2n}$ is less than unity, equation (11) can be re-written as:

$$A_{soil-surface} = A_0 \sqrt{\frac{1}{1-R^4\beta^4}} \tag{12}$$

In comparison, the amplitude of ground motions experienced by structures founded directly on the rock surface can be represented by equation (13).

$$A_{rock-surface} = 2A_i \tag{13}$$

where the factor of 2 represents the surface effects at the interface between rock and air.

The peak displacement ratio (*PDR*), which is the ratio of the wave amplitude, as calculated from equations (12) and (13) is hence represented by:

$$PDR = \frac{A_{\text{soil-surface}}}{A_{rock\text{-surface}}} = \frac{2\alpha}{1+\alpha}\sqrt{\frac{\beta}{1-R^4\beta^4}} \tag{14}$$

Spectral Ratio (SR)

The response of linear elastic single-degree-of-freedom (SDOF) systems found on the soil surface is considered next. The modeling is based on systems with natural period matching the site natural period. The amplification of the system's response, which is represented by the "*f* " factor in equation (2), has been found to be sensitive to the rate of energy dissipation in both the soil and the structure. The empirical function of equation (15) was developed by the authors (Tsang *et al.*, 2006b) in a parametric study to investigate the trends.

$$f(\alpha) = \alpha^{0.3} \le 2.3 \tag{15}$$

The upper limit of 2.3 is to reflect the observation that f becomes insensitive to changes in the value of α when $\alpha > 16$. An expression for estimating the value of *SR* is finally obtained by combining equations (14) and (15) and is shown by equation (16).

$$SR = f(\alpha)\cdot\frac{2\alpha}{1+\alpha}\sqrt{\frac{\beta}{1-R^2\beta^2}} \tag{16}$$

It is noted that *SR* is basically a function of (i) ratio α of the impedance contrast; and (ii) half-period damping factor β (a function of soil damping ratio ζ). It is noted that R in itself is also a function of α.

VERIFICATION AND SENSITIVITY STUDY

Verification with SHAKE

Shear wave analyses using program *SHAKE* have been undertaken on some twenty soil columns to analyze the values of *PDR* and *SR* for comparison with results obtained using equations (14) and (16) (Tsang *et al.*, 2006b). The analyses covered the following parameter values: (i) bedrock response spectral velocity (RSV_{Tg} = 20 – 400 mm/s) (ii) initial soil *SWV* (V_s = 100 – 500 m/s), (iii) initial site natural period (T_i = 0.12 – 2.4 s), (iii) soil plasticity index (*PI* = 0, 15, 30 and 50%) and (iv) *SWV* of the bedrock half-space (V_R = 500 – 3500 m/s). It is shown in Fig. 3 that the accuracy is found to be remarkably good, with around 95% of the estimates being within 20% of the results obtained from *SHAKE* analyses. The ± 20% error can also be regarded as the 95% confidence limits. The *SR* estimates are subjected to greater potential errors (an average of 9.5%), compared to the *PDR* estimates, due to uncertainties in the resonance factor *f*. This accuracy of predicting *SR* is considered very good, given the additional uncertainties in the resonance factor *f* [equation (15)] and the high level of randomness in the generation of the response spectra.

The sensitivity of the non-linear *PDR* estimates [equation (14)] to each of the input parameters has been investigated (Tsang *et al.*, 2006b). The value of *PDR* has been found to be most sensitive to variations in the shear stiffness of both the soil and bedrock materials, and is least sensitive to variations in the soil plasticity. The relatively minor effects of plasticity is reflected in IBC-2006 (in which soil plasticity has not been parameterized).

The high sensitivity of site response behavior to variations in the shear stiffness of both the soil and bedrock is also indicative of the importance of the accurate modeling of the impedance contrast at the soil-bedrock interface. This phenomenon is further reaffirmed by seismological theory. This observation can be used to justify the utilization of the soil *SWV* parameter for determining the site coefficients in code provisions (whilst the bedrock *SWV* is seldom parameterized). The importance of bedrock rigidity will be further discussed in Section 5.

It is also observed that the effects of the level of ground shaking on site responses are not as significant as that of the shear stiffness of the soil and bedrock materials. This is considered to be the result of the trading-offs between the degradation of the soil shear stiffnesses (which leads to greater impedance contrast, and hence, higher level of site amplification) and the material non-linearity of soils (which leads to higher damping within the soil layers, and hence, lower level of site amplification). It is believed to be an important phenomenon for displacement response (the subject matter of the formula developed), which is controlled by longer period wave components. This finding is consistent with the empirical study of Ni *et al.* (2000), in which significant non-linear response behavior of the soil could not be observed for site periods greater than 0.3 sec.

Whilst the dominant effects of ground shaking intensity on site response behavior has always been emphasized in high seismicity regions, it was revealed by studies undertaken by the authors that the impedance contrast between the soil and bedrock is a very critical factor in regions of low-to-moderate seismicity (but typically neglected in most site response models). This points to the need of developing a suitable technique for estimate non-linear site response behavior and expressing results in terms of the peak ground displacement (*PGD*) or response spectral displacement (*RSD*), as opposed to the more conventionally used parameters of peak ground acceleration (*PGA*) or response spectral acceleration (*RSA*). It is concluded that the simple, yet comprehensive, formulae presented in this paper can significantly reduce uncertainties in the estimation of non-linear site responses and be suited to worldwide applications by virtue of its robustness and generalities.

The sensitivity of the value of *SR* to variations in input parameters has also been investigated. As for *PRD*, the value of *SR* is most sensitive to variations in the shear stiffness of both the soil and bedrock materials. It is noted that, as the impedance ratio (α) plays an even more important role in the estimation of *SR*, significant errors in the prediction of site responses could be expected if the shear stiffness of both the soil and bedrock materials cannot be obtained with good accuracy. (Errors are up to 36 and 23%, respectively, should the soil and bedrock *SWV* values be held constant).

Comparison with 1994 Northridge Earthquake Recordings

Further verification analyses have been undertaken using empirical data. The usual practice to investigate non-linearity effects is the use of *PGA*, while *PGD* and spectral parameter at longer period ranges are seldom parameterized. Moreover, in most studies, soil *SWV* has not been provided and local bedrock condition has been ignored. These have translated into difficulties in obtaining suitable data for verification purposes.

Figure 4 shows the comparison of the non-linear spectral ratio (*SR*) estimated using equation (16) with that inferred from empirical data presented and analyzed in Borcherdt (2002). The data were recorded by strong-motion recorders at more than 200 sites during the Northridge earthquake of 17 January 1994, which provides abundant data of high shaking level (base acceleration levels up to 0.5g) for quantifying non-linear effects. Detailed site information (soil *SWV* in particular), peak velocity and displacement motions at the bedrock level have been presented, in addition to the peak ground accelerations. The weighted averaged shear wave velocity in the upper 30 m of the soil sediments, $\bar{V}_{30}$, ranges widely from 200 to 1300 m/s. Site coefficients inferred from data recorded on soil sites and the adjacent rock outcrops were grouped in predetermined azimuth-distance bins, in order that effects of variations in the source radiation pattern and crustal propagation path are minimized. There are totally 20 reference rock stations, in which 8 are on granitic or metamorphic rock and 12 on sedimentary rock. The results were presented as the short-period (0.3 sec) and mid-period (1.0 sec) spectral amplification ratio, namely, $F_{s,S0.3}$ and $F_{m,S1.0}$, respectively, whilst the latter is considered as appropriate to compare with the *SR* formula developed in this study. As the spectral amplification ratio tabulated in the original paper were normalized to Site Class B with $\bar{V}_{30}$ = 850 m/s, the bedrock shear wave velocity V_R in employing equation (16) can accordingly be set at 850 m/s. The wide range of peak velocities recorded on surface of rock outcrops, from around 20 to 450 mm/s, has been used to verify the formula for the complete range of level of shaking considered in this study.

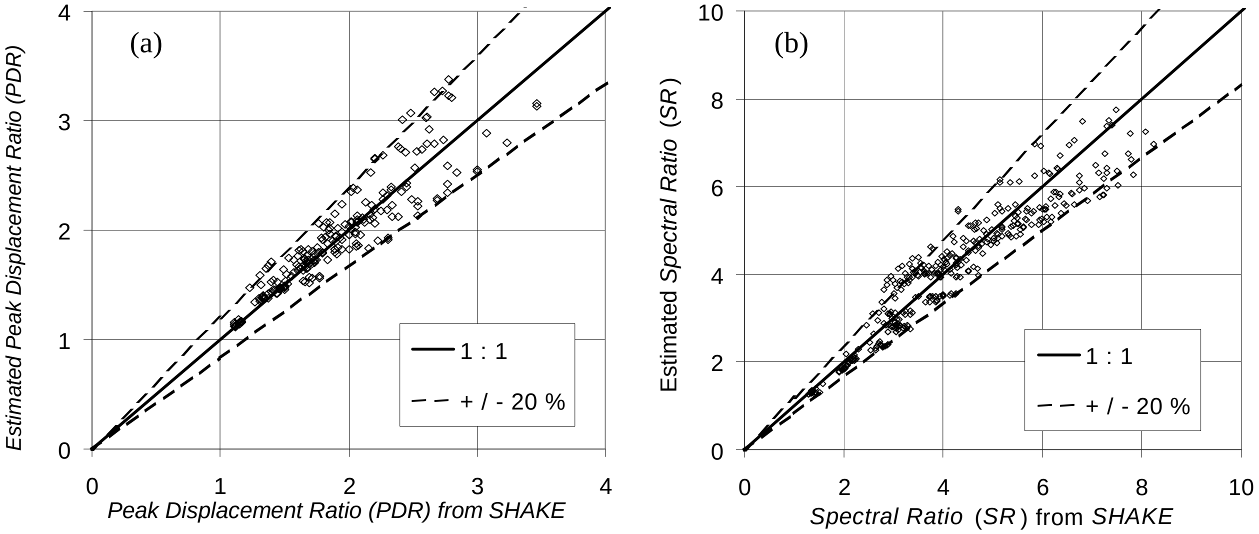


Fig. 3. Correlation of (a) peak displacement ratio (PDR) and (b) spectral ratio (SR) [defined in equation (2)] estimated using equations (14) and (16) and the computed values from SHAKE.

It is shown in Fig. 4 that empirical data is within 50% agreement of that estimated by equation (16), with most estimates being within 50% of the empirical data (and with the outliers which represent 5% of the data points removed from the correlation). This accuracy is considered very good, given the great uncertainties due to the complex nature of the local geological conditions, dynamic soil properties and analysis methodology represented in the data set.

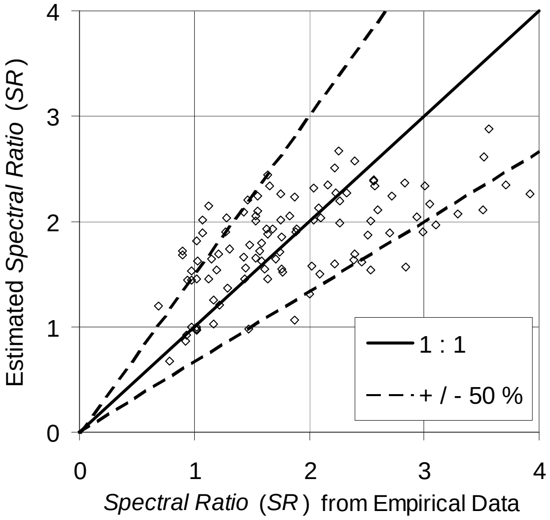


Fig. 4. Estimated non-linear spectral ratio (SR) from equation (16) compared with empirical data (Borcherdt, 2002) recorded in 1994 Northridge earthquake.

CONSTRUCTION OF THE DISPLACEMENT DESIGN RESPONSE SPECTRUM

A simple procedure for constructing displacement design spectrum, based on the procedure developed herein, is summarized as follows:

1. Obtain the basic parameters from normal site investigation: initial V_{si} and *PI*, thickness *H* of soil layer; bedrock V_R. Initial site natural period T_i can be computed by equation (1). The value of RSD_{Ti} can be read off from a response spectrum for rock conditions.

2. Calculate the soil damping ratio, by equation (17) (Tsang *et al.*, 2006a), with $\lambda = 1$ (as initial estimate); and then β by equation (6), and the actual reduction factor λ, by equation (18).

$$\zeta = 12.5 + 6.5\log(R_\gamma \lambda \psi) - 0.13PI$$

$$\text{where} \quad \psi = \frac{RSV_{Tg}}{V_s} = \frac{RSD_{Tg}}{H}\frac{\pi}{2} \qquad (17)$$

R_γ is the ratio of the effective shear strain to maximum shear strain, which has been empirically found to vary between about 0.5 to 0.7 (0.6 has been used in this study).

The reduction factor λ is needed to account for the effects of bedrock rigidity:

$$\lambda = \frac{\alpha}{1+\alpha}\sqrt{\frac{1-\beta^4}{1-R^4\beta^4}} \qquad (18)$$

Equation (17) may be bounded by a “practical” minimum damping ratio ζ_{pi} and an upper bound damping ratio ζ_{ub}:

$$\zeta_{pi}(\%) = 2.5 + 0.03 \cdot PI(\%) \leq 6.8 \qquad (19a)$$

$$\zeta_{ub}(\%) = 17.5 - 0.07 \cdot PI(\%) \geq \zeta_{pi} \qquad (19b)$$

3. Calculate the degraded soil V_s, by equation (20):

$$\frac{V_s}{V_{si}} = \frac{T_i}{T_g} = \frac{1}{1+R_\gamma \lambda \psi \mu} \qquad (20)$$

The actual shifted site natural period T_g, can then be computed by equation (21) (Tsang *et al.*, 2006a), using the degraded soil V_s and the revised RSD_{Ti}.

$$T_g / T_i = 1 + R_\gamma \lambda \psi \mu \qquad (21)$$

where μ is the plasticity factor which has the values of 1.6 (for sand with PI = 0%), 0.9 (PI = 15%), 0.4 (PI = 30%) and 0.2 (PI = 50%).

4. The impedance ratio α [equation (4)], reflection coefficient *R* [equation (5)] soil damping ratio ζ [equation (17)], and damping factor β [equation (6)] are now known and hence the spectra ratio (*SR*) can be calculated using equation (16).

5. The bi-linear displacement design spectrum is finally constructed from the calculated value of *SR* as shown in Fig. 1.

Figure 5 shows an example of comparing a recorded *RSD* with the idealized bi-linear model. The dotted and dashed lines are respectively the *RSD* for rock site and soil site recorded in Oakland Outer Harbour of the Central San Francisco Bay during the 1989 Loma Prieta, California earthquake (Dickenson *et al.*, 1991). The initial weighted-average $V_{S,i}$ is around 305 m/s resulted from loose silty sand fills overlying soft Bay Mud, medium dense fine sands, medium stiff to stiff Old Bay Mud, and a deep profile of stiff older alluvial silty clays extending to the depth of around 130 m. The value of *PI* and V_R were suggested to be 50% and 2000 m/s respectively. It is shown that the idealized bi-linear model can effectively capture the resonance peak displacement demand and the corresponding site natural period.

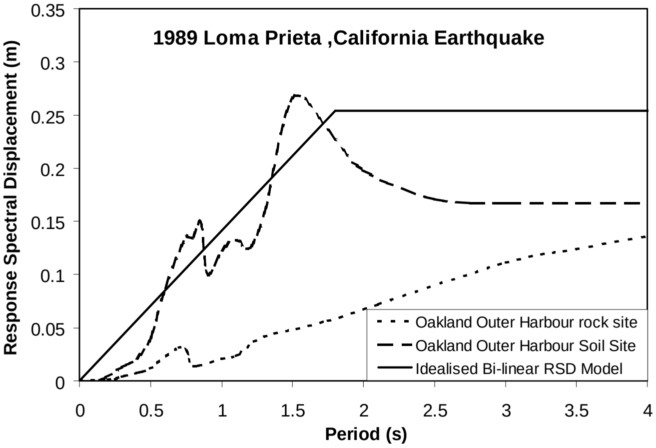


Fig. 5. Comparison between the idealized bi-linear RSD model with recordings from the 1989 Loma Prieta, California earthquake (Dickenson et al., 1991).

DISCUSSION

Shear Wave Velocity Measurement

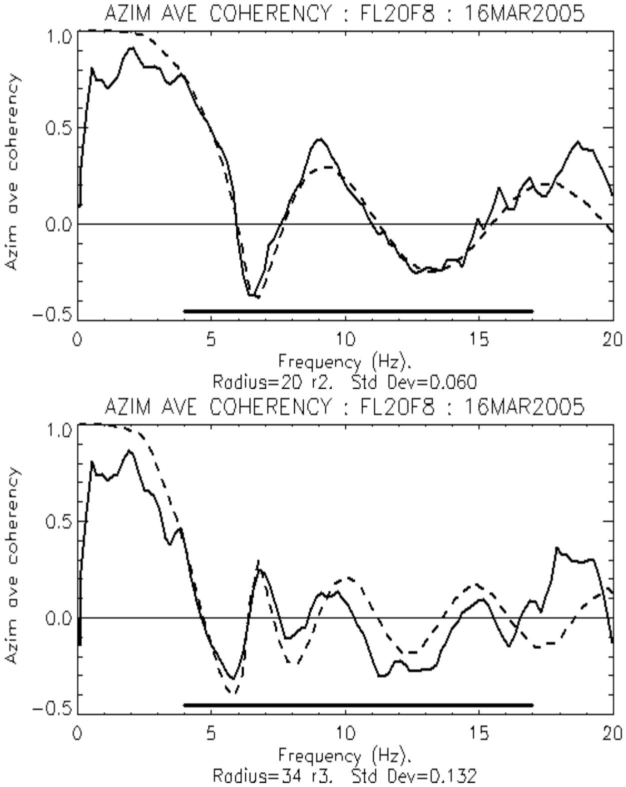
The commonly adopted site classification schemes in major codes of practices are based on *SWV* averaged over a certain depth in the soil sediments. However, the importance of accurately modeling the *SWV* profile (down to bedrock level) at a soil site has been ascertained in a recent study (Asten *et al*., 2005).

The microtremor array method with the spatial auto-correlation (SPAC) processing technique has been used widely in the estimation of the *SWV* profile of Quaternary sediments (Asten, 2005 and references therein). The method appears to be well suited to applications in urban areas. Its advantages include non-invasiveness (no drilling required), inexpensive and speedy acquisition of data, and the ability to provide *SWV* information over a wide range of depths (which can be up to or over a hundred meter depending on the dimension of the array of geophones used in the survey). SPAC is recommended to be used as a common method for measuring *SWV* information of a site forming part of the seismic hazard assessment.

A sensitivity study for the proposed model revealed that the potential response behavior of a site in an earthquake is the most sensitive to the *SWV* of both the soil and bedrock materials (Tsang *et al.,* 2006b). This is also indicative of the importance of the seismic impedance contrast at the soil-bedrock interface [equation (4)]. Hence, not only the soil *SWV* has to be accurately measured, the bedrock *SWV* is as important to be parameterized in the estimation of site seismic hazard.

An example site in north-west Melbourne is used herein as case-study to illustrate how the *SWV* profile of a site can be obtained by the SPAC method which involves the use of an array of geophones in capturing synchronized signals. This method enables the *SWV* profile of a site to be de-lineated, and must be distinguished from the more commonly used, and simpler, method of estimating only the site natural periods based on the measurement of the horizontal/vertical spectral ratios (HVSR) of transmitted signals received by only one geophone.

With the SPAC method, the *SWV* profile of a site is obtained by calibration. First, a (model) coherency spectrum is generated analytically for an assumed *SWV* profile. The model spectrum is then compared with the averaged coherency spectrum as measured from the array of geophones. The model *SWV* profile is refined iteratively until the measured coherency spectrum matches with the modeled spectrum. Examples of the model-measured coherency spectra of the case-study site obtained from the hexagonal array of seven geophones are shown in Figs. 6(a) and 6(b) for different array configurations (with array radius of 20 m). The *SWV* profile determined iteratively by the calibration procedure is summarized in Table 1.



Solid line – field measured coherency spectrum
Broken line – modeled coherency spectrum

Fig. 6. Model and measured coherency spectra in SPAC method: (a) array radius of 20 m and station separation of 20 m; (b) array radius of 20 m and station separation of 34.6 m.

The resolution of the *SWV* measurements at different depth ranges can be optimized by configuring the geophones with different array dimensions. For example, measurements from the smaller array [Figs. 6(a) and 6(b)] were best used in constraining the *SWV* in the upper 5 m of the sand/silt sediments whereas measurements from the larger array (not shown) were used in constraining the *SWV* of the underlying (soft) Coode Island silt sediments and the gravel sediments. The *SWV* velocity of the Silurvian (basement) mudstone was estimated from similar surveys undertaken for other sites in Melbourne (Roberts *et al.,* 2004).

Table 1. *SWV* model of case-study site

Layer	H (m)	V_P (m/s)	V_S (m/s)	ρ (t/m^3)	Geology
1	2	800	190	1.8	sand/silt
2	3	1600	190	2.0	sand/silt
3	6.5	1600	140	2.0	Coode silt
4	95	2100	600	2.4	gravels
5		3100	1500	2.4	basement

Effects of Bedrock Shear Rigidity

Bedrock *SWV*, V_R, is seldom parameterized in code provisions. Sensitivity studies undertaken by the authors revealed that the effects of the intensity of shaking on the potential site response behavior is actually not as significant as that of the shear rigidity of the bedrock materials (even though site factors in some major codes of practices are expressed as functions of intensity).

Nevertheless, it is recognized that the average near-surface conditions in Californian bedrock has already been implicitly considered in the site amplification models in IBC-2006. However, it is well-known that the upper crustal structure of the Central and Eastern North America (CENA) and that of California can be extremely different, with a *SWV* at 30m depth of 2800 and 850m/s respectively. Hence, site coefficients should ideally be developed specifically for each region. A recent study by the authors (Chandler *et al.,* 2006) demonstrated the large variation of bedrock conditions even within a small city, Hong Kong, in which the near-surface *SWV* ranges between 1000 and 2500m/s. It is found that an error of over 50% can be resulted if the value of V_R has not been parameterized in the estimation of site response behavior.

Figure 7 shows the variation of the estimated *SR* against the bedrock shaking level in terms of the spectral velocity of a bedrock spectrum. A series of curves is shown for a possible range of V_R (from 500 to 3500 m/s). An additional hypothetical case of infinitely rigid bedrock has also been superimposed onto the figure to show the upper bound of *SR*. It is shown that *SR* varies greatly with the bedrock *SWV*. For rock RSV = 100 mm/s, *SR* varies from 1.3 to 3.8 for the range of possible V_R, which urges the importance of accurately estimating V_R for calculating site response.

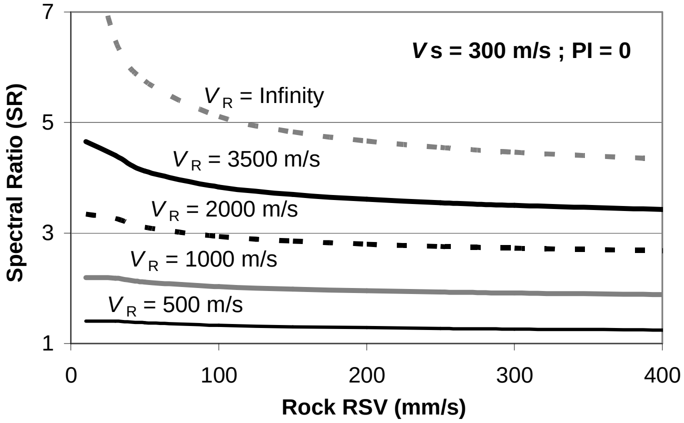


Fig. 7. The effects of bedrock rigidity on soil response.

Having acknowledged the importance of considering the bedrock rigidity in calculating site response, it is, however, noted that there is no documented method to estimate the value of V_R for site response analysis. This parameter is commonly required for a standard site response analysis, such as using program *SHAKE*. To the best of the knowledge of the authors, there has been no discussions over the estimation (or measurement) of the value of V_R , with due considerations given to the feasibility in engineering implementations.

Undoubtedly, the simplest way of modeling is to assume that bedrock is a uniform half-space (possessing homogeneous properties which do not vary with depth) and hence the value of V_R can be based upon the properties of rocks sampled immediately below the overlying sediments. It is however recognized that crustal materials are actually heterogeneous in nature, as the acoustic impedance increases with depth where the rock crusts become more compact (Faust, 1951; Chandler *et al.,* 2005). There is no consensus amongst scientists and practitioners over the value of "effective depth" into bedrock at which the value of V_R could be measured.

In the light of this, the authors propose a method for addressing this important, yet underrated, element of uncertainties in site response analyses. It is proposed herein that the value of the effective depth (D_R) be determined using the *Resonant-Period Equivalence* (RPE) Principle in conjunction with the well-established *Quarter-Wavelength* (QWL) Method (Tsang *et al.*, 2008).

As is well-known, the fundamental resonant period (RP) of the site (T_g) can be estimated using equation (1), in which the soil thickness H is equal to the quarter-wavelength (QWL) of the multiply reflected waves which dominate the site response and has frequency f_g (= 1 / T_g). While H is measured up from the soil-bedrock interface, the effective depth into bedrock (D_R) is measured down. Significantly, both H and D_R are associated similarly with the dominant site period (T_g), or site frequency (f_g), as defined by equation (1) and (22) respectively.

$$f_g = \frac{1}{\sum_{i=1}^{N} \frac{d_i}{V_i} \times 4} \qquad (22)$$

where $D_R = \sum_{i=1}^{N} d_i$, i is the layer number in bedrock, each having finite thickness d_i and shear wave velocity V_i. N is the total number of bedrock layers considered for computing the effective depth. The value of the equivalent bedrock *SWV* (V_R) can be computed by dividing the effective depth (D_R) by the total traveling time of the seismic shear wave, using equation (23).

$$V_R = \frac{D_R}{\sum_{i=1}^{N} \frac{d_i}{V_i}} \qquad (23)$$

The alternative method of evaluating V_R is by calibration and is much more time consuming as it involves site response analysis of two soil column models: (i) model in which bedrock is represented by a half-space of homogeneous materials and (ii) model in which the variation in the shear rigidity of the rock crust with increasing depth is accurately represented. The value of V_R in model (i) can be obtained by trial and error until the site amplification behavior represented by the two models matches.

It is shown in Fig. 8 that estimates from the proposed RPE principle (using equations (22) and (23) and those from calibration are remarkably good, with around 95% of the estimates having less than 15% discrepancies. The ± 15% error can also be regarded as the 95% confidence limits. Details and verifications of the RPE principle can be found in Tsang *et al.* (2008).

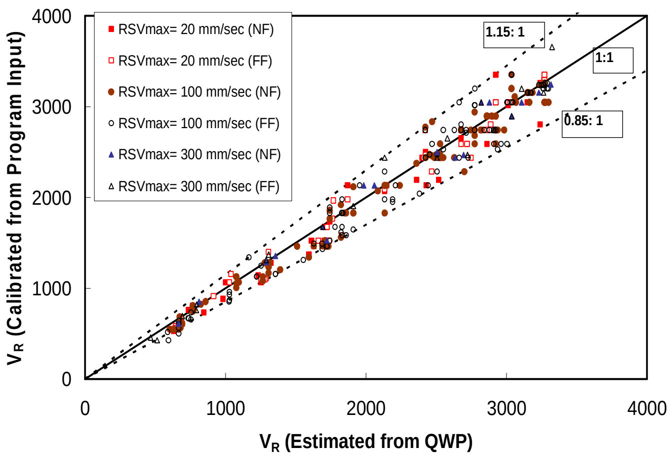


Fig. 8. Bedrock shear rigidity estimated by the proposed resonant-period equivalence (RPE) principle compared with the correct value calibrated using SHAKE.

CONCLUSIONS

1. The effects of the occurrence of soil resonance on the site-specific seismic hazard can be represented by the *PDR* and *SR* parameters which are defined by equations (14) and (16) respectively. The model so proposed enables the site amplification factor to be calculated by a simple manual procedure.

2. In the proposed procedure, both PDR and SR can be estimated as a function of the ratio of impedance contrast α [equation (4)] and the hysteretic damping factor β [equation (6)].

3. Verification analyses based on comparison with results obtained from program *SHAKE* and from recordings of ground shakings in the 1994 Northridge earthquake have been undertaken to support the proposed model.

4. The microtremor array method with SPAC processing technique has been recommended for obtaining *SWV* of both the soil and bedrock materials for input into site response analysis.

5. A simple and effective heuristic model has been introduced for estimating the effective shear rigidity, and hence effective *SWV*, of horizontally stratified bedrock materials.

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