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Fifth International Conference on
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DYNAMIC ANALYSIS OF PILES UNDER LATERAL HARMONIC VIBRATION

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ABSTRACT

This paper presents a new mathematical approach for the analysis of harmonically vibrating horizontal, linear, elastic uniform pile. The soil properties may vary from layer to layer. No separation is allowed at the soil-pile interface. The pile is modeled as a number of cylindrical segments connected by rigid nodes. The length of each segment is chosen such that the effects of the soil inhomogenity are accounted for. The governing differential equation for an arbitrary pile segment is obtained and solved. According to the pile support types such as pinned, fixed and free conditions, first an arbitrary appropriate value for either toe force, bending moment, rotation, or displacement is assumed. The governing differential equation is then solved from the lower pile segment to the top one. The stiffness of the whole pile-soil system will then be computed. It is shown that the slenderness ratio, the stiffness ratio and toe fixity are the governing parameters affecting the stiffness of the soil-pile system. The new analytical model, which is verified using existing numerical and analytical solutions, is more efficient than the equivalent numerical solutions for example finite eminent methods.

INTRODUCTION

Horizontal vibration of piles occurs with footing and structures exposed to dynamic force such as those produced by machines, wind and earthquake. Various techniques have been used to describe the behavior of piles under dynamic loads. These are continuum approach (Novak, 1974; Novak and Aboul-Ella, 1978; and Nogami, 1980), boundary element method (Kaynia and Kausel, 1982; Sen, Kausel, and Banerjee, 1985), lumped-mass method (Penzien, 1975; El Naggar and Novak, 1994), and finite element solutions (Blaney, Kausel, and Roesset, 1976; Wolf and von Arx, 1982; Chow, 1985). The response of a pile to external excitation and the pile stiffness and damping associated with the response are a result of the interaction between the elastic pile and the soil surrounding it. This paper attempts to offer a continuum method based on elasto-dynamic theory of Novak (1974) for the analysis of piles under dynamic horizontal vibrations. The elastic continuum models are convenient for developing explicit solutions, which can accommodate the dynamic effects such as inertial forces and damping. However, the inclusion of non-linear behavior in these models is very difficult. The method requires considerably less computational effort than the equivalent rigorous solutions. The approach presented in this paper was used for vertically loaded piles and called "segment by segment method", SMM (Ghazavi et al., (1997a, b; Ghazavi, 2002).

ANALYTICAL MODEL

The characteristic effects of surrounding soil on the pile response are determined using stiffness and damping parameters of soil-pile system. These effects can be taken into account if a proper soil reaction is employed. The soil reaction on the pile is represented by springs and dashpots, which are modeled using elasto-dynamic theory or elastic half-space theory. The pile is divided into segments connected rigidly at nodes connected by stiff spring, which are characterized using linear elastic theory. The pile is divided into some segments (Fig. 1).

The interaction of the soil and the pile is then determined for each segment according to the characteristics of soil. This interaction can be demonstrated by a complex displacement, shear force, rotation and bending moment at the nodes at each end of the segment. These must be identical to the same displacement, shear force, rotation, and moment at the end of the adjacent segment. This procedure is performed from the lowest pile segment and extends to the next upper segment. This manner is continued to reach the topmost segment. That is why this procedure is called the SSM (Ghazavi et al., 1997a, b; Ghazavi, 2002; Ghazavi, 2007; Ghazavi, 2008). Figure 1 shows the interaction between pile segment-soil and subsequent segments.

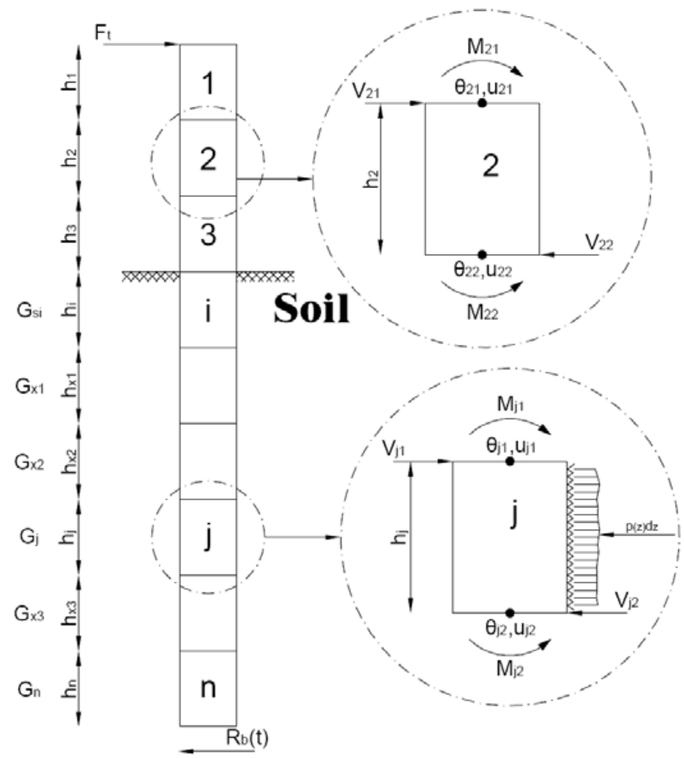


Fig1. Discretisation of pile for dynamic analysis in horizontally inhomogeneous media using S.S.M

In the analysis, it is assumed that the soil reaction associated with a given displacement of a pile segment within a give soil layer is identical to that of an infinite rigid pile undergoing a uniform displacement of the same properties as the soil of that layer. This assumption is essential to the solution and will be examined subsequently using other, existing solutions. This assumption has also been used by other researcher (Novak and Aboul-Ella, 1978). It is noted that Novak and Aboul-Ella (1978) used finite element method (FE) for analysis of piles embedded in inhomogeneous soil and subjected to lateral harmonic vibrations.

The soil response with time to motion of the pile toe, R_b(t), is taken as that of a viscoelastic half-space to rigid, massless, circular disc of radius r'_0 undergoing harmonic vibration. This can be expressed as:

$$R_b(t) = G_b r_b\,[Cu_1(\alpha'_0, \nu, D) + iCu_2(\alpha'_0, \nu, D)]u_b(t) \qquad (1)$$

where G_b is the soil shear modulus at the pile toe, Cu_1, Cu_2 are dimensionless complex parameters given in the form of polynomial expressions (Veletsos and Verbic, 1973), $u_b(t)$ is the toe horizontal displacement; r_b is pile radius at the pile tip, $a'_0 = \frac{r_b \omega}{v_b}$, where V_b is shear wave velocity of soil below the tip, ω is circular frequency, ν is Poisson's ratio, D is material damping. By considering the typical embedded segment j at depth z, shown in Figure1 and on the basis of the above assumption, the following governing dynamic differential equation of a soil-pile system subjected to harmonic lateral load can be obtained:

$$\begin{aligned} &m_{pj}(z)\frac{\partial^2 u(z,t)}{\partial t^2} + c_{pj}(z)\frac{\partial u(z,t)}{\partial t} \\ &+ E_p I_p(z)\frac{\partial^4 u(z,t)}{\partial z^4} + G_{sj} S_{uj}(z) u(z,t) = 0 \end{aligned} \qquad (2)$$

where m_{pj} the pile mass per unit length, c_{pj} is the damping coefficient of the pile material, $E_p I_p$ is the bending stiffness of the pile, G_{sj} is the shear modulus of the soil surrounding the pile, *u(z,t)* is the local displacement, and S_{uj} is a complex dimensionless soil resistance parameter defined by Novak (1974) as a function of dimensionless frequency, a_0 and Poisson's ratio, ν. It is noted that $a_0 = \frac{r_0 \omega}{V_S}$, where V_S the shear velocity of the soil surrounding the pile is, r_0 is the pile radius. The four terms in Equation (2) represent the inertia force due to lumped mass of the pile, the damping force of pile material, the lateral interaction between pile segments, and the soil resistance, respectively. For harmonic vibration, the displacement *u(z,t)* is given by:

$$u(z,t) = u(z)e^{i\omega t} \qquad (3)$$

where u(z) is the complex amplitude at depth z.

$$u(z) = u_1(z) + iu_2(z) \qquad (4)$$

Combining Equations (2) and (3) gives

$$\begin{aligned} &E_p I_p(z)\frac{\partial^4 u(z,t)}{\partial z^4} + u(z)[G_{sj} S_{u1} - m_{pj}\omega^2 \\ &+ i(c_{pj}\omega + G_{sj} S_{u2})] = 0 \end{aligned} \qquad (5)$$

The above equation can be solved explicitly. The solution for the displacement at a point at vertical distance z below the upper node of segment j is given by:

$$u(z) = A\cosh\left(\xi_j \frac{z}{h_j}\right) + B\sinh\left(\xi_j \frac{z}{h_j}\right) + C\cos\left(\xi_j \frac{z}{h_j}\right) + D\sin\left(\xi_j \frac{z}{h_j}\right) \quad (6)$$

$$\xi_j = h_j \sqrt[4]{\frac{1}{E_p I_p(z)}[m_{pj}\omega^2 - G_{s\,j} S_{u1} - i(c_{pj}\omega + G_{sj} S_{u2})]} \quad (7)$$

Where A, B, C, D are integration constants determined using appropriate boundary conditions.
In a laterally loaded prismatic pile, the rotation, shear force, and bending moment transmitted by pile at a point at a vertical distance z below top node of segment j, is given by:

$$\theta(z) = -\frac{du(z)}{dz} \quad (8a)$$

$$M(z) = -E_p I_P(z) \frac{d^2 u(z)}{dz^2} \quad (8b)$$

$$V(z) = -E_p I_P(z) \frac{d^3 u(z)}{dz^3} \quad (8c)$$

If the displacement, rotation, shear force and bending moment transmitted by the pile at node 2 of segment j are know, the integration constants A, B, C, and D can be calculated. Thus, the displacement, rotation, bending moment and shear force at node 1 of segment j are respectively given by:

$$u_1 = \frac{V_2 h_j^3(\sinh\xi_j - \sin\xi_j) + \xi_j^2 E_p I\theta_2 h_j(\sinh\xi_j + \sin\xi_j)}{2E_p I\xi_j^3} \quad (9a)$$
$$\underline{+\xi_j^3 E_p I u_2(\cosh\xi_j + \cos\xi_j) + M_2 h_j^2 \xi_j(\cos\xi_j - \cosh\xi_j)}$$

$$\theta_1 = \frac{V_2 h_j^3(\cos\xi_j - \cosh\xi_j) - \xi_j^2 E_p I\theta_2 h_j(\cos\xi_j + \cosh\xi_j)}{-2E_p I\xi_j^2 h_j} \quad (9b)$$
$$\underline{+\xi_j^3 E_p I u_2(\sin\xi_j - \sinh\xi_j) + M_2 h_j^2 \xi_j(\sinh\xi_j + \sin\xi_j)}$$

$$M_1 = \frac{V_2 h_j^3(\sinh\xi_j + \sin\xi_j) + \xi_j^2 E_p I\theta_2 h_j(\sinh\xi_j - \sin\xi_j)}{-2\xi_j h_j^2} \quad (9c)$$
$$\underline{+\xi_j^3 E_p I u_2(\cosh\xi_j - \cos\xi_j) - M_2 h_j^2 \xi_j(\cosh\xi_j + \cos\xi_j)}$$

$$V_1 = \frac{-V_2 h_j^3(\cosh\xi_j + \cos\xi_j) - \xi_j^2 E_p I\theta_2 h_j(\cosh\xi_j - \cos\xi_j)}{-2h_j^3} \quad (9d)$$
$$\underline{-\xi_j^3 E_p I u_2(\sinh\xi_j + \sin\xi_j) + M_2 h_j^2 \xi_j(\sinh\xi_j - \sin\xi_j)}$$

According to the support conditions, the analysis is carried out sequentially from the lowest segment to the top segment.
As seen, the SSM is very simple and efficient to use compared with FE. In the FE analysis, mass, damping, and stiffness matrixes are to be generated for pile elements and then the global matrix is made. In the SSM, there is no need to generate these matrixes.

<u>1.1 Pinned-Ended Pile</u>

At this support, initially an arbitrary value for the toe shear force and rotation is assumed, then bending moment, displacement, shear force and rotation for the node 1 of the lowest segment are calculated using Equations (9a)-(9d) and applied of node 2 of the next higher segment as boundary The corresponding shear force, displacement, rotation and bending moment for node 1 of the same segment are then calculated using (9a)-(9d).The procedure continues in this manner from segment to segment until the pile head displacement, u_h, shear force, V_h, bending moment, M_h, and rotation, θ_h, are determined. The dynamic complex stiffness of the foundation then calculated and hence the stiffness and damping parameters can be established as:

$$k_{uu} = \frac{V_h}{u_h} \quad (10a)$$

Where k_{uu} dynamic complex horizontal stiffness and

$$k_{\theta\theta} = \frac{M_h}{\theta_h} \quad (10b)$$

Where $k_{\theta\theta}$ is dynamic complex rotation stiffness and also

$$k_{u\theta} = \frac{M_h}{u_h} \text{ Or } k_{u\theta} = \frac{V_h}{\theta_h} \quad (10c)$$

Where $k_{u\theta}$ is dynamic complex cross-stiffness.

The complete derivation of the complex stiffness is given by Novak (1974) and is not repeated here.
Decomposing $k_{uu}, k_{\theta\theta} and k_{u\theta}$ for pinned-ended pile, in to real and imaginary parts leads to:

$$(k_{uu})_{real} = \frac{E_p I_p}{r_0^3} f_{11,1}$$

(11a)

$$(k_{uu})_{imaginary} = \frac{E_p I_p}{r_0^2 V_S} f_{11,2}$$

$$(k_{\theta\theta})_{real} = \frac{E_p I_p}{r_0} f_{7,1}$$

(11b)

$$(k_{\theta\theta})_{imaginary} = \frac{E_p I_p}{V_S} f_{7,2}$$

$$(k_{u\theta})_{real} = \frac{E_p I_p}{r_0^2} f_{9,1}$$

(11c)

$$(k_{u\theta})_{imaginary} = \frac{E_p I_p}{r_0 V_S} f_{9,2}$$

where $f_{11,1}, f_{7,1} and f_{9,1}$ are named dimensionless stiffness and $f_{11,2}, f_{7,2} and f_{9,2}$ are named dimensionless damping.

<u>1.2 Fixed-Ended and Free-Ended Pile</u>

For a fixed-ended pile, first a zero value is assumed for the pile toe displacement and rotation. Arbitrary values are here considered for the shear force and bending moments. The corresponding shear force, bending moment, displacement, and rotation of node 1 of the lowest segment are computed and assumed to be for node 2 of the second last segment. The procedure will be continued in this manner to reach the topmost pile segment. Finally, the complex stiffness and damping parameters are computed for pile-soil system using Equations (10a)-(10c).

For a free-ended pile, a zero value is assumed for shear force and bending moment for the pile toe. Also arbitrary values are assumed for displacement and rotation of pile toe. The corresponding shear force, bending moment, displacement, and rotation of node 1 of the lowest segment are computed and assumed to be for node 2 of the second last segment. The procedure will be continued in this manner to reach the topmost pile segment. Finally, the complex stiffness and damping parameters are computed for pile-soil system using Equations (10a)-(10c).

The SSM method may be suitable for the layered soil and includes much less calculations effort than the finite element and rigorous method.

VERIFICATION OF MODEL

The SSM can be validated using the work example given by Novak (1974) and Novak and Aboul-Ella (1978). They assumed the following parameters:
For pile, they assumed:
Density, $\rho_p = 2500 \frac{kg}{m^3}$
Young's modulus, $E_p = 1.96 \times 10^7 kpa$ and hence longitudinal wave velocity, $V_c = \sqrt{\frac{E_p}{\rho_p}} = 2800 \frac{m}{s}$
For the soil, they considered:
Density, $\rho = 1750 \frac{kg}{m^3}$
Poisson's ratio, $\nu = 0.4$
Figures 2 to 5 compares data extracted from the presented SSM approach with those reported by Novak (1974).

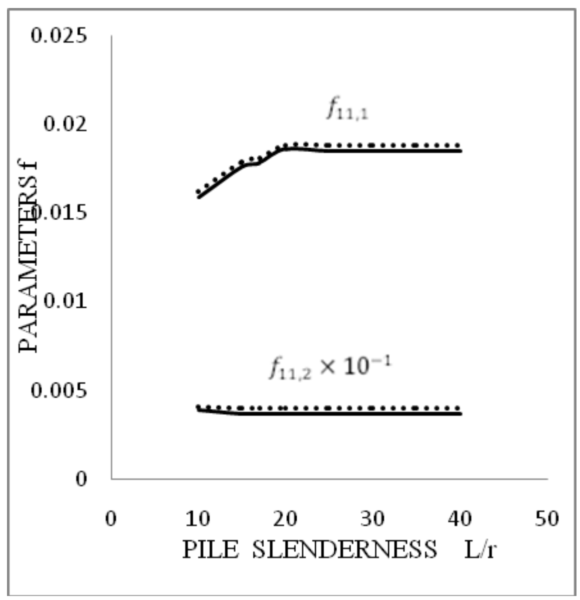


Fig2.Variation of horizontal stiffness and damping dimensionless parameters with slenderness for pinned-ended piles, $(\frac{\rho}{\rho_p} = 0.7, \nu = 0.4, a_0 = 0.3, \frac{V_s}{V_c} = 0.03)$

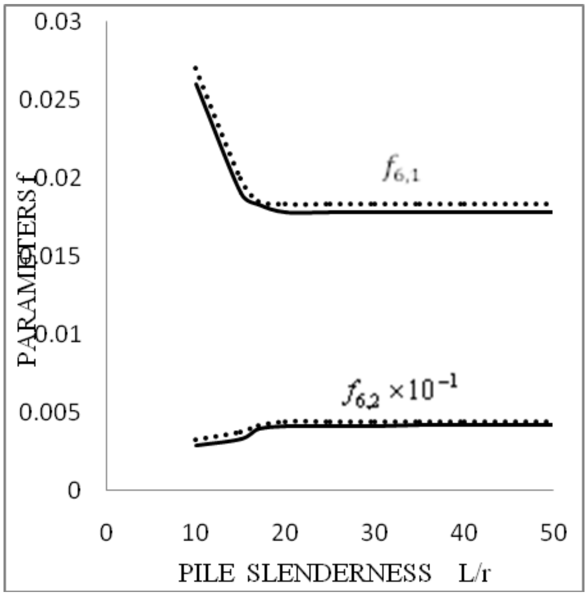


Fig3.Variation of horizontal stiffness and damping dimensionless parameter with slenderness ratio for fixed-ended piles, $(\frac{\rho}{\rho_p} = 0.7, \nu = 0.4, a_0 = 0.3, \frac{V_s}{V_c} = 0.03)$

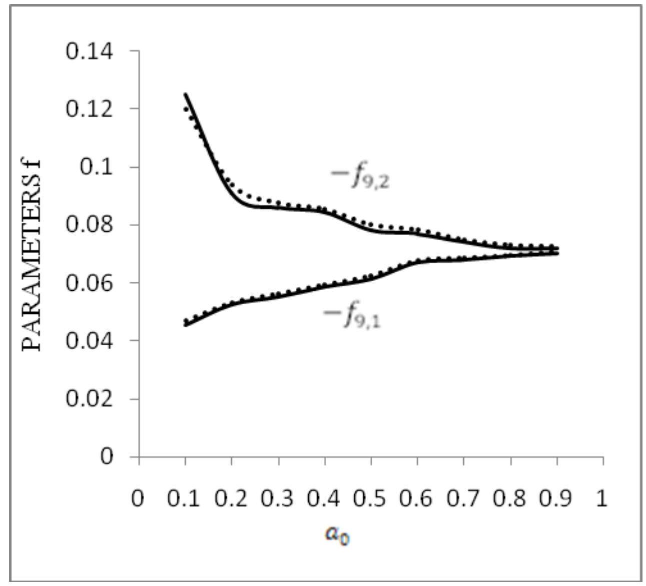


Figure 5. Variation of cross stiffness and damping dimensionless parameter with dimensionless frequency for pinned-ended piles, $(\frac{\rho}{\rho_p} = 0.7, \nu = 0.4, \frac{L}{r_0} = 40, \frac{V_s}{V_c} = 0.03)$

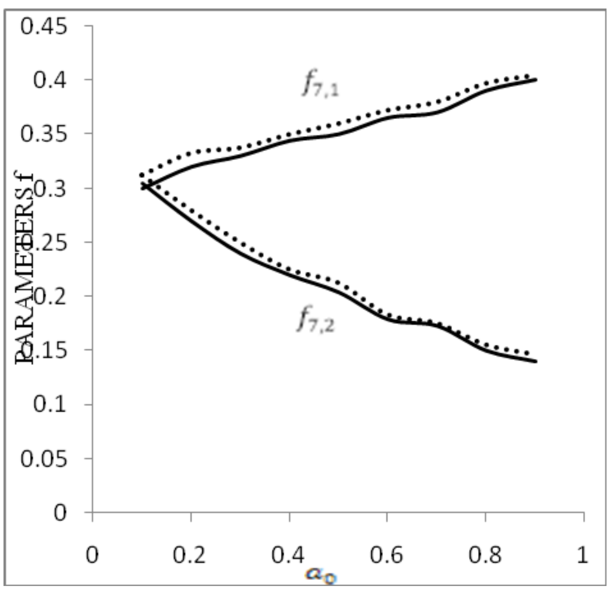
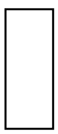
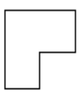
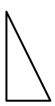
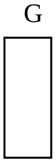
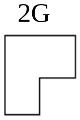
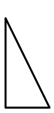


Fig4.Variation of rotation stiffness and damping dimensionless parameter with dimensionless frequency for pinned-ended piles, $(\frac{\rho}{\rho_p} = 0.7, \nu = 0.4, \frac{L}{r_0} = 40, \frac{V_s}{V_c} = 0.03)$

For inhomogeneous soil in the vertical direction, the concrete pile under dynamic loading with free –ended toe, the length and radius of pile are 6.1 m and 0.15 m, respectively. The SSM results have been compared with those obtained from the finite element method presented by Novak and Aboul-Ella (1987). Table 1 and 2 shows different soil profiles used for comparison. In these profiles, the soil shear moduli may be constant, triangular, and varying along the depth. The average value of the shear modulus was taken constant in three profiles. The shear wave velocity of the soil was assumed to be 91.5m/s, Poisson's ratio, $\nu = 0.3$ and material damping, δ=0.1.The SSM method has been examined in piles under axial vibration (Ghazavi, 2002) and the results were satisfactory. This method has originated from continuum method and has been profited of soil and structural dynamics rudiments. The effects on soil-pile system stiffness and damping due to surrounding soil have been calculated properly in aforementioned method.

Table 1 and 2. Comparison between Results for Stiffness and Damping Parameters from the SSM and Aboul-Ella (1978) for $a_0 = 0.3$

	Dimensionless Stiffness		
Soil Profiles	Horizontal Stiffness	Rotation Stiffness	Cross Stiffness
G 	0.01678**	0.34274	-0.05548
	0.01673*	0.3427	-0.05542
2G  G	0.02751	0.39773	-0.07579
	0.027495	0.397711	-0.075771
 G	0.00842	0.28841	-0.03726
	0.0084	0.28839	-0.037245

	Dimensionless Damping		
Soil Profiles	Horizontal Damping	Rotation Damping	Cross Damping
G 	0.04652**	0.26899	-0.09219
	0.0465*	0.26893	-0.09216
2G  G	0.06355	0.27182	-0.10808
	0.06354	0.2718	-0.108078
 G	0.03111	0.25455	-0.07417
	0.0311	0.25453	-0.07415

[Note: * represent result for the SSM and ** those from Novak and Aboul-Ella (1978)]

CONCLUSIONS

A simple approach, called SSM, has been presented in this paper for determination of stiffness and damping parameters of laterally loaded piles subjected to harmonic vibrations. The soil-pile interaction in this method is modeled within each segment and applied via the segment nodes to the analysis of the adjacent segment. Therefore, the stiffness and damping parameters for the whole pile-soil system are determined. The method has been validated using an existing solution for the analysis of pinned, fixed, and free ends of piles. A parametric study has been carried out and shown that the soil stratification can affect significantly the stiffness and damping characteristics of piles. As demonstrated, the SSM is an efficient and simple method for analysis of piles under harmonic vibration. In particular, the effects of the soil inhomogeneity in the vertical direction even with complicated stratifications can be easily captured. This method involves less computational work than available numerical method based on the FE.

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