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Oscillation of second-order p-Laplace dynamic equations with a nonpositive neutral coefficient

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ABSTRACT

We establish some new oscillation criteria for a class of second-order p-Laplace dynamic equations with a nonpositive neutral coefficient on a time scale. The results obtained supplement and improve those reported in the literature.

1. Introduction

In this paper, we study the oscillatory behavior of a second-order neutral dynamic equation with a p-Laplacian like operator

$$\left(r(t)|z^{\Delta}(t)|^{p-2}z^{\Delta}(t)\right)^{\Delta} + q(t)|x(\delta(t))|^{p-2}x(\delta(t)) = 0 \tag{1.1}$$

on a time scale $\mathbb{T}$, where $z(t) := x(t) - a(t)x(\tau(t))$ and $p > 1$ is a constant. The p-Laplace equations have applications in continuum mechanics as seen from [1–3]. The increasing interest in oscillation and nonoscillation of solutions to various classes of differential equations and dynamic equations is motivated by their applications in the natural sciences and engineering. We refer the reader to [4–21] and the references cited therein.

We assume that the following assumptions are satisfied:

(H_1) $\tau \in C_{rd}(\mathbb{T}, \mathbb{T})$, $\tau(t) \le t$, τ is strictly increasing, and $\lim_{t\to\infty} \tau(t) = \infty$;
(H_2) $\delta \in C_{rd}(\mathbb{T}, \mathbb{T})$, δ is strictly increasing, and $\lim_{t\to\infty} \delta(t) = \infty$;
(H_3) $r, a, q \in C_{rd}(\mathbb{T}, \mathbb{R})$, $r(t) > 0$, $0 \le a(t) \le a_0 < 1$, and $q(t) > 0$, where $a_0 > 0$ is a constant.

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential or difference equations. Several authors have studied the theory of dynamic equations on time scales; see, e.g., [22–25] and the references cited therein. Since we are interested in oscillatory properties, we assume throughout this paper that the given time scale $\mathbb{T}$ is unbounded above. We assume $t_0 \in \mathbb{T}$ and it is convenient to assume $t_0 > 0$, and define the time scale interval of the form $[t_0, \infty)_{\mathbb{T}}$ by $[t_0, \infty)_{\mathbb{T}} := [t_0, \infty) \cap \mathbb{T}$.

By a solution of (1.1) we mean a nontrivial real-valued function $x \in C_{rd}[T_x, \infty)_{\mathbb{T}}$, where $T_x \in [t_0, \infty)_{\mathbb{T}}$, which has the property that $r|z^{\Delta}|^{p-2}z^{\Delta} \in C^1_{rd}[T_x, \infty)_{\mathbb{T}}$ and satisfies (1.1) for $t \in [T_x, \infty)_{\mathbb{T}}$. The solutions vanishing in some neighborhood

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of infinity will be excluded from our consideration. A solution x of (1.1) is said to be oscillatory if it is neither eventually positive nor eventually negative; otherwise, it is nonoscillatory. Eq. (1.1) is called oscillatory if all its solutions oscillate.

In what follows, we briefly comment on the related results that motivate our study. Most oscillation results reported in [4,5,7–10,16–20] for Eq. (1.1) have been obtained under the assumptions that $p = \gamma + 1$ and $a(t) \le 0$, where γ is a ratio of odd positive integers. Assuming $a(t) \ge 0$, Erbe et al. [10,11] and Karpuz [12] proved several oscillation criteria for particular cases of Eq. (1.1); see [10, Theorems 2.6–2.10], [11, Theorem 3.4 and Corollary 3.5], and [12, Theorem 8]. In particular, Karpuz [12, Theorem 8] proved that every solution of a second-order neutral delay dynamic equation

$$(x(t) - a(t)x(\tau(t)))^{\Delta\Delta} + q(t)x(\delta(t)) = 0$$

oscillates or tends to zero asymptotically provided that $0 \le a(t) \le 1$, $\limsup_{t\to\infty} a(t) < 1$, and $\int_{t_0}^{\infty} q(t)\Delta t = \infty$ (where $\int_{t_0}^{\infty} q(t)\Delta t := \lim_{t\to\infty}\int_{t_0}^{t} q(s)\Delta s$).

Recently, Agarwal and Bohner [26], Agarwal et al. [27], Bohner [28], Bohner et al. [29], Karpuz [30], Şahiner and Stavroulakis [31], and Zhang and Deng [32] established some sufficient conditions which ensure that a first-order delay dynamic inequality

$$x^{\Delta}(t) + q(t)x(\tau(t)) \le 0$$

has no eventually positive solutions.

We stress that theorems in [10–12] cannot ensure that all the solutions of second-order dynamic equations with a non-positive neutral coefficient are oscillatory. The purpose of this paper is to develop a new method for the analysis of the oscillation of Eq. (1.1) via comparison principles.

2. Oscillation results

In this section, we study the oscillatory behavior of (1.1) relating oscillation of this equation to the existence of positive solutions to the associated first-order dynamic inequalities. In what follows, all functional inequalities are assumed to hold eventually, that is, for all t large enough. We also let $\tau^{-1} \in C_{rd}(\mathbb{T}, \mathbb{T})$ be the inverse function of τ.

Theorem 2.1. *Assume* (H_1)–(H_3) *and let*

$$\int_{t_0}^{\infty} r^{-1/(p-1)}(t)\Delta t = \infty \tag{2.1}$$

and

$$\int_{t_0}^{\infty} q(t)\Delta t = \infty. \tag{2.2}$$

Suppose there exists a function $\alpha \in C_{rd}(\mathbb{T}, \mathbb{T})$ *such that* $\alpha(t) > t$ *and* $\lim_{t\to\infty} \tau^{-1}(\delta(\alpha(t))) = \infty$. *If the first-order dynamic inequality*

$$u^{\Delta}(t) + Q(t)u\left(\tau^{-1}(\delta(\alpha(t)))\right) \le 0 \tag{2.3}$$

has no eventually positive solutions, where

$$Q(t) := \frac{1}{a_0}\left(\frac{1}{r(t)}\int_t^{\alpha(t)} q(s)\Delta s\right)^{1/(p-1)}, \tag{2.4}$$

then Eq. (1.1) *is oscillatory.*

Proof. Assume (1.1) has a nonoscillatory solution x on $[t_0, \infty)_{\mathbb{T}}$. Without loss of generality, we can suppose that there exists a $t_1 \in [t_0, \infty)_{\mathbb{T}}$ such that $x(t) > 0$, $x(\tau(t)) > 0$, and $x(\delta(t)) > 0$ for all $t \in [t_1, \infty)_{\mathbb{T}}$. Eq. (1.1) yields

$$\left(r(t)|z^{\Delta}(t)|^{p-2}z^{\Delta}(t)\right)^{\Delta} = -q(t)|x(\delta(t))|^{p-2}x(\delta(t)) < 0 \quad \text{for all } t \in [t_1, \infty)_{\mathbb{T}}. \tag{2.5}$$

Then $r|z^{\Delta}|^{p-2}z^{\Delta}$ is strictly decreasing on $[t_1, \infty)_{\mathbb{T}}$. Therefore, z and z^{Δ} are of constant sign eventually. We claim that x is bounded. Assume now that x is unbounded. Along the same lines as in [10, Theorem 2.6], there exists a $t_2 \in [t_1, \infty)_{\mathbb{T}}$ such that

$$z(t) > 0 \quad \text{and} \quad z^{\Delta}(t) > 0 \quad \text{for all } t \in [t_2, \infty)_{\mathbb{T}}. \tag{2.6}$$

Using (1.1) and (2.6), we conclude that, for all $t \in [t_2, \infty)_{\mathbb{T}}$,

$$\left(r(t)(z^{\Delta}(t))^{p-1}\right)^{\Delta} + q(t)z^{p-1}(\delta(t)) \le 0.$$

It follows from (2.6) that there exist a $t_3 \in [t_2, \infty)_{\mathbb{T}}$ and a constant $c > 0$ such that $z(\delta(t)) \ge c$ for all $t \in [t_3, \infty)_{\mathbb{T}}$. Thus, we obtain, for all $t \in [t_3, \infty)_{\mathbb{T}}$,

$$\left(r(t)(z^{\Delta}(t))^{p-1}\right)^{\Delta} \le -c^{p-1}q(t) < 0.$$

Integrating this inequality from t_3 to t, we have

$$r(t)(z^{\Delta}(t))^{p-1} - r(t_3)(z^{\Delta}(t_3))^{p-1} \leq -c^{p-1}\int_{t_3}^{t} q(s)\Delta s.$$

The latter inequality and condition (2.2) yield $\lim_{t\to\infty} r(t)(z^{\Delta}(t))^{p-1} = -\infty$, which contradicts $z^{\Delta} > 0$. Hence, x and z are bounded. Next, we prove that $z^{\Delta} > 0$ eventually. If not, then there exists a $t_4 \in [t_3, \infty)_{\mathbb{T}}$ such that $z^{\Delta}(t) < 0$ for all $t \in [t_4, \infty)_{\mathbb{T}}$. Inequality (2.5) implies that, for all $t \in [t_4, \infty)_{\mathbb{T}}$,

$$z^{\Delta}(t) \leq -\left(-\frac{c_0}{r(t)}\right)^{1/(p-1)} < 0,$$

where $c_0 := r(t_4)|z^{\Delta}(t_4)|^{p-2}z^{\Delta}(t_4) < 0$. The latter inequality and condition (2.1) imply that $\lim_{t\to\infty} z(t) = -\infty$, which contradicts the fact that z is bounded, and hence $z^{\Delta} > 0$ eventually. Then, $z < 0$ eventually. On the other hand, we get by the definition of z that

$$x(t) = \frac{x(\tau^{-1}(t)) - z(\tau^{-1}(t))}{a(\tau^{-1}(t))},$$

and so

$$x(\delta(t)) = \frac{x(\tau^{-1}(\delta(t))) - z(\tau^{-1}(\delta(t)))}{a(\tau^{-1}(\delta(t)))} \geq -\frac{z(\tau^{-1}(\delta(t)))}{a(\tau^{-1}(\delta(t)))} \geq -\frac{z(\tau^{-1}(\delta(t)))}{a_0}.$$

Then, by virtue of (1.1), we see that

$$\left(r(t)(z^{\Delta}(t))^{p-1}\right)^{\Delta} + \frac{q(t)}{a_0{}^{p-1}}\left(-z(\tau^{-1}(\delta(t)))\right)^{p-1} \leq 0.$$

Integrating the latter inequality from t to $\alpha(t)$ ($\alpha(t) > t$), we find

$$r(\alpha(t))(z^{\Delta}(\alpha(t)))^{p-1} - r(t)(z^{\Delta}(t))^{p-1} + \frac{1}{a_0{}^{p-1}}\int_{t}^{\alpha(t)} q(s)\left(-z(\tau^{-1}(\delta(s)))\right)^{p-1}\Delta s \leq 0,$$

and hence

$$-r(t)(z^{\Delta}(t))^{p-1} + \frac{1}{a_0{}^{p-1}}\int_{t}^{\alpha(t)} q(s)\left(-z(\tau^{-1}(\delta(s)))\right)^{p-1}\Delta s \leq 0.$$

By virtue of the latter inequality and the fact that $(-z)^{\Delta} < 0$, we deduce that

$$-r(t)(z^{\Delta}(t))^{p-1} + \frac{1}{a_0{}^{p-1}}\left(\int_{t}^{\alpha(t)} q(s)\Delta s\right)\left(-z(\tau^{-1}(\delta(\alpha(t))))\right)^{p-1} \leq 0,$$

which yields

$$z^{\Delta}(t) \geq -\frac{1}{a_0}\left(\frac{1}{r(t)}\int_{t}^{\alpha(t)} q(s)\Delta s\right)^{1/(p-1)} z\left(\tau^{-1}(\delta(\alpha(t)))\right).$$

Writing the latter inequality in the form

$$-z^{\Delta}(t) - \frac{1}{a_0}\left(\frac{1}{r(t)}\int_{t}^{\alpha(t)} q(s)\Delta s\right)^{1/(p-1)} z\left(\tau^{-1}(\delta(\alpha(t)))\right) \leq 0. \tag{2.7}$$

Setting $u := -z > 0$, then u is a positive solution of the first-order dynamic inequality (2.3). This contradiction completes the proof. ■

On the basis of Theorem 2.1 and [27, Theorem 3.1], one can obtain the following result.

Corollary 2.1. *Assume* (H_1)–(H_3), (2.1), *and* (2.2). *Suppose also that there exists a function* $\alpha \in C_{rd}(\mathbb{T}, \mathbb{T})$ *such that* $\alpha(t) > t$, $\tau^{-1}(\delta(\alpha(t))) < t$, *and* $\lim_{t\to\infty} \tau^{-1}(\delta(\alpha(t))) = \infty$. *If*

$$\limsup_{t\to\infty}\sup_{\lambda\in E} \lambda \exp_{-\lambda Q}\left(t, \tau^{-1}(\delta(\alpha(t)))\right) < 1,$$

where Q *is as in* (2.4) *and*

$$E := \{\lambda : \lambda > 0, 1 - \lambda Q(t)\mu(t) > 0\},$$

then Eq. (1.1) *oscillates.*

Theorem 2.2. *Assume* (H_1)–(H_3), (2.1), *and* (2.2)*. Suppose that there exists a function* $\alpha \in C_{rd}(\mathbb{T}, \mathbb{T})$ *such that* α *is nondecreasing,* $\alpha(t) > t$, $\tau^{-1}(\delta(\alpha(t))) < t$, *and* $\lim_{t\to\infty} \tau^{-1}(\delta(\alpha(t))) = \infty$. *If*

$$\limsup_{t\to\infty} \int_{\tau^{-1}(\delta(\alpha(t)))}^{t} Q(s)\Delta s > 1, \tag{2.8}$$

where Q *is as in* (2.4)*, then Eq.* (1.1) *is oscillatory.*

Proof. As in the proof of Theorem 2.1, we obtain (2.7). Define $u := -z$. Then, we get

$$u^{\Delta}(t) + Q(t)u\left(\tau^{-1}(\delta(\alpha(t)))\right) \le 0,$$

where $u > 0$ and $u^{\Delta} = -z^{\Delta} < 0$. Therefore, we have

$$\begin{aligned} 0 &\ge u(t) - u\left(\tau^{-1}(\delta(\alpha(t)))\right) + \int_{\tau^{-1}(\delta(\alpha(t)))}^{t} Q(s)u\left(\tau^{-1}(\delta(\alpha(s)))\right)\Delta s \\ &\ge u(t) + \left[\int_{\tau^{-1}(\delta(\alpha(t)))}^{t} Q(s)\Delta s - 1\right] u\left(\tau^{-1}(\delta(\alpha(t)))\right), \end{aligned}$$

which contradicts (2.8). The proof is complete. ■

3. Examples

The following two examples illustrate the applications of the main results in this paper.

Example 3.1. For $t \ge 1$, consider a second-order neutral delay differential equation

$$\left(x(t) - \frac{1}{2}x(t-2\pi)\right)'' + \frac{1}{2}x(t-4\pi) = 0. \tag{3.1}$$

Let $p = 2$, $r(t) = 1$, $a(t) = a_0 = 1/2$, $\tau(t) = t-2\pi$, $\delta(t) = t-4\pi$, $q(t) = 1/2$, and $\alpha(t) = t+\pi$. Then $\tau^{-1}(\delta(\alpha(t))) = t-\pi$ and $Q(t) = \int_t^{\alpha(t)} q(s)\Delta s/a_0 = \int_t^{\alpha(t)} q(s)\mathrm{d}s/a_0 = \pi$. By virtue of Ladde et al. [13, Theorem 2.1.1], we obtain that the first-order differential inequality

$$u'(t) + \pi u(t-\pi) \le 0$$

has no eventually positive solutions. Hence, by Theorem 2.1, every solution of (3.1) oscillates. It is not difficult to verify that $x(t) = \sin t$ is an oscillatory solution of this equation.

Example 3.2. For $\mathbb{T} := \overline{2^{\mathbb{Z}}} = \{2^k : k \in \mathbb{Z}\} \cup \{0\}$, consider a second-order neutral delay dynamic equation with a p-Laplacian like operator

$$\left(t|z^{\Delta}(t)|^{p-2}z^{\Delta}(t)\right)^{\Delta} + q_0\left|x\left(\frac{t}{8}\right)\right|^{p-2} x\left(\frac{t}{8}\right) = 0, \tag{3.2}$$

where $z(t) := x(t) - a_0x(t/2)$, $a_0 \in (0, 1)$, $p \in [2, \infty)$, and $q_0 > 0$ are constants. Let $r(t) = t$, $a(t) = a_0$, $\tau(t) = t/2$, $\delta(t) = t/8$, $q(t) = q_0$, and $\alpha(t) = 2t$. Then $\tau^{-1}(\delta(\alpha(t))) = t/2$ and $Q(t) = {q_0}^{1/(p-1)}/a_0$. It follows from Bohner and Peterson [24, Theorem 5.68] that condition (2.1) holds. It is not hard to verify that all the assumptions of Theorem 2.2 are satisfied. Therefore, we deduce that (3.2) is oscillatory.

4. Summary

We suggested several oscillation criteria for Eq. (1.1) under the assumptions that (2.1) holds, $\tau(t) \le t$, and $0 \le a(t) \le a_0 < 1$. The results obtained in this paper complement and improve [10, Theorems 2.6–2.10], [11, Theorem 3.4 and Corollary 3.5], and [12, Theorem 8], since our results can insure that all the solutions of (1.1) are oscillatory.

Three interesting problems for future research can be formulated as follows.

(P1): Is it possible to establish the oscillation criteria for (1.1) in the case where $\int_{t_0}^{\infty} r^{-1/(p-1)}(t)\Delta t < \infty$?

(P2): Suggest a different method to investigate (1.1) in the case where $\tau(t) \ge t$.

(P3): Develop a different method to study (1.1) in the case where $a(t) \ge 1$.

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