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A PRACTICAL MODEL FOR ADVANCED NONLINEAR ANALYSIS OF EARTHQUAKE EFFECTS IN CLAY SLOPES

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ABSTRACT

Presented in this paper is an effort in providing an advanced yet practical tool with a reasonable level of complexity for modeling of clays in realistic geotechnical engineering problems. SANICLAY model is a Simple ANIsotropic CLAY plasticity model that has been developed by Dafalias et al. (2006). The SANICLAY model provides successful simulation of both undrained and drained rate-independent behavior of normally consolidated clays, and to a satisfactory degree of accuracy of overconsolidated clays. An associated flow rule extension of the SANICLAY model has been employed in the present study, trading simplicity for some accuracy in simulations. The model requires just three constants more than those of the Modified Cam-Clay model, all of which can easily be calibrated from well-established laboratory tests. In order to make the model applicable to practical problems in geotechnical engineering, this simple version of SANICLAY model has been efficiently integrated in FLAC3D program. An illustrative example describing earthquake behavior of saturated clayey slope using the simple form of the SANICLAY model is presented and discussed.

INTRODUCTION

Stability evaluation of slopes under earthquake loading is one of the most challenging issues in geohazard studies. Traditional methods of seismic slope stability assessment are based on quasi-static approach. Earthquakes generate vibrations and mass inertia forces, which at times cause large shear stresses. In moderate to severe seismic regions the quasi-static approach usually indicates failure in slopes that have marginal static Factor of Safety (FoS). The duration of earthquake load, however, is short and in most such cases the slope experiences some permanent displacements without total failure. This means that the focus of the seismic slope stability assessment must be on estimating the earthquake-induced deformations, rather than computing a pseudo-static FoS as commonly done by many geotechnical engineers. A more realistic approach is therefore to allow for soil nonlinearity and set limits for acceptable displacements. Stress-deformation analyses with numerical tools are becoming more common because they can provide insight into the nonlinear behavior of the material.

Numerical methods (mainly finite difference and finite element) have been applied to several case studies. Pestana and Nadim (2000) introduced a finite element program for the solution of the one-dimensional wave propagation problem in the case of an infinite slope. Havenith et al. (2002, 2003), Bourdeau et al. (2004), Crosta et al. (2005) and Chugh and Stark (2006) presented two-dimensional models with finite difference method of cases from Kyrgyzstan, El Salvador, California, using simple constitutive models such as Mohr Coulomb model. Bourdeau et al. (2004) noticed that two-dimensional numerical models give smaller failed areas than pseudo-static and static slope stability analyses. Chugh & Stark (2006) found similar displacements in their results as obtained with the Newmark displacement-based method. Loukidis et al. (2003) compared the finite element method in a linear approach with pseudo-static evaluations and obtained similar results for the selected cases. Azizian and Popescu (2006) used two- and three-dimensional non-linear finite element models for earthquake slope stability assessments for quantitative study on the limits of applicability of the 2D, plane strain analysis assumptions. Sigaran-Loria (2007) conducted detailed sensitivity analyses on the effects of earthquake frequencies and amplitudes in different slopes using a nonlinear finite element and Mohr- Coulomb model.

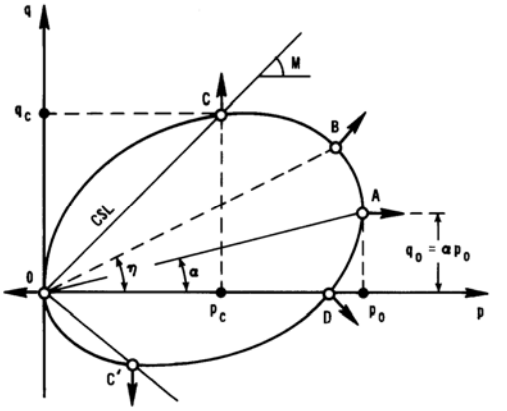
Nadim et al. (2007) presented the main mechanisms contributing to instability of clay slopes under seismic loading. Through one-dimensional analyses they showed that an appropriate approach for assessing seismic stability of clay slopes is to focus on estimating the earthquake-induced shear strains.

A realistic three-dimensional simulation of the nonlinear material subjected to irregular earthquake loading is very complex owing to all types of wave propagation and boundary conditions. For such a comprehensive analysis to be still applicable to engineering problems, one needs reliable computational tools. Such tools need to address a number of challenges such as capabilities for 3D analyses, dynamic loading and handling the boundary conditions in a realistic way, fully coupled solid-fluid interaction, and inclusion of structural elements. Last but not least is constitutive behavior of soil matrix, which is the crucial part of a successful numerical simulation. Most of the advanced numerical efforts, particularly in geotechnical earthquake engineering, are conducted either within very simple frameworks that do not allow for sufficient insight into many important features of the system response, or within very complex frameworks that may not be applicable for real engineering problems. One of the key points in successful and applicable research in this area is to seek adequate sophistication in modeling, and to match sophistication of model to availability of data and needs of application.

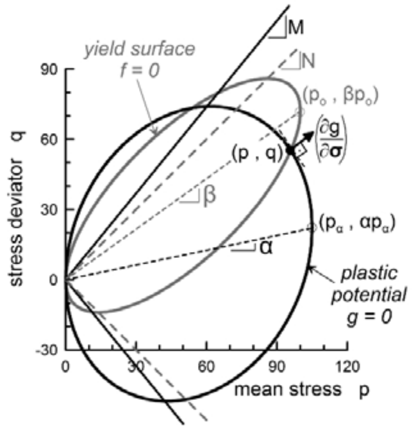
In the present development the nonlinear finite difference code FLAC3D (Itasca Consulting Group, Inc., 2006) is selected as the main computational platform. FLAC3D (Fast Lagrangian Analysis of Continua in three Dimensions) is a dynamic code that uses an explicit time integration scheme and large strain formulation, and is well suited for calculating dynamic stability problems. In the present study, constitutive behavior of the soil matrix is characterized by a Simple ANIsotropic CLAY plasticity model (SANICLAY) that has been developed by Dafalias et al. (2006). In order to make the model applicable to practical problems in geotechnical engineering, the SANICLAY model is efficiently integrated in FLAC3D program. The model implementation is successfully verified for different stress paths. The resulting framework presents a practical tool for advanced numerical simulation of fully coupled nonlinear dynamic behavior of soil in a Boundary Value Problem. The employed constitutive model is used in numerical simulation of the seismic response of a generic slope in order to investigate its performance and features. Despite the fact that the implementation is within a general 3D framework, for simplicity of presentation a 2D section of the slope has been simulated under a sinusoidal-type acceleration time history. The importance of anisotropy to the mechanism of accumulation of displacements in clay slopes is illustrated in this study. It should be emphasized that the objective is not to reproduce the measured response of a certain case history, rather to illustrate the capabilities of developed tool using the rigorous yet practical constitutive model that is introduced in this numerical framework.

CONSTITUTIVE MODEL

An accurate estimation of the soil strength is crucial in the assessment of slope stability. The shear strength is not a unique property of soil, but is affected by a number of factors, each with different effects and variable influence. Advanced geotechnical design on soft clays has often been based on using isotropic elastoplastic soil models, such as the modified Cam Clay (MCC) model (Burland, 1990). Natural soft clays, however, almost always have certain degree of anisotropy due to the structure formed during deposition and subsequent one-dimensional (oedometric) consolidation along with development of chemical bonds between particles due to various processes. The existing and evolving anisotropy may be critical for design. Neglecting the effect of anisotropy and its evolution in soil behavior may lead to incorrect predictions of soil response under loading (see, e.g. Leroueil et al., 1979 and Zdravkovic et al., 2002). Anisotropy can be accounted for by rotational hardening, which implies a rotation of the yield and plastic potential surfaces.



(a) $f = g = (q - p\alpha)^2 - (M^2 - \alpha^2)p(p_0 - p) = 0$



(b)
$$f = (q - p\beta)^2 - (N^2 - \beta^2)p(p_0 - p) = 0$$
$$g = (q - p\alpha)^2 - (M^2 - \alpha^2)p(p_\alpha - p) = 0$$

Fig. 1. Schematic diagram of the anisotropic yield surface and plastic potential in the p-q space in (a) Dafalias (1986) model, and (b) Dafalias et al. (2006) model (SANICLAY).

Dafalias (1986) proposed what can be considered to be the simplest possible energetic extension of the Modified Cam Clay (MMC) model from isotropic to anisotropic response, introducing in the rate of plastic work expression a contribution coupling the volumetric and deviatoric plastic strain rates. The resulting plastic potential surface in the triaxial p-q stress space, which for associative plasticity serves also as a yield surface, is a rotated and distorted ellipse. The amount of rotation and distortion portrays the extent of anisotropy, and is controlled by an evolving variable α, which is scalar-valued in triaxial and tensor-valued in multiaxial stress space. Fig. 1(a), adopted from Dafalias (1986), shows the configuration of the yield surface in this model. The parameter M refers to the critical stress ratio and was assumed as a constant in this model.

More recently Dafalias et al. (2006) proposed the SANICLAY model based on their earlier work in which a non-associated flow rule allows the simulation of softening response under undrained compression following oedometric consolidation. Although built on the general premises of critical state soil mechanics, the model induces a critical state line in the void ratio-mean effective stress space, which is a function of anisotropy. Fig. 1(b) presents the configuration of the yield surface and plastic potential in this model. The parameter M here is again the critical stress ratio, however its value varies between M_c and $M_e=mM_c$ by means of the Lode angle. The parameter β is the rotational hardening variable of the yield surface, and N is a constant, similar in nature to M, but taken the same in compression and extension. Another modification in this version of the model, comparing to the work of Dafalias (1986) and others who followed his suggestion for the plastic potential, e.g. Wheeler et al. (1999,2003), is addressing an important requirement on $|\alpha|<M$ so that the equation of plastic potential can be satisfied for real values of q and p. In order to address this requirement, Dafalias et al. (2006) employed a bounding surface technique in the formulation of the SANICLAY model, whereby the increments of α were made proportional to the stress-ratio 'distance' $(\alpha^b-\alpha)$ from its properly defined bounding 'image' α^b on a bounding surface which must not be exceeded, so that α freezes when $\alpha^b=\alpha$.

Formulation of the SANICLAY model in multiaxial stress space is presented and discussed in detail by Dafalias et al. (2006). The SANICLAY model provides successful simulation of both undrained and drained rate-independent behavior of normally consolidated clays and, to a satisfactory degree of accuracy, of overconsolidated clays. The SANICLAY model requires merely three constants more than those of the Modified Cam-Clay model, all of which can easily be calibrated from well-established laboratory tests.

The SANICLAY model has been calibrated for Boston Blue clay (Papadimitriou et al., 2005) and Lower Cromer Till (Dafalias et al. 2006) and details of the performance of the model with different constitutive features have been extensively examined. An example of the extensive verification process for the SANICLAY model is presented in Fig. 2.

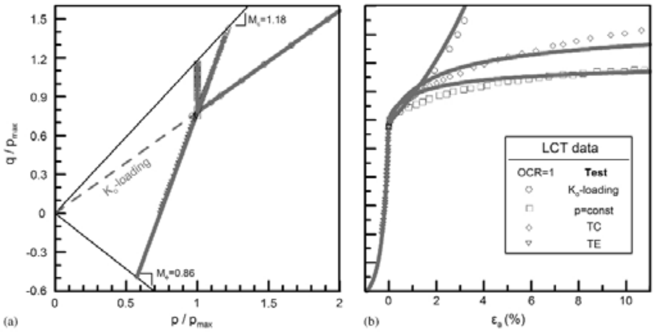


Fig. 2. An example of the SANICLAY model verification: Comparison of data and simulations for drained probe triaxial tests on K_o-consolidated samples of LCT and OCR=1 (from Dafalias et al. (2006)).

The employed formulation of the model in the present study takes advantage of the simple framework of MCC, and with perhaps the simplest possible approach adds the very important features of Lode angle dependency and anisotropy to the MCC model. The new constitutive features can be de-activated, if so desired by the user, simply by selecting appropriate values for certain model constants. In this way the developed model can be simplified back to the MCC model. The present formulation is basically the SANICLAY model of Dafalias et al. (2006) with all of its enhancements comparing to Dafalias (1986), except that it does not include the non-associated flow rule mechanism.. The resulting model is simple, convenient and rational yet significantly improves the MCC model in describing some essential features of response in natural clays. It is aimed to become a tool for solution of boundary value problems encountered in geotechnical engineering.

In addition to the regular five parameters of MCC model, the present form of SANICLAY has one constant $m= M_e/M_c$ for characterization of initial anisotropy (Lode angle dependency), and the following two constants for anisotropy (rotational hardening): C which controls the rate of rotational hardening, and x which controls the value of K_0 and can even be obtained by a closed-form analytical expression. Similar to MCC the model has a variable size for the yield surface (p_0), and in addition has a tensor variable $\boldsymbol{\alpha}$ for the degree of orientation of the yield surface (anisotropy).

MODEL IMPLEMENTATION IN FLAC3D

The SANICLAY model in its simple form as it was explained in the previous section, has been numerically implemented in the three-dimensional explicit finite difference program FLAC3D (Itasca Consulting Group, Inc., 2006) via its UDM option. The constitutive model is written in C++ and compiled as a DLL file (Dynamic Link Library) that can be loaded whenever it is needed.

Accuracy of the numerical implementation of a constitutive model in a numerical framework is tied to the employed integration scheme. Various numerical techniques - explicit, refined explicit, and implicit - have already been proposed and extensively discussed in the literature (see e.g., Potts and Gens (1985); Sloan (1987); Borja and Lee (1990); Crisfield (1991); Jeremic and Sture (1997); Sloan et al. (2001)). Implicit integration is the most accurate approach, however it could be computationally complex and run-time extensive in particular for more advanced constitutive models. The choice of the more efficient algorithm depends on the constitutive law and the numerical code where it will be implemented. Because of the explicit nature of the global solution algorithm used in FLAC3D, using implicit algorithm for constitutive integration should preferably be avoided, or else a simple analysis may appear computationally very extensive. In fact, very small increments must be used anyway because this is prerequisite for computational stability of the global solution. Therefore, the stress update algorithm should have a computational effort as low as possible. In the current work, an explicit integration scheme with a drift correction method and an optional substepping technique has been adopted for the model implementation (Taiebat et al., 2009b).

VERIFICATION AND VALIDATION

Prediction of mechanical behavior comprises the use of computational model to foretell the state of a physical system under consideration under conditions for which the computational model has not been validated (Oberkampf et al., 2002). Confidence in predictions relies heavily on proper verification and validation (V&V) processes.

Verification is the process of determining that a model implementation accurately represents the developer's conceptual description and specification, so it is a Mathematics issue. Verification provides evidence that the model is solved correctly. Verification is also meant to identify and remove errors in computer coding and verify numerical algorithms and is desirable in quantifying numerical errors in computed solution. Validation is the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model; therefore, it is a Physics issue. Validation provides evidence that the correct model is solved. Validation serves two goals, namely, (a) tactical goal in identification and minimization of uncertainties and errors in the computational model and (b) strategic goal in increasing confidence in the quantitative predictive capability of the computational model.

The V&V procedures are the primary means of building confidence and credibility in modeling and computational simulations. The employed mechanisms in the SANICLAY model with different levels of complexity have been examined for capturing the results of a wide range of element tests on Boston Blue clay, Lower Cromer Till, and Bothkennar clay (Papadimitriou et al., 2005, Dafalias et al. 2006, Taiebat et al., 2009a) with the aim of model validation. In addition, accuracy of the numerical implementation of the model in FLAC3D has been verified in Taiebat et al. (2009b) using a comprehensive and independent constitutive driver developed based on the proposed approach by Bardet and Choucair (1991).

NUMERICAL SIMULATIONS

In order to illustrate various features of the SANICLAY model in a boundary value problem, the response of a saturated clay slope under an idealized earthquake excitation has been simulated using this constitutive model integrated within the framework of FLAC3D. Partial results of these simulations are presented and discussed in this section.

Figure 3 shows the geometry and finite difference mesh of a 5H:1V slope of saturated clay in FLAC3D. The newly implemented SANICLAY model has been used to characterize the response of clay in the simulation. The model is discretized using 1700 brick zones in FLAC3D. For simplicity, the grid points have been fixed in the y direction and thus the problem is reduced to only two dimensions (x-z).

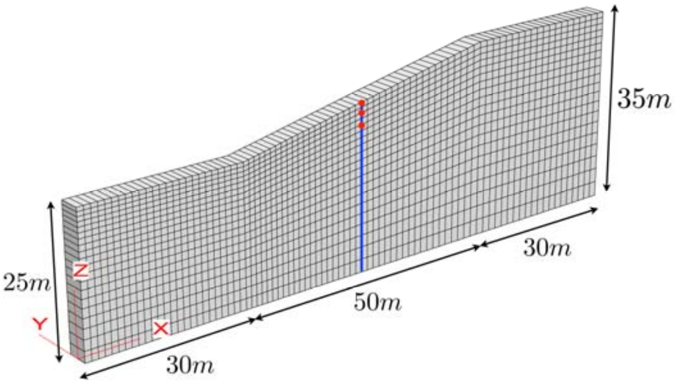


Fig. 3. Model geometry and the finite difference mesh and the positions of the monitoring points.

The analysis starts with applying the initial stresses and pore pressures under self-weight loading using a K_0 value of 0.6. Here is where the unique capability of the SANICLAY model in realistic prediction of the K_0 value becomes important. This has become possible with the aid of the rotational hardening mechanism and using the flexibility that the constant x brings to the model. This flexibility allows SANICLAY to be calibrated via the parameter x to whatever the measured value of K_0, unlike the MCC model, which is known for overestimating the K_0 value. The slope remains stable under self-weight. The input excitation is then applied in form of horizontal acceleration at the base of the model. Figure 4 shows the time history of the idealized input acceleration with a frequency of 2 Hz and a maximum amplitude of 0.25g. Ghosh and Madabhushi (2003) concluded that responses with simple input motions are easier to understand than to motions with wide frequency ranges, such as real earthquakes.

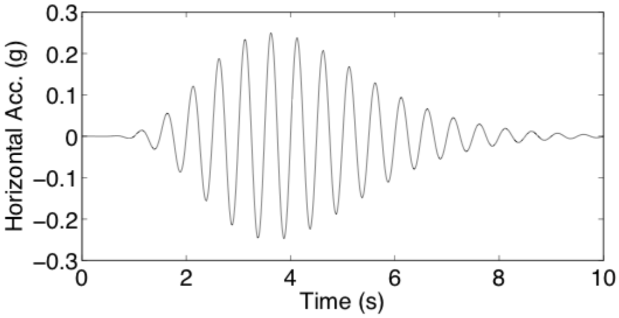


Fig. 4. Time history of input motion, a_x, at the base of the model.

The SANICLAY model parameters that are used in the present simulation are presented in Table 1. These parameters are adopted from Dafalias et al. (2006) where the SANICALY model with non-associated flow rule had been calibrated for the data of Lower Cromer Till (Gens, 1982). With the present assumption for associated flow rule, the value of N is set equal to the critical stress ratio M.

Table 1. Model constants used in the simulation.

Model constant		Value
Elasticity	κ	0.009
	ν	0.2
Critical state	M_c	1.18
	m	0.73
	λ	0.063
Hardening	C	16
	x	1.56

The initial deviatoric stress state (**s**), i.e. the deviatoric stress under self-weight loading, is used to estimate the initial value of the internal tensor variable **α** as **s**/x at each zone. The initial size of yield surface is set to $p_0 = Rp_{0,n}$ with R = 1.2 and $p_{0,n}$ the corresponding value of p_0 for normally consolidated state at the present stress state, i.e. having the stress point on the yield surface. Therefore a small value of overconsolidation has been introduced for the material state at the beginning of the shaking phase. The values of the model internal variables at the beginning of the shaking phase are presented in Table 2.

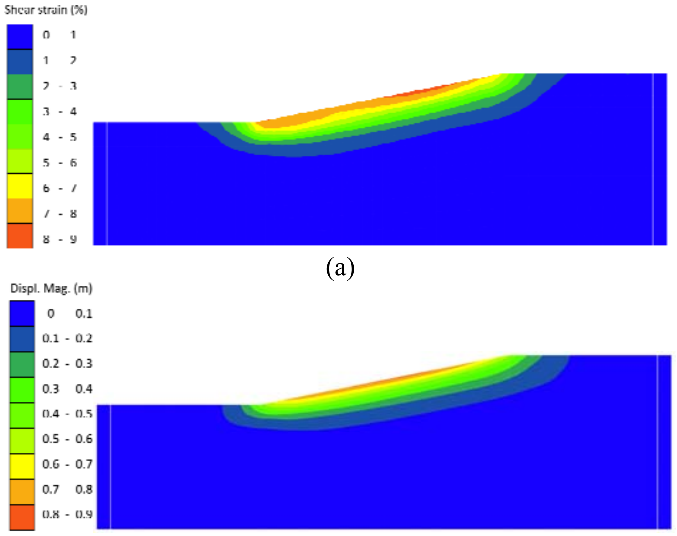
Table 2. Initial values of the model internal variables used in the simulations.

Model internal variable		Value
Size of the YS	p_0	$1.2p_{0,n}$
Orientation of the YS	**α**	**s**/x

Formulation of FLAC3D allows for conducting any sophisticated solid-fluid interaction analysis. The examined problem in this study, i.e. seismic response of a saturated clayey slope, is essentially an undrained problem and this allows switching off the water flow in the analysis. A proper constitutive model together with the built-in equations for modeling of solid-fluid interaction allows the time-dependent pore pressure changes in clayey ground. For the dynamic analysis specifying the Free-Field boundary condition feature of FLAC reduces wave reflections at the boundaries of the mode. This approach enforces the free-field motion in such a way that boundaries retain their non-reflecting properties - i.e., outward waves are properly absorbed.

A number of representative results of this simulation are presented in Figs. 5-7 and explained in the following. Figs. 5(a) and 5(b) show contours of maximum shear strain and displacement magnitude, respectively, at the end of shaking (t=10s). In particular, Fig. 5(a) shows that the maximum shear strain at the end of shaking reaches to about 9% in the slope. The magnitude of (downslope) displacement reaches up to about 0.85m at the end of shaking as it is presented in Fig. 5(b).

Time histories of the horizontal displacements at three different depths of 0, 3m and 5m in the middle section of the slope are presented in Fig. 6. This figure shows the accumulation of lateral displacements at these monitoring depths during the 10s of shaking. The amount of accumulated displacement is a function of depth, and as expected, increases towards the surface. While generation of permanent lateral displacement is a result of yielding in the slope, it is not an indication of slope failure. The size of permanent displacement, however, is used by some engineering guidelines to decide on stability of a slope under earthquake loading.



(a)

(b)

Fig. 5. Contours of (a) maximum shear strain and (b) displacement magnitude at t=10s.

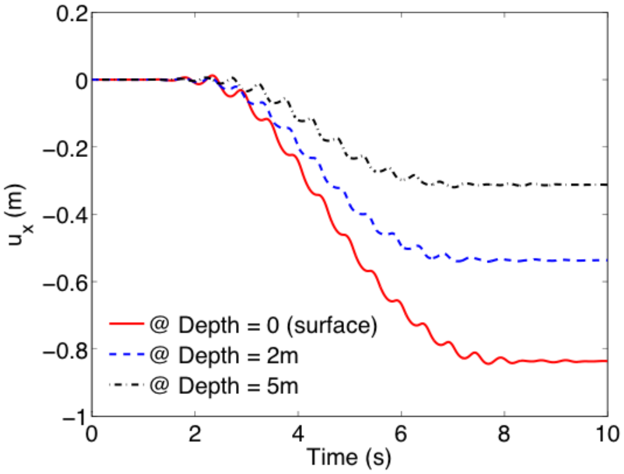
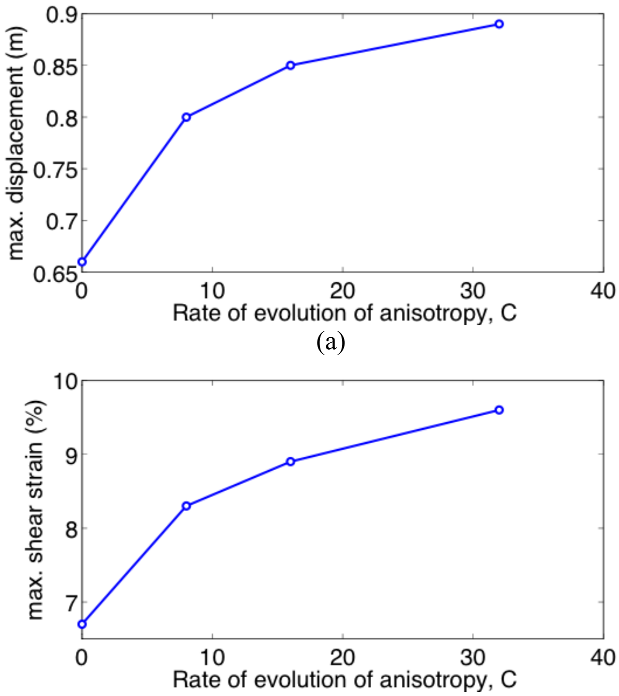


Fig. 6. Comparison of the time histories of horizontal displacement u_x at different depths in the middle part of the slope.

For assessing the importance of the anisotropic (rotational) hardening in the model, one may compare results of simulation without anisotropic hardening (C = 0) with those of the present simulation with anisotropic hardening (C = 16). The analyses show that in the first case the maximum shear strain does not exceed 6.6% while in the second case this value shows an increase of about 25% and reaches up to 8.3%. The maximum value of displacement in the slope for the first case (without anisotropy, C=0) is about 0.65m while in the second case (with anisotropy, C=16) with a 30% increase it reaches as high as 0.85m.



(a)

(b)

Fig. 7. Effect of the anisotropic hardening on the max. values of (a) shear strain (%) and (b) displacement (m) in the slope.

Fig. 7 displays in more detail the effect of the rate of anisotropic hardening by variation of the parameter C in a range of 0 to 30 (typical range). The maximum values of displacements and shear strains in the slope increase with the rate of evolution of anisotropy (constant C) as illustrated in this figure. Effect of the parameter C on the time histories of the horizontal displacement (u_x) for three monitoring points at the middle part of the slope is shown in Fig. 8. It is again clear from this figure that the horizontal displacement in the slope at each time increase with the rate of evolution of anisotropy. It should be noted that the response in the present problem is mainly controlled by one-sided cyclic plasticity (except for the toe of the slope). The effect of anisotropy may appear even more important in problems where two-sided cyclic plasticity is more pronounced.

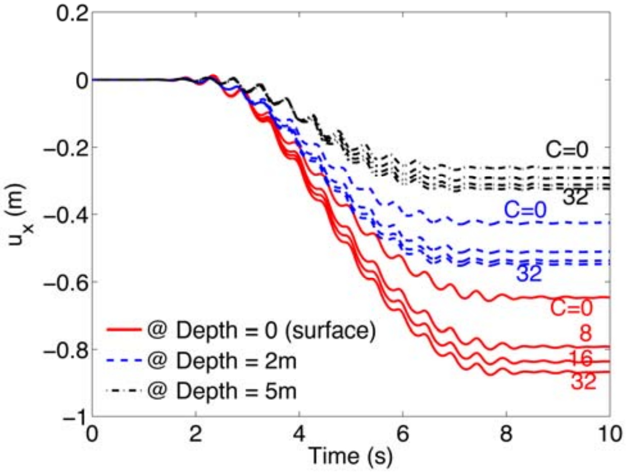


Fig. 8. Effect of the anisotropic hardening on the time histories of horizontal displacement u_x at the surface of the middle part of the slope.

SUMMARY

The ultimate goal of constitutive modeling is its application to practical problems. To this end the corresponding constitutive model should be comprehensive enough in order to account for main features of material response, and at the same time be simple enough for understanding, calibration, and application. In addition the model should be efficiently implemented in a proper numerical framework that can handle different types of loading and boundary conditions. For certain problems of interest in geotechnical engineering the numerical program also needs to have a three dimensional formulation, and properly handle the solid-pore fluid interaction and dynamic problems.

A simple and practical version of the advanced SANICLAY model is employed for characterization of the response of clays. This model includes a number of key mechanisms that are essential in prediction of response in clays, such as Lode angle dependency and anisotropic hardening, in a simplified approach with the hope to make it attractive for real applications. Capabilities of the main features of the model are already validated against a number of laboratory results. The model has been efficiently integrated in FLAC3D program that is well-known in both research and practical communities of geotechnical engineering and especially in the field of geotechnical earthquake engineering. The implementation details have been extensively verified in the numerical framework.

The verified and validated numerical framework is a versatile and powerful tool for modeling of response in many desired boundary value problems. In order to demonstrate the importance of a number of key features of the model in simulation of boundary value problems, the resulting numerical framework has been used in modeling the seismic response in a slope problem. The general 3D implementation allows the developed framework to be used for modeling of any 3D problem. Because the focus is mainly on the constitutive model in the chosen numerical framework, for simplicity of discussions the slope is assumed to behave in plain strain and therefore a 2D geometry of the slope has been studied using the 3D zones (elements). The numerical study incorporates the solid-fluid interaction and dynamic analysis features in FLAC3D. The model acquires the reduced form of the classical MCC model, by setting proper model constants. The influence of anisotropy in the simulation of the boundary value problem has been demonstrated in this numerical study. When anisotropy and Lode angle dependency are deactivated, in which case the model becomes the classical MCC model, the analysis may under-predict the rate of accumulation and the final level of plasticity, which manifests in the developed permanent shear strains and the resulting deformations. This also includes the level of excess pore pressures developed due to volumetric shear strains in the model and could be of interest in certain problems.

In terms of model constants, the use of just three additional ones than what is required in the MCC, namely the m, C and x, the model can capture the important feature of anisotropic hardening. Besides the more accurate evaluation of plastic shear strains, this feature allows the correct prediction of the K_0 by the model, something that is missing in the MCC model. More refinement of the model to account for cyclic loading applicable to seismic problems while maintaining its simplicity are underway. However the present framework meets a good portion of the practical needs in many problem of interest in Geotechnical Engineering.

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