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ASSESSING CROSS ANISOTROPY OF SMALL-STRAIN STIFFNESS USING THE RESONANT COLUMN APPARATUS

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ABSTRACT

Almost all soils exhibit cross-anisotropic stiffness to some extent. However, measuring the cross anisotropic properties of soils is difficult because of the need to determine the 3 independent stiffness parameters E_v, E_h, G_{vh}, and the associated Poisson's ratios, ν_{vh} and ν_{hh}. Current techniques that are employed, for example using bender elements or field geophysics, are not always reliable, whilst preparing specimens in different orientations and subsequent testing using standard laboratory techniques has practical constraints.

The resonant column is a laboratory apparatus that has been extensively used to measure the torsional stiffness (G_{vh}). Relatively recent development has also allowed the Stokoe resonant column to measure Young's modulus from flexural excitation of the specimen. The apparatus has also been used to determine E_v through axial oscillation. Thus a modified resonant column apparatus can apply four different excitations (flexure in two directions, torsion and longitudinal excitation) to a soil.

This paper reports a series of dynamic finite element numerical simulations of physical tests in the resonant column apparatus, carried out to model both the apparatus and a cross-anisotropic soil specimen. Forward modelling has been carried out to determine the impact of different degrees of anisotropy on the resonant frequencies of 'specimens' with their axes of anisotropy aligned in different directions relative to the vertical axis of the apparatus. Methods of determining the elastic parameters from these data are assessed.

INTRODUCTION

Small strain stiffness parameters such as Young's moduli and shear moduli (G, E) are essential when modelling the deformation behaviour of soils. Within geotechnical design, soil stiffness is nearly always thought of as isotropic, using moduli that are the same in all directions. However it has long been recognized that in reality the small strain stiffness of natural soils is anisotropic, that is $G_v \neq G_h$ and $E_v \neq E_h$, where the subscript v and h relate to the vertical and horizontal direction in which the stiffness is measured.

This elastic anisotropy can usefully be categorized as either inherent anisotropy or stress-induced anisotropy. Inherent anisotropy develops during deposition of the material, due to particles becoming aligned with the plane of deposition. The degree of anisotropy is generally greater, all other things being equal, in clay rich soils (which have platy particles) than in sands (which tend to have more rounded particles). Stress induced anisotropy occurs when an anisotropic loading is applied to the soil. It has been shown that for sands the degree of anisotropy is low when subjected to an isotropic stress at small strains (Kuwano and Jardine, 2002, Tatsuoka and Kohata, 1995) but increases as the strain increases and the loading conditions become anisotropic (Bellotti et al., 1996, Kuwano and Jardine, 2002).

A variety of different measurement methods are available to determine small strain stiffness, both within the field (such as seismic cross-hole and down-hole profiling, and surface-wave testing) or through laboratory tests (such as triaxial, resonant column, and bender element tests). In practice the majority of these tests can only measure stiffness in one plane, and therefore different tests are required to determine anisotropic behaviour. The combination of down-hole and cross-hole test data can be used to derive a number of stiffness parameters, however factors such as poor wall coupling, large hole spacing, spatial variability of the ground, and background noise can make interpretation of the data difficult. In the laboratory the use of local strain triaxial testing (e.g. Clayton and Khatrush, 1986) with bender elements (e.g. Pennington et al., 1977) has provided the capability to measure soil stiffness in a variety of planes. However, local strain measuring systems do not always have very small strain resolution, and bender element data is dependent on a number of factors such as input waveform and frequency, signal-to-noise ratio, mode conversions and near field effects, that make the results variable (Yamashita et al., 2009) and sometimes unreliable.

The resonant column apparatus is typically used to determine the shear modulus of soils by measuring the resonant frequency of the soil/apparatus system under torsional vibration. Modifications to the original resonant column apparatus and testing method have allowed flexural and axial vibration to be applied to a sample, to derive values of Young's modulus. Using a variety of modes of vibration allows a number of different stiffness values to be derived, which might potentially allow anisotropic stiffness to be determined. To investigate the value of this, a series of finite element analyses, modelling both the resonant column and the soil sample, have been carried out, and resonant frequencies determined. These frequencies have been used with the classical resonant column equations to derive the stiffness of the modelled soil, and these have then been compared with the soil properties used in the finite element model. This paper reports the results of these analyses and discusses the potential to use the resonant column to determine anisotropic soil properties.

THEORY

To determine anisotropic stiffness parameters it is necessary to adopt a model of elastic behaviour, and to develop methods of analysing the results of the test. Both exist in the literature.

Anisotropy

For an isotropic material stiffness is constant in all directions. Only two soil parameters are required. Commonly the parameters are Young's modulus (E) with Poisson's ratio (ν), but in some applications shear modulus (G) and bulk modulus (K) are used. For an isotropic material the relationship ratio between shear modulus, G, measured in torsion, and Young's modulus, E, measured in either flexure of longitudinal vibration should be a function of Poisson's ratio, and since

$$E = G \, . \, 2\,(1+\nu) \qquad (1)$$

E / G can be expected to vary from about 2.5 for an unsaturated or dry material with isotropic stiffness (assuming $\nu \approx 0.25$) to 3 ($\nu = 0.5$) for a saturated, undrained isotropic material.

However, in general, soils are not isotropic. For a completely anisotropic behaviour, the stiffness in each orthogonal plane is different, that is $G_v \neq G_{h1} \neq G_{h2}$ and $E_v \neq E_{h1} \neq E_{h2}$, where v, $h1$ and $h2$ represent the vertical, first horizontal and second horizontal directions respectively, as shown in Figure 1. A total of 21 independent parameters are required to describe the stiffness of a fully anisotropic material. However in most cases soils can be assumed to be transversely isotropic. In a transversely isotropic soil stiffness in the horizontal plane is assumed to be isotropic, and the vertical direction is the axis of anisotropy (such that $G_v \neq G_{h1}$, $G_{h1} = G_{h2}$ and $E_v \neq E_{h1}$, $E_{h1} = E_{h2}$). To describe a transversely isotropic elastic material five independent parameters are required (Love, 1927). Commonly these are Young's modulus (E_v and E_h); the Poisson's ratio linking strains in the horizontal directions to the vertical direction (ν_{vh}), Poisson's ratio linking strain in one horizontal direction to the other (ν_{hh}); and the shear modulus in the vertical plane (G_v). The shear modulus in the horizontal plane, G_h, is calculated from $G_h = E_h / 2(1 + \nu_{hh})$).

Since E_v and G_v are independent parameters, in theory the ratio E_v / G_v can adopt any value for a transversely isotropic material.

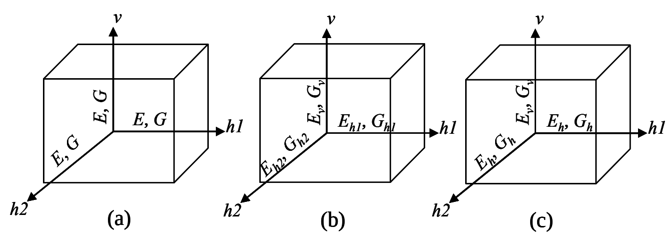


Fig. 1. Properties of material in different directions (a) for isotropic (b) for anisotropic and (c) for cross-anisotropic materials.

Resonant Column Apparatus

The test procedure for a fixed-free RC involves vibrating a cylindrical column of soil and measuring the amplitude of the vibration during a frequency sweep. The resonant frequency is then obtained from the frequency response curve, determined from the peak amplitude observed. As shown in Figure 2 various vibration modes can be applied to the soil (torsional, flexural (bending) and longitudinal) and from the resonant frequencies so obtained the shear modulus, flexural Young's modulus and vertical Young's modulus can be calculated.

For the routine interpretation of a fixed-free resonant column test the specimen is assumed to be elastic, homogeneous and isotropic and fixed at its base, with the drive system (drive mechanism, end platen, etc.) fixed to the top of the sample assumed to be a lumped mass. The solution for the torsional mode of vibration is given as (Richart et al., 1970).

$$\frac{I}{I_o} = \frac{\omega_n l}{V_s} \tan \frac{\omega_n l}{V_s} \qquad (2)$$

from which,

$$G = \rho V_s^2 \qquad (3)$$

where
I_o is the mass polar moment of inertia of lumped mass attached to the free end,
I is the mass polar moment of inertia of specimen,
l is the length of the specimen
ω_n is the circular resonant frequency ($=2\pi f$) from torsional vibration of the specimen,
V_s is the calculated shear wave velocity of the soil,
ρ is the density of the specimen, and
G is the inferred shear modulus of the soil.

The solution for the flexural mode of vibration is given as (Cascante et al., 1998).

$$\omega_f^2 = \frac{3E_{flex} I_b}{\left[\frac{33}{140} m_T + \sum_{i=1}^{N} m_i h\left(h_{0i}, h_{1i}\right)\right] l^3} \qquad (4)$$

where

$$h\left(h_{0i}, h_{1i}\right) = 1 + \frac{3\left(h_{1i} + h_{0i}\right)}{2l} + \frac{3}{4} \frac{\left(h_{1i}^2 + h_{1i} h_{0i} + h_{0i}^2\right)}{l^2} \qquad (5)$$

I_b and E_{flex} are the second moment of inertia and the flexural modulus of the specimen respectively,
N is the number of masses, m_i evenly distributed between h_{oi} and h_{1i}, added during calibration;
m_T is the mass of the specimen, and
ω_f is the circular resonant frequency from flexural vibration of the specimen.

Finally the solution for the longitudinal mode of vibration is given as (Kohoutek, 1981).

$$\frac{m_T}{M} = \sqrt{\frac{m_T l \omega^2}{EA}} \tan \sqrt{\frac{m_T l \omega^2}{EA}} \qquad (6)$$

where
M is the total lumped mass attached at the free end of the specimen,
E is the Young's modulus of the specimen,
A is the cross sectional area of the specimen and ω is the circular resonant frequency from longitudinal vibration of the specimen

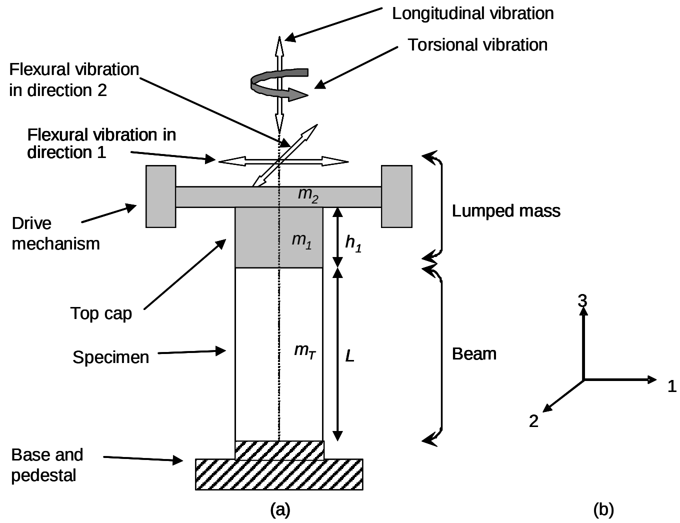


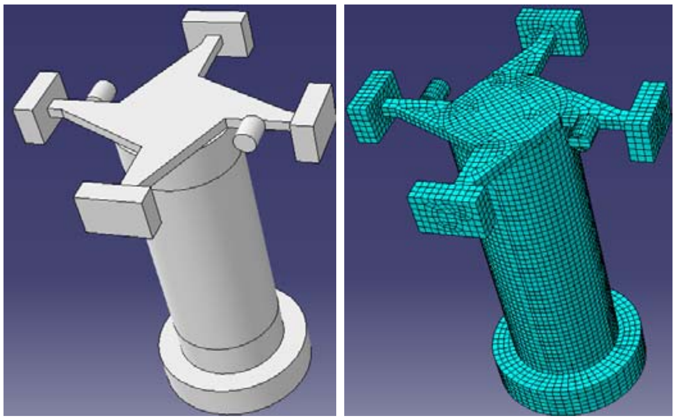
Fig. 2. (a) Different mode of vibration of a specimen in RCA. (b) Axis representation for RCA.

NUMERICAL MODELLING

To assess the effects anisotropy has on the stiffness values derived from a resonant column test a finite element model of the resonant column and a specimen with known cross-anisotropic properties was developed. Clayton et al. (2009) had previously developed an FE model of a Stokoe' resonant column using the finite element software ABAQUS (version 6.8). They were able to validate their model against measured laboratory test results on aluminium bars. They showed that the FE model accurately predicted the behaviour of a calibration bar of known properties. Their model was therefore adopted as a starting point for this work.

Modifications were made to include a specimen (diameter 70mm and length 140mm) with defined elastic transversely isotropic properties. The geometry and density of the individual components of the drive mechanism (drive plate, magnets, accelerometer and counter weight) accurately replicate the complex geometry of the physical drive mechanism of a Stokoe resonant column apparatus (Figure 3).

The specimen and drive mechanism were modelled separately and merged together to ensure all connections were rigid. The model was carefully partitioned such that the model was built using predominately hexagonal elements of size 4mm x 4mm x 4mm; however some triangular prisms (wedges) were used in transition regions. 22646 elements were used, of which 12110 elements were for the specimen alone. The number of elements and element sizes were chosen to minimize the effect of meshing error on resonant frequency, as suggested by Clayton et al. (2008). Natural frequency extraction (sometimes referred to as ‘frequency analysis’) was used to calculate the natural modes of vibration and the corresponding natural frequencies by computing the eigenvectors and the eigenvalues of the model.



(a) (b)

Fig. 3. Modelled resonant column apparatus with a sample, (a) before partition, and (b) after meshing.

A number of different specimens were analyzed. One set of analyses (‘iso’) modelled an isotropic specimen, with a further two sets being undertaken to include specimen anisotropy. In the first anisotropic model (‘vc’) the plane of isotropy was horizontal, equivalent to a specimen cut vertically from a typical transversely isotropic soil. In second anisotropic model (‘hc’) the plane of isotropy was vertical, equivalent to a specimen cut horizontally from a transversely isotropic soil, before being mounted vertically in the apparatus, as shown in Figure 4.

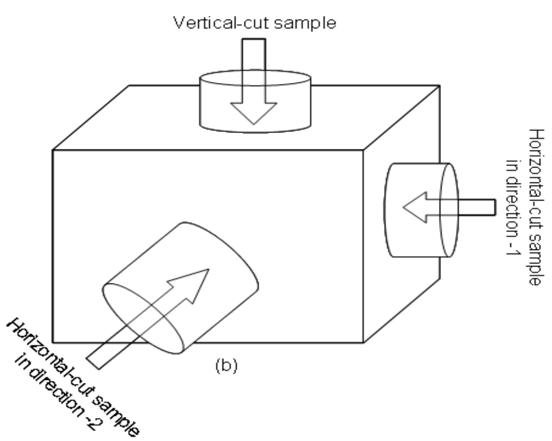


Fig. 4. Different sampling directions in a block sample. For a cross-anisotropic material sampling in horizontal direction 1 and 2 are same.

RESULTS AND DISCUSSION

The first set of analyses was conducted on isotropic specimens (‘iso’) with varying aspect (height to diameter) ratios. G_{iso}, E_{iso}, $E_{flex1iso}$ and $E_{flex2iso}$ were calculated from resonant frequencies for the respective torsional, longitudinal and flexural modes of vibration using Equations 2 - 6, where ‘iso’ denotes isotropic material, and ‘1’ and ‘2’ denote the two different flexural directions. Since the specimen was isotropic, $E_{flex1iso}$ and $E_{flex2iso}$ were equal and were denoted as $E_{flexiso}$.

The reason for testing an isotropic specimen with differing length/ diameter ratios was to investigate the validity of Equations 3 & 5. In the derivation of Equation (4) it is assumed that shear stiffness does not contribute significantly to deformations, and therefore to the resonant frequency in flexure. Classical elastic (Timoshenko) beam bending theory suggests that the contribution of shear stiffness is negligible where the length/diameter ratio of a beam is greater than 6. Since in a standard resonant column test the length/diameter ratio is of the order of 2, it was thought that significant errors might exist in the routine calculation of E_{flex}.

In the derivation of Equation 6 the effects of platen stiffness and lateral restraint are not taken into account. For short specimens it was thought that these might be significant, leading to over prediction of Young’s modulus.

Figure 5 shows the ratios of the $E_{flex1iso}$, E_{iso}, and G_{iso}, calculated from the extracted frequencies obtained from torsional, longitudinal and flexural mode of vibration respectively, by using Eqs. 2 – 6, to those input as material properties. It can be seen that the calculated value of $E_{flexiso}$ is dependent on *l/d* ratio, and for the conventional resonant column *l/d* ratio (=2) an error of around 10% will occur. Thus in using the resonant column in flexural vibration mode any derived value of *E* will need to be a corrected. The effect of aspect ratio on the shear modulus determined from torsional vibration appears negligible. The effect on Young’s modulus derived from longitudinal vibration remains small down to aspect ratios of unity.

In the second set of numerical analyses, the specimen was modelled as a vertically cut transversely isotropic material (‘vc’), with the axis of isotropy in the horizontal plane. G_{vc}, E_{vc}, $E_{flex1vc}$ and $E_{flex2vc}$ were calculated from Eqs. 2 - 6, where the subscript ‘*vc*’ represents vertical cut sample. As might be expected, G_{vc} and E_{vc} values computed from the resonant frequencies in torsion and in longitudinal vibration were equal to the input values of G_v and E_v respectively. However, given the contribution of shear stiffness to the flexural stiffness of a 2:1 aspect ratio specimen identified (above) for the isotropic 'specimen', the values of Young's modulus calculated from the resonant frequency under flexural excitation (E_{flexvc}) differed from those in longitudinal excitation, and depended both on the input value E_v and G_v, as shown in Figure 6.

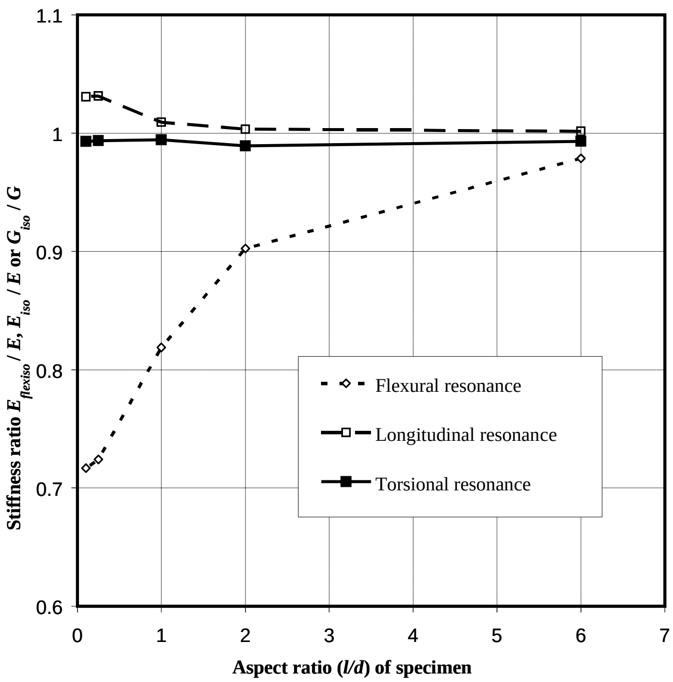


Fig. 5. Ratio of calculated stiffness to the defined stiffness for an isotropic material with slenderness ratio.

A third set of numerical analyses were performed to model a horizontally cut ('hc') specimen where the plane of isotropy is perpendicular during resonant column testing, as shown in Fig. 4. Similar cross-anisotropic parameters to those used in the second set of analyses were chosen. G_{hc}, E_{hc}, $E_{flex1hc}$ and $E_{flex2hc}$ were calculated from Equations 2 - 6, where the subscript '*hc*' represents horizontally cut specimen.

From these analyses it could be seen that, irrespective of the values taken by all other parameters, E_{hc} was equal to E_h. $E_{flex1hc}$ and $E_{flex2hc}$ were equal to 0.9-0.93 E_h for the analyses that were conducted, but (by comparison with the effects shown in Fig. 6) shear stiffness in the plane of flexure may have a significant effect

G_{hc} was not equal to G_h or G_v. This stems from the fact that in the plane of torsional shear vibration, the shear properties of the specimen are not uniform, but vary from G_h to G_v. The value of G_{hc} inferred from the torsional resonant frequency results from a combination of shear moduli G_h and G_v, and from our analyses is approximately equal to the square root of the product of the shear moduli in the vertical and horizontal planes (Figure 7).

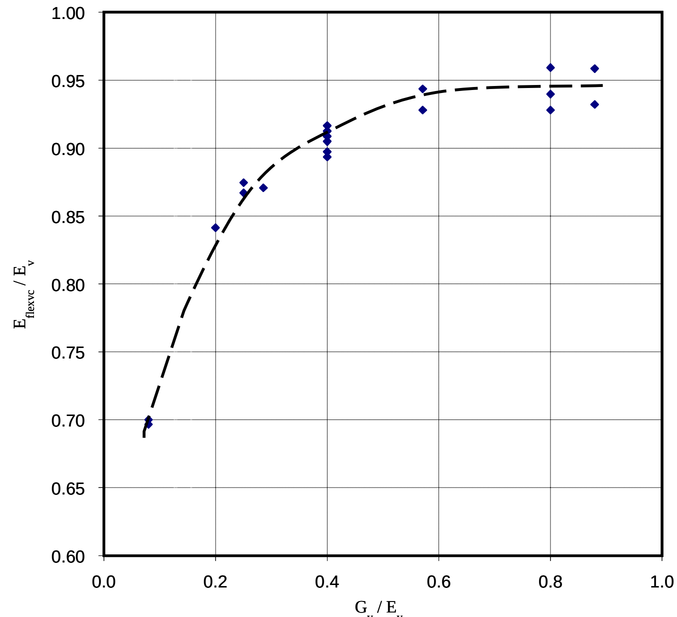


Fig. 6. Effect of vertical shear modulus on Young's modulus inferred from flexural vibration.

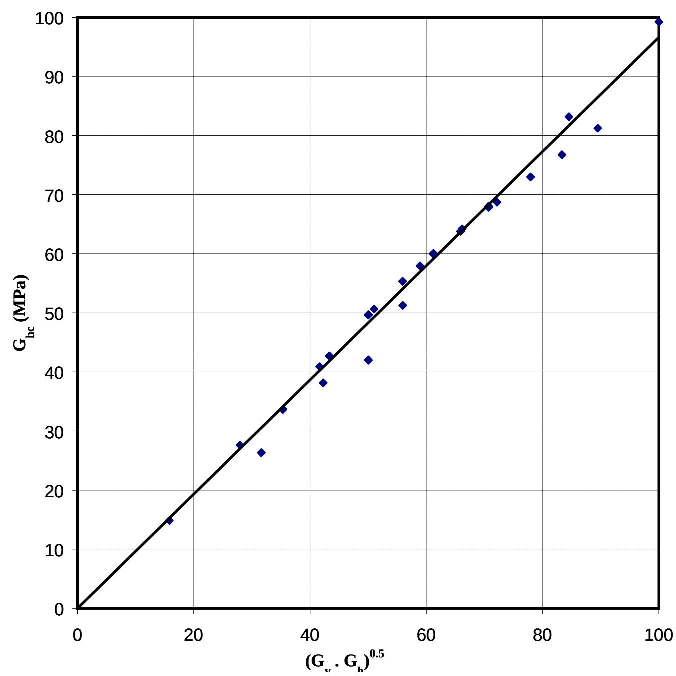


Fig. 7. Relationship between Shear modulus determined from horizontally cut specimens, and shear moduli values.

These results suggest that, at least in principle, resonant column testing of both vertical and horizontal specimens cut from a transversely isotropic soil can allow the degree of anisotropy to be quantified. G_v can be determined from torsional resonance of vertically cut specimens, as usual. E_v can be determined from longitudinal resonance, and from flexural resonance and (with correction) Cascante's equation. E_h can be determined from horizontally cut specimens. The shear stiffness in the horizontal plane (G_h) can be deduced from G_v and the measured value of G_{hc}, since

$$G_h = G_{hc}^2 / G_v \qquad (7)$$

In practice, values of shear moduli are likely to be more reliable than values of *E*, since the former are measured in torsion, and will be little affected by bedding effects, which may reduce measured values of Young's modulus for stiffer materials.

CONCLUSIONS

The standard (torsional) resonant column test determines the independent shear modulus in the vertical plane, G_v, when carried out on a conventional vertically-cut specimen. This value would equal one-third of the Young's modulus (*E*) for an isotropic undrained specimen, but the ratios G_v / E_v and G_v / E_h will vary if the material is transversely isotropic.

For an isotropic material the use of flexural vibration to derive *E* using Eq. 4 introduce errors of about 10 % when the aspect ratio (l/*d*) of the specimen is (as usual) around 2. This error in E_{flex} occurs because no shear deformation is included in deriving Eqs. 4 – 5. However, although longitudinal vibration (and the use of Eq. 6) may provide a more accurate method for calculating *E*, more complex apparatus is required.

Numerical analyses of the resonant column apparatus with transversely isotropic soil shows that the degree of anisotropy of a soil can only be deduced if both vertical and horizontal cut specimens are tested. When this is done a maximum of four independent variables can be obtained with reasonable accuracy from flexural (or longitudinal) and torsional vibration of vertically and horizontally cut specimens. Values of E_v and E_h can be measured using longitudinal vibration of vertical and horizontal cut specimens respectively. Flexural vibration can also be used to calculate these stiffnesses, although the effect of shear stiffness in the plane of distortion needs to be borne in mind. Shear stiffness in the vertical plane can be obtained using torsional vibration of a vertical cut specimen. Shear stiffness in the horizontal plane can determined using Equation (7), once the values of G_v and G_{hc} have been obtained.

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