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Frequency-Domain Turbo Equalization for MIMO Underwater Acoustic Communications

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Abstract— **This paper investigates a low-complexity frequency-domain turbo equalization (FDTE) based on linear minimum mean square error (LMMSE) criterion for single-carrier (SC) multiple-input multiple-output (MIMO) underwater acoustic communications (UAC). The receiver incorporates both the equalizer and the decoder which exchange the extrinsic information on the coded bits for each other to implement the iterative detection. The channel impulse responses (CIRs) required in the equalization are estimated in the frequency domain (FD) by inserting the well-designed pilot blocks which are frequency-orthogonal Chu sequences. The proposed SC-MIMO-FDTE architecture is applied to the fixed-to-fixed underwater data gathered during SPACE08 ocean experiments in October 2008, where multiple transducers and hydrophones are deployed in communication ranges of 200m and 1000m, and the channel bandwidth is 9.765625 kHz. The phase shift keying (PSK) signals are transmitted from multiple transducers in various block sizes. The proposed transceiver has been demonstrated to improve the bit-error-rate (BER) performance significantly by processing the QPSK data blocks with block length of 1024 in 200m and 1000m ranges. The average BERs obtained by turbo detection with 3 iterations can achieve approximately 1.4×10^{-4} for the 200m system and 4.4×10^{-5} for the 1000m system.**

I. Introduction

High data rate with low BER is an important objective for underwater acoustic communications, which is very challenging due to limited transmission bandwidth, severe frequency selective fading, and time-varying Doppler shift [1]-[4]. Recently, MIMO technology combined with multicarrier transmission [5]-[7] and single carrier transmission with frequency domain equalization (SC-FDE) [8], [9] have been investigated to increase the information data rates and combat the severe inter-symbol interference (ISI). Furthermore, various channel coding schemes, such as space-time trellis codes (STTC), layered space-time codes (LSTC), turbo codes [10], and low-density parity-check codes [11], have been applied to the UAC to lower the BER and increase the reliability of the communications over shallow water in medium range underwater channels. These aforementioned algorithms have been tested by undersea experimental data and displayed very good performance in various communication tasks.

In this paper, frequency-domain turbo LMMSE equalization is employed for SC-MIMO underwater communications. The concept of LMMSE turbo equalization was provided in [12] and [13], which was developed in the time domain for single-input single-output (SISO) systems with multi-level modulations. The time-domain turbo equalization (TDTE) was further extended to MIMO frequency selective channels in [14]. The TDTE has been applied to radio frequency (RF) wireless communications to improve the performance of data detection. The frequency-domain turbo equalization (FDTE) was investigated in [15]-[17], and these work are focusing on SISO systems. This paper considers to extend the SISO FDTE scheme [15] and modify the RF MIMO FDTE scheme [18],[19] to fit into MIMO underwater acoustic communications. The proposed algorithm will be employed to process real-world undersea experimental data. In this turbo detection, the channel equalization and channel decoding are iteratively applied on the same block of data, and the extrinsic information on the coded bits are exchanged between the two modules. Different from the traditional LMMSE FDE, FDTE takes into account the *a-priori* information of the transmitted symbols and output the estimation of the extrinsic information for the coded bits which are provided to the decoder as *a-priori* information. The decoder which is implemented by maximum-*a-posterior* (MAP) criterion will generate the extrinsic bit information that is fed back to the equalizer. This processing will be repeated depending on the number of iterations, and the BER performance will be enhanced steadily. The channel estimation is performed in the frequency domain by inserting the frequency-orthogonal Chu sequence at the front of each block. Experiment results, which are obtained by processing QPSK data blocks of 200m and 1000m systems collected during SPACE08, indicate that the average BER performance for both systems can achieve 10^{-4} level for 200m range and 10^{-5} for 1000m range by using the FDTE with 3 iterations when the channel bandwidth is 9.765625 kHz and the carrier frequency is 13 kHz. As opposed to other non-iterative detection methods like Viterbi hard decision (HD) and Viterbi soft decision (SD), the FDTE outperforms these methods.

Throughout the paper, we use $[\cdot]^T$, $[\cdot]^H$, and $(\cdot)^{-1}$ to denote the matrix transpose, Hermitian transpose, and inverse, respectively.

II. Transceiver Structure

We consider a MIMO underwater acoustic communication system with Q transmit transducers and P receive hydrophones. The structure of the transceiver is shown in Fig.1. At each transmitter, a bitstream is coded by a convolutional encoder with coding rate R, and the coded bits are permuted by a random interleaver to generate a bit sequence which is assumed to be independent. This assumption is guaranteed by the random interleaver. The interleaved coded bits are then

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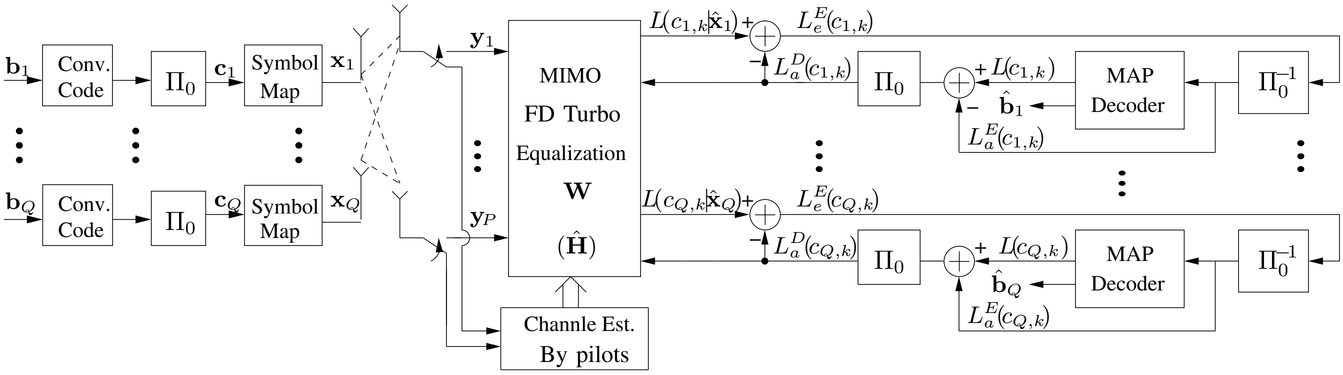

Fig. 1. Structure of transceiver for single-carrier MIMO underwater communications

mapped into 2^M-ary symbols according to a symbol alphabet set $\mathcal{S} = \{\alpha_1, \cdots, \alpha_{2^M}\}$, where α_m has unit average power, which is mapped by the bit pattern $[d_{m,1} \cdots d_{m,M}]$. The modulated symbols are grouped into blocks with length of N, and N_{zp} zeros are appended at the end of each data block in order to avoid inter-block interference (IBI) and make frequency-domain method applicable at receivers. The zero-padded data blocks are transmitted from all transducers simultaneously at the same carrier frequency over the underwater acoustic channels.

The received signals measured by hydrophones are synchronized first and then demodulated to baseband signals. Overlap-add operations are conducted on the received blocks, and the tasks of channel estimation and equalization are all performed in the frequency domain. The estimated channels obtained by pilot blocks are employed in the FDTE which generates *a-posteriori* information $L(c_{q,k}|\hat{\mathbf{x}}_q)$ corresponding to the coded bits $c_{q,k}$ based on the estimated symbols $\hat{\mathbf{x}}_q$, where $q = 1, \cdots, Q$, and $k = 1, \cdots, MN$. The information $L(c_{q,k}|\hat{\mathbf{x}}_q)$ is defined as the log-likelihood ratio (LLR) as

$$L(c_{q,k}|\hat{\mathbf{x}}_q) = \log \frac{P_r\big(L(c_{q,k}=0|\hat{\mathbf{x}}_q)\big)}{P_r\big(L(c_{q,k}=1|\hat{\mathbf{x}}_q)\big)}. \quad (1)$$

The extrinsic information $L_e^E(c_{q,k})$ provided by the equalizer can be gleaned by subtracting the *a-priori* information $L_a^D(c_{q,k})$ transferred by the decoder. The deinterleaved extrinsic information will be provided to the decoder as the *a-priori* information to generate the soft bit information $L(c_{q,k})$. The decoder is implemented in MAP criterion by the BCJR algorithm proposed in [20]. The extrinsic information gleaned by the decoder will be interleaved to feed back to the equalizer as the *a-priori* information. This iterative detection scheme is performed independently for each data stream, but the *a-priori* information from all MAP decoders will be combined to equalize the received blocks in the equalizer.

III. Frequency-domain Turbo Equalization and Channel Estimation

We use $\mathbf{x}_q = [x_{q,1} \cdots x_{q,N}]^T$ to denote the data block transmitted by the q-th transducer, where $q = 1, \cdots, Q$, and $x_{q,i} \in \mathcal{S}$ corresponds to the interleaved coded bits $[c_{q,k} \cdots c_{q,k+M-1}]$, $k = M(i-1)+1$. We define $\mathbf{F}_N$ to be the DFT matrix of size $N \times N$, i.e., its (i,j)-th element is given by $\exp\left(\frac{-j2\pi(i-1)(j-1)}{N}\right)$. Then the frequency-domain representation of $\mathbf{x}_q$ can be written as $\mathbf{X}_q = \mathbf{F}_N \mathbf{x}_q$. In the same way, $\mathbf{y}_p$ and $\mathbf{Y}_p$ denote the received data block and its frequency domain representation at the p-th hydrophone, respectively. Hence, we have the system model expressed by frequency tones as follows [8]:

$$\mathbf{Y}_n = \sum_{q=1}^{Q} \mathbf{H}_{q,n} X_{q,n} + \mathbf{V}_n, \quad n = 1, \cdots, N \quad (2)$$

where n represents the frequency tone, $X_{q,n}$ is the n-th element of $\mathbf{X}_q$, $\mathbf{Y}_n = [Y_{1,n}, \cdots, Y_{P,n}]^T$ is the n-th tone of the received block on the P hydrophones, $\mathbf{V}_n = [V_{1,n}, \cdots, V_{P,n}]^T$ is the frequency-domain representation of white Gaussian noise, and $\mathbf{H}_{q,n} = [\lambda_{1,q} H_{1,q;n}, \cdots, \lambda_{P,q} H_{P,q;n}]^T$ is the channel frequency responses on the n-th tone for the q-th transducer. Here, $\lambda_{p,q} = \frac{1}{N}\sum_{i=1}^{N} \exp(j2\pi f_{p,q} iT + \theta_{p,q})$ in which $f_{p,q}$ is the Doppler drift for the (p,q)-th subchannel, $\theta_{p,q}$ is the initial phase error, and T is the symbol period [8].

Before performing turbo equalization, the mean and variance of $x_{q,i}$ should be calculated based on the *a-priori* information $L_a^D(c_{q,k})$.

$$\begin{aligned} \mu_{q,i} &= \mathrm{E}[x_{q,i}] = \sum_{\alpha_m \in \mathcal{S}} \alpha_m \cdot P_r(x_{q,i} = \alpha_m) \\ &= \sum_{\alpha_m \in \mathcal{S}} \alpha_m \prod_{k=M(i-1)+1}^{Mi} P_r(c_{q,k} = d_{m,(k \bmod M)}) \end{aligned} \quad (3)$$

$$\nu_{q,i} = \sum_{\alpha_m \in \mathcal{S}} |\alpha_m|^2 P_r(x_{q,i} = \alpha_m) - |\mu_{q,i}|^2 \quad (4)$$

where $d_{q,k} \in \{0,1\}$ is determined by α_m. For example, a quadrature phase shift keying (QPSK) symbol alphabet is selected to map the bit pattern [00, 01, 11, 10] into $\mathcal{S} = [1, j, -1, -j]$ correspondingly. Then the mean of $x_{q,i}$ can be calculated by the following equation:

$$\mu_{q,i} = \frac{1}{2}\Big(\tanh\big(\frac{1}{2}L(c_{q,2(i-1)+1})\big) + \tanh\big(\frac{1}{2}L(c_{q,2i})\big)\Big) + j \cdot \frac{1}{2}\Big(\tanh\big(\frac{1}{2}L(c_{q,2(i-1)+1})\big) - \tanh\big(\frac{1}{2}L(c_{q,2i})\big)\Big) \quad (5)$$

and the variance is given by:

$$\nu_{q,i} = 1 - |\mu_{q,i}|^2. \quad (6)$$

A. Low Complexity FDTE

We extend the FDTE of SISO systems introduced in [15] to the MIMO case. This extended MIMO FDTE is also a modified version of [18]. The equalizer incorporates the soft information of the coded bits to compute the soft symbols $\mu_{q,i}$. Then DFT is applied to convert the time-domain soft symbols to frequency domain which is represented by $\mathbf{S}_q$. The equalizer coefficients are derived by using minimum mean square error (MMSE) criterion which can be expressed as:

$$\mathbf{W}_{q,n} = K_q^{-1}\mathbf{U}_{q,n}, \qquad n = 1, \cdots, N \quad (7)$$

where $\mathbf{U}_{q,n} = \Big(\sum_{q=1}^{Q} \bar{\nu}_q \hat{\mathbf{H}}_{q,n}\hat{\mathbf{H}}_{q,n}^H + \sigma^2\mathbf{I}_P\Big)^{-1}\hat{\mathbf{H}}_{q,n}$, $\bar{\nu}_q = \frac{1}{N}\sum_{i=1}^{i=N-1}\nu_{q,i}$ and $K_q = \big(1 + \frac{1-\bar{\nu}_q}{N}\sum_{i=1}^{N}\mathbf{U}_{q,n}^H\hat{\mathbf{H}}_{q,n}\big)$. σ^2 is the average noise power, and $\mathbf{I}_P$ is the unit matrix with the size of $P \times P$.

Apply the coefficients to the received blocks in the frequency domain and obtain the soft cancelation signal given by

$$Z_{q,n} = \mathbf{W}_{q,n}^H\Big[\mathbf{Y}_n - \sum_{q=1}^{Q}\hat{\mathbf{H}}_{q,n}\mathbf{S}_{q,n}\Big]. \quad (8)$$

Then the equalized symbol for the i-th data of the q-th branch can be expressed as:

$$\hat{x}_{q,i} = \frac{1}{N}\mathbf{F}_N^H \cdot \mathbf{Z}_q + \frac{\mu_{q,i}}{N}\sum_{n=1}^{N}\mathbf{W}_{q,n}^H \cdot \hat{\mathbf{H}}_{q,n} \quad (9)$$

with $\mathbf{Z}_q = \big[Z_{q,1}, \cdots, Z_{q,N}\big]^T$.

B. Coded bit extrinsic LLR computation

An assumption is made that the equalized symbol is subject to Gaussian distribution given the transmitted symbol, which is represented by:

$$\hat{x}_{q,i} = \rho_q x_{q,i} + \eta_q \quad (10)$$

where η_q is $\mathcal{N}(0, \sigma_\omega^2)$. The conditional probability density function (pdf) $P(\hat{x}_{q,i}|x_{q,i} = \alpha_m)$ is represented by:

$$P(\hat{x}_{q,i}|x_{q,i} = \alpha_m) = \frac{1}{\pi\sigma_\omega^2}\exp\Big(-\frac{|\hat{x}_{q,i} - \rho_q\alpha_m|^2}{\sigma_\omega^2}\Big). \quad (11)$$

The extrinsic LLR for the coded bit can be calculated by:

$$L_e^E(c_{q,k}) \quad (12)$$

$$= \frac{\sum_{\alpha_m: d_{m,k\mathrm{mod}M}=0} P(\hat{x}_{q,i}|\alpha_m)\prod_{\forall k': k' \neq k} P_r(c_{q,k'} = d_{m,k'\,\mathrm{mod}M})}{\sum_{\alpha_m: d_{m,k\mathrm{mod}M}=1} P(\hat{x}_{q,i}|\alpha_m)\prod_{\forall k': k' \neq k} P_r(c_{q,k'} = d_{m,k'\,\mathrm{mod}M})}. \quad (13)$$

C. Frequency-domain channel estimation

The time-varying channels are estimated in the frequency domain by inserting a pilot block prior to the data block. The pilot blocks transmitted by all antennas are overlapped in the time domain but orthogonal in the frequency domain. This bandwidth-efficient method can be achieved by repeating and rotating the Chu sequence in designing the pilot blocks for each antenna. With this method, a length-$N_p/Q (N_p/Q \geq L)$ Chu sequence is generated as the basic training sequence denoted by $\mathbf{s_t}$. The length-N_p pilot sequence of the first antenna can be constructed by simply duplicating $\mathbf{s_t}$ Q times. As a result, $Q-1$ zeros are inserted between adjacent frequency tones of the basic Chu sequence to shape a comb-like spectrum. Then $Q-1$ phase-shifted sequences can be constructed as the pilot blocks of the other $Q-1$ antennas, whose i-th symbol is multiplied by $e^{j2\pi(i-1)(q-1)/N_P}$ with $q = 2, \cdots, Q$. As a result, the phase-shifted Chu sequences are orthogonal in the QN_p frequency tones. At receivers, the received pilot blocks are converted to the frequency domain to estimated all the QP subchannels by using the method introduced in [21]. The detail is omitted here for brevity.

IV. Experimental Results

Experimental data were collected during the SPACE08 experiment conducted by Woods Hole Oceanographic Institution (WHOI), in October 2008. Six receiver systems are deployed in six different locations away from the transducers: 80-meter southeast and southwest, 200-meter southeast and southwest, and 1000-meter southeast and southwest. We will present detailed results for the 200-meter and 1000-meter systems equipped with 2 transducers in the transmitter and 12 hydrophones in the receiver. Binary information bits were encoded by a rate-1/2 convolutional encoder with coding generators $(1+D+D^2+D^3, 1+D^2+D^3)$. The encoded bits were interleaved in a random fashion, and modulated to PSK symbols with a bandwidth of 9.765625 kHz, and the modulated data symbols were transmitted at a carrier frequency of 13 kHz. The sampling frequency is set to $1e7/256 = 39.0625$ kHz and the baseband received data were sampled with 2 samples/symbol. Hence, $2N$-point FDTE was employed to improve the performance of data processing [22].

In this conference paper, we will focus on the QPSK data blocks with length of N=1024 symbols. The length of pilot symbols N_p = 240 for two transmitters and the MIMO channel impulse responses (CIRs) can be estimated by the frequency-domain channel estimation method introduced in section III. The multipath fading channel lengths were estimated to span 90 symbols for the 200-meter systems and 80 symbols for the 1000-meter systems. The gap (padded zeros) between adjacent data blocks was selected to be 120-symbol interval (12.3 ms duration), which is large enough to avoid IBI. The typical normalized amplitude responses of the channels estimated by pilot blocks corresponding to 2 transmitters for the 1000m system are plotted in Fig. 2 and Fig. 3. The counterparts of the amplitude responses for 200m system are shown in Fig.

4 and Fig. 5. As shown in these figures, more than one peak appeared in the CIRs, and the highest peak is not located at the beginning of the CIRs which indicate that the channels are non-minimum phase. Another observation from these figures is that the CIRs for the 200m system have more power than the ones for 1000m system, which matches the real situation well.

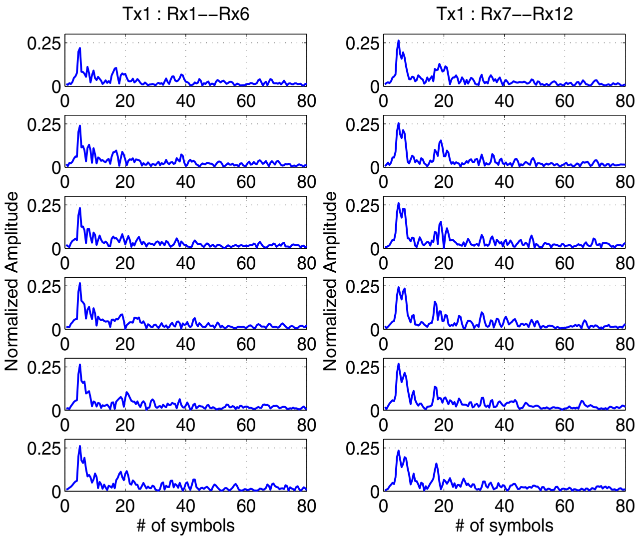

Fig. 2. CIRs of twelve channels for Tx1 in the 1000m southwest system.

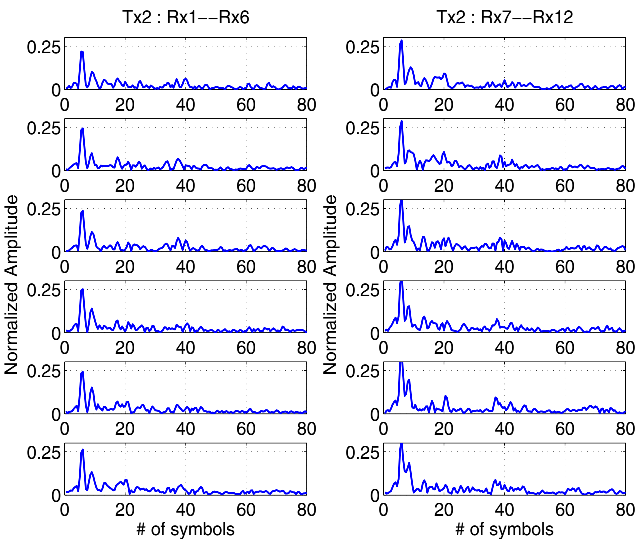

Fig. 3. CIRs of twelve channels for Tx2 in the 1000m southwest system.

In evaluating the BER performance, we compared the non-iterative detection and iterative detection schemes. For non-iterative detections, the equalizer and decoder are independent from each other, which implies that they work separately and no information is fed back from decoder to equalizer. The equalizer part was carried out by MMSE FDE algorithms

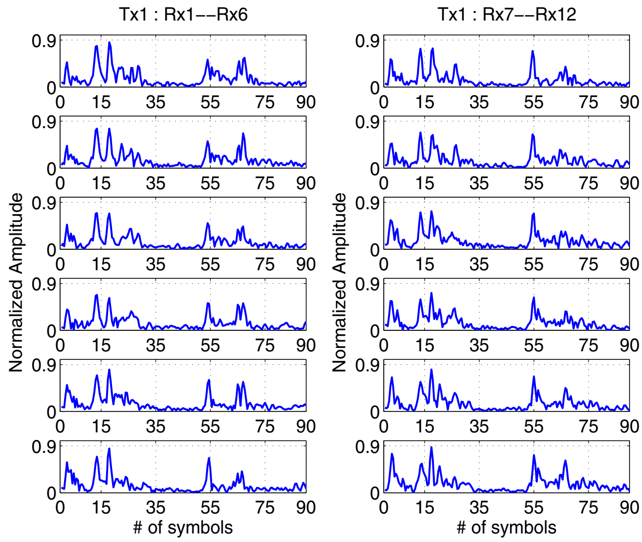

Fig. 4. CIRs of twelve channels for Tx1 in the 200m southeast system.

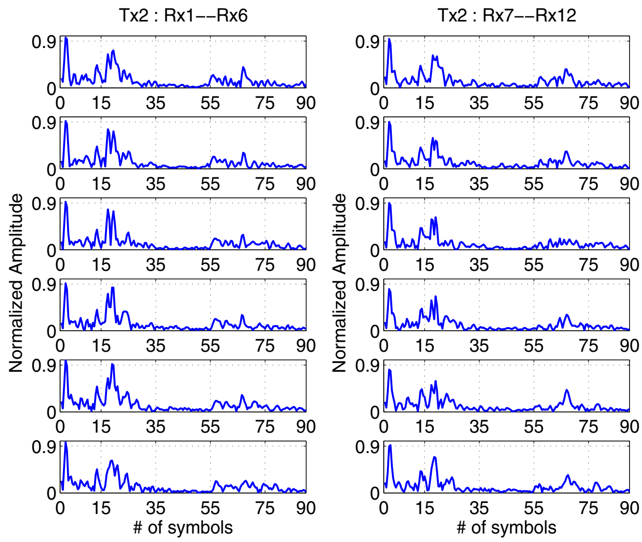

Fig. 5. CIRs of twelve channels for Tx2 in the 200m southeast system.

described in [8], and the equalized symbols are decoded by Viterbi hard-decision (HD) and soft-decision (SD) decoding algorithms to recover the information bits. For iterative detection, the information bits are detected by the cooperation of channel equalization and channel decoding. Table I and Table II list the coded BER of different detection schemes for 200m and 1000m systems with QPSK modulation and $N = 1024$. As listed in the two tables, the FDTE with 1 iteration has a similar average BER performance with the Viterbi SD method, and they both achieve better results than Viterbi HD. The FDTE with 3 iterations provides the best detection performance among these methods, which achieves BER in the order of 10^{-4} for 200m systems and 10^{-5} for 1000m systems. With the increasing of iteration number, the performance tend to be improved steadily.

Table I: BERs of the 200m systems for QPSK, $N = 1024$

Index of frame	Viterbi HD	Viterbi SD	FDTE 1 iter	FDTE 3 iters
1	0.0117	2.0751e-3	1.6798e-3	4.9407e-4
2	6.7194e-3	5.4348e-4	2.4704e-4	0
3	2.2233e-3	2.9644e-4	0	0
4	8.1028e-3	1.4822e-3	2.4704e-4	0
5	0.0178	1.8775e-3	4.9407e-4	0
6	0.0893	0.0344	0.0251	9.3874e-4
7	0.0782	0.0366	0.0328	0
8	0.0710	0.0232	0.0146	0
9	0.134	0.0602	0.0448	0
10	5.7312e-3	3.9526e-4	9.8814e-5	0
Average	0.0425	0.0143	0.0121	1.4328e-4

Table II: BERs of the 1000m systems for QPSK, $N = 1024$

Index of frame	Viterbi HD	Viterbi SD	FDTE 1 iter	FDTE 3 iters
1	9.1403e-3	1.3340e-3	6.9170e-4	2.4704e-4
2	7.9051e-3	2.9644e-4	4.4466e-4	1.9763e-4
3	0.0121	1.3834e-3	3.9526e-4	0
4	2.8656e-3	9.8814e-5	0	0
5	2.4209e-3	1.9763e-4	9.8814e-5	0
6	7.4111e-4	4.9407e-5	4.9407e-5	0
7	2.4209e-3	1.4822e-4	0	0
8	1.0870e-3	4.9407e-5	0	0
9	2.5198e-3	9.8814e-5	0	0
10	4.4466e-4	0	0	0
Average	4.1645e-3	3.6561e-4	1.6798e-4	4.4467e-5

V. CONCLUSION

In this paper, a low-complexity frequency-domain turbo equalization scheme is presented to process the undersea data collected in MIMO communication systems during SPACE08 ocean experiments conducted in October, 2008. The time-varying underwater acoustic MIMO channels were estimated by the frequency orthogonal pilot blocks at the front of each data block in the frequency domain. The estimated channels were employed in the iterative turbo detection. The FDTE is proceeding iteratively with the exchange of coded bit extrinsic information between the channel equalizer and the channel decoder. For channel bandwidth being 9.765625 kHz with carrier frequency of 13 kHz, it has been shown via experimental data that the average bit error rate of two-transducer twelve-hydrophone system with QPSK modulation can achieve 10^{-4} for 200m and 10^{-5} for 1000m communication systems.

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