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SUMMARY In this paper, the horizontally stratified soil layers are simplified as a shearing type multidegree of freedom linear system. The transfer function of the system is determined in accordance with the statistic analysis of the result from the mode superposition method of soil layer earthquake response. By referring to the method of finding the extreme value in probabilistic theory, a response spectrum corresponding to the power spectral density of soil layers is given. At the same time, distributions of maximum response values of acceleration, displacement, shearing strain and shearing stress of the soil layers along the depth are given according to the root mean square procedure.

FOREWORD

In case of approximately horizontal strata the effect of local soil conditions on earthquake ground motion can often be solved by simplifying the soil profile as a shear beam system. At present, there are several kinds of methods of solution, among which the mode superposition method is well-known to engineers. Idriss, Seed and Whiteman have made a comparison between the result of this method and that of the exact solution, and a detailed discussion of its adaptability, error and choosing the number of masses has been carried out. We have analyzed the earthquake response of soil layer in Beijing area by this method. The input motion on bed-rock is acceleration record at Gianan Station in the aftershock of Tangshan earthquake in Hopei province (1976), however, its amplitude and period have been made proper adjustments. In the analysis, the nonlinearity of soil layer is considered by means of the linearization method.

The variation of modulus and damping in soil according to strain amplitude was determined by field and laboratory tests. This method has also been used in analyzing the seismic damages to typical buildings in Beijing Area during Tangshan Earthquake (1976). There seems to have certain coincidence between the result of the analysis and the actural seismic damages. The result obtained from this method and that from visco-elaspoplastic wave propagation method are mostly identical except for slight differences.

However, sometimes it is not necessary to calculate the earthquake response for the whole time process, but to know its maxmum response is enough for earthquake engineering. In this paper, the soil layer is dealt with as a shear type system of lumped masses. In accordance with the statistic analysis of the result from the mode superposition method of soil layer earthquake response and considering the non-linearity of soil layer by modification of the natural period of soil layer, with reference to the charactertics of input seismic excitation, the transfer function of the system is determined. Assuming the input be a stationary Gaussian random process, the output power spectral density of the ground surface or any intermediate soil layer from the known input power spectral density on the basement can found out and by referring to the method of finding extreme value in probabilistic theory, it is not impossible to find out the response spectrum corresponding to the power spectral density. In this paper, this is attained by the adoption of the method proposed by Kaul. As to the distribution of the maximum values of acceleration, displacement, shear strain and shear stress of the soil layer along its depth, it will be given according to the root mean square procedure. It can be seen below that the means adapted in this paper (A method) is simple, and the results thus obtained with the results obtained from the mode superposition in deterministic analysis (B method) and from the visco-elastoplastic wave propagation (C method) are identical.

SOME STATISTICAL RESULTS OBTAINED FROM MODE SUPERPOSITION ANALYSIS

Results from large amount of calculations show that there is a good statistical relation between predominant period of soil layer ($T_0 = \sum 4h_i/V_{si}$) and the natural period of soil layer obtained in the mode superposition analysis. Fig.1 shows statistical results of T_0 and the first three natural periods.

This is the results of calculations carried out by using the acceleration record on the basement rock at Gianan Station in Hopei Province in the aftershock of Tanshan Earthquake in 1976, the maximum amplitude and predominant period of the input accelerogam being adjusted to 100 gal and 0.3 sec. respectively. Same calculations have also been carried out on the earthquake acceleration record of Liaoning Da Shi-qiao (1975) made on the ground surface. Same adjustments as mentioned above have been made on its peak acceleration and predominant peroid. The results obtained are represented by small circles in Fig. 1. It seems that as long as the maximum acceletion and predominant period of the input accelerograms are the same, there would be very little relation between the natural period of the soil layer and the time history of input seismic excitation. We assume the natural period varies primarily with the maximum acceleration of input excitation and predominant period for a given soil layer. Thus two coeffcients ξ_A, ξ_T are introduced in this paper, and the results obtained from Fig.1 are modified according to the maximum acceleration of input seismic excitation actually used and the predominant peroid, i.e.

$$T_j' = T_j \xi_A \xi_T$$

where T_j - natural period obtained from Fig.1;
T_j' - natural period of soil layer after modification

Coefficients ξ_A and ξ_T are mean value obtained from response analysis of a number of typicale soil profile by Qianan acceleration record (1976) while maintaining the input motion with predominant period of 0.3 sec. or maximum acceleration of 100 gal change respectivily the amplitude or period of the acceleration record used in the analysis. Results of calculations show that there is little difference between the modified coefficients ξ_A and ξ_T of the first three models. Values of ξ_A and ξ_T can be obtained froms Fig.2.

From results obtained through analysis of fifty soil profiles, it is found that in spite of the great difference in the soil profile, the parameter of input seismic wave used are not entirely identical, but when the depth of the soil layers are normalized, then the products $\varphi_j(n)$ of mode function of soil profile and their corresponding mode participation ceofficients do not differ greatly, thus their mean values can be adopted. Table (1) shows the average results of $\varphi_j(n)$ and their standard deviation at respective depth.

As to the damping of soil layers, due to the nonuniformity of soil layers, their numerical values of each part are different. In the analysis of mode superposition, the determination of mode damping is necessary. We adopted the mode weighting method proposed by Whitman. Furthermore, we have all added 6% of damping as Idriss had done for the viscous damping of soil layer. With regard to radiant damping, we have not yet found any good wayto consider. Reslts of calculations show that in the range of the parameters we have used, all the mode damping obtained from the above method has little relation with the soil layer profile or the input seismic excitation parameters. Hence, the mean damping ratio of 0.125 has been used for all mode damping.

DETERMINATION OF SOIL LAYER ACCELERATION RESPONSE SPECTRUM BY PROBABILISTIC METHOD

Based on the research of strong earthquakes motion by Knai, H. Tajimi used the following functions to express spectral density of ground motion.

$$S_{\ddot{x}_g}(\xi_g, \omega_g, \omega) = \frac{1 + 4\xi_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\xi_g^2(\omega/\omega_g)^2} S_0$$

Actually, this is considering the soil layer as a single degree of freedom system, whereas in this paper, the soil is regarded as a shear type linear system of multi-masses. But considering the nonlinearity of soil layer according to method mentioned above. If the input of the seismic motion is considered as stationary random process, it would not be difficult to obtain a formula similar to that of H. Tajimi. Then the generalized coordinates for the jth mode of soil layer will satisfy the motion equation;

$$\ddot{q}_j + 2\xi_j\omega_j\dot{q}_j + \omega_j^2 q_j = 2\xi_j\omega_j\dot{x}_j + \omega_j^2 x_g \qquad (1)$$

where: $\ddot{q}_j$, $\dot{q}_j$ and q_j are the generalized coordinates of the absolute acceleration, velocity and displacement for the j th mode respectively; $\dot{x}_j$, x_j are the seismic velocity and displacement of the input motion on basement respectively; ω_j and ξ_j are the circular frequency and model damping ratio for the jth mode respectively.

Absolute acceleration of nth soil layer may be expressed as:

$$\ddot{X}_n(t) = \sum_j r_j Z_j(n)\ddot{q}_j(t) = \sum_j \varphi_j(n)\ddot{q}_j(t) \qquad (2)$$

where: r_j - mode participation coefficient;
Z_j - mode function;
φ_j - product of mode function and mode participation coeffcient obtained afore.

Its corresponding auto-correlation function is:

$$R_{\ddot{x}_n}(\tau) = E[\ddot{X}_n(t)\ddot{X}_n(t+\tau)]$$

$$= E\Big[\sum_j\sum_k \varphi_j(n)\ddot{q}_j(t)\varphi_k(n)\ddot{q}_k(t+\tau)\Big] \qquad (3)$$

By means of the two pairs of Fourier Transform Algorithmic relations, i. e. relations between spectral density and autocorrelation function as well as the transfer function of time domain and that of frequency domain, it would not be difficult to obtain the relational expression of input and output spectral density of the linear system of lumped masses:

$$S_{\ddot{x}_n}(P) = \sum_j\sum_k\Big[\varphi_j(n)\varphi_k(n)H_j(iP)H_k(-iP)S_{\ddot{x}}(P)\Big] \qquad (4)$$

the damping is not large and the different between periods for various modes is comparatively large, thus correlated terms in formula (4) may be neglected, then the formula may be simplified as:

$$S_{\ddot{x}_n}(P) \approx \sum_j |H_j(iP)|^2 \varphi_j^2(n) S_{\ddot{x}}(P) \qquad (5)$$

where: $S_{\ddot{x}_n}$ (P) --acceleration response power spectral density of the nth soil layer;
$S_{\ddot{x}}$ (P) --acceleration power spectral density of input motion on basement rock;

$$|H_j(iP)|^2 = \frac{\omega_j^4 + 4\xi^2 P^2\omega_j^2}{(\omega_j^2 - P^2)^2 + 4\xi^2 P^2 \omega_j^2}$$

where: P -- the circular frequency of input wave;

ω_j -- circular frequency of soil layer after modification.

As it is well known, in the determinante analysis, the value of seismic response spectral value at given period is the maximum response of single-degree-of-freedom system with the same period. In case of random process, it may be consider to be such a value, on the probability distribution of the peak response of single-degree-of-freedom system, that correspond to the exceedance probability r. Many scholars have made studies in this respect. Kaul supposed that the input was a stationary zero-mean Gauss random process, and on the basis of results obtained by Rice, cartwright and Longnet-Hijgins, he set up the relationship between power spectral density and acceleration response spectrum, and took the exceedance probability r= 0.15 according to general seismic record analysis. This result will be adapted in this paper to analyze earthquake response of soil layer.

The above mentioned formula(5) has already given the spectral density $S_{\ddot{x}n}$, then the response of the single degree of freedom system subjected to excitation of $S_{\ddot{x}n}$ may be expressed as:

$$S(\omega,p)=H(p,\omega)\sum_j|H_j(ip)|^2\varphi_j^2(n)S_{\ddot{x}}(p) \qquad (6)$$

Where: $H(p,\omega)=\frac{\omega^4+4\xi^2p^2\omega^2}{(\omega^2-p^2)+4\xi^2p^2\omega^2}$

ω, ξ -- circular frequency and damping ratio of single-degree-of-freedom system.

Since it is supposed as a stationary zero-mean Gauss random process, so the probability density function of maximum value of single-degree-of-freedom system response may be expressed as:

$$p(\eta)=\frac{1}{\sqrt{2\pi}}\left[\varepsilon e^{-\eta^2/2\varepsilon^2}+(1-\xi^2)\eta e^{-\eta^2/2}\int_{-\infty}^{[\eta(1-\xi^2)^{1/2}]/\xi} e^{-\ddot{x}^2/2}\,d\ddot{x}\right] \qquad (7)$$

Where: $\varepsilon=\frac{m_0m_4-m_2^2}{m_0m_4}$; $\eta=\ddot{x}/m_0^{1/2}$;

$\ddot{x}$ -- absolute acceleration response of single-degree-of-freedom system;

$$m_k=\int_{-\infty}^{+\infty}p^kS(p,\omega)dp \qquad k=0,2,4$$

If the number of occurence of peak response in time T of response process is large enough, after some simplification, it is easy to find the maximum η value corresponding to exceedance probability r:

$$\eta_{max}=\left\{-2\ln\left[-\pi(m_0/m_2)^{1/2}\ln(1-r)/T\right]\right\}^{1/2} \qquad (8)$$

Apparently, the acceleration response spectrum may be expressed as:

$$A(\omega)=\eta_{max}m_0^{\frac{1}{2}} \qquad (9)$$

We have made some calculations on soil layer profiles according to formula (6) and (9), and compared their results with those obtained from mode method. Fig.(3) shows one of the examples. At the same time, the results obtained by visco-elastoplastic wave propagation method are also given in this illustration. It seems that the results of analyses made by the three methods are identical.

GROUND MOTION VARIES WITH THE DEPTH

Now we will find the variation of maximum values of acceleration, displacement, shear strain and shear stress of soil layer along its depth. In this case, input spectral density on basement may be used to find the acceleration response spactrum on bed-rock. Naturally, its response spectrum should correspond to the damping of the soil layer. With the acceleration response spectrum available, the maximum value of accelertion, displacement, shear strain and shear stress could be obtained by root mean square procedure. Maximum acceleration of any soil layer n may be calculated by the following equation:

$$a_{max}(n)=\sqrt{\sum_{j=1}^{s}[\varphi_j(n)A_b(\omega_j)]^2} \qquad (10)$$

Where: $A_b(W_j)$ -- acceleration response spectrum bed-rock. In the same way, maximum values of displacement, shear strain, and shear stress of the nth soil layer may be obtained by the following equation:

$$d_{max}(n)=\sqrt{\sum_{j=1}^{s}[\frac{1}{\omega_j^2}\varphi_j(n)A_b(\omega_j)]^2} \qquad (11)$$

$$\gamma_{max}(n)=\frac{d_{max}(n+1)-d_{max}(n)}{h_n} \qquad (12)$$

$$\tau_{max}(n)=G(n)\gamma_{max}(n) \qquad (13)$$

Where: h_n -- thickness of nth soil layer;

$G(n)$ -- shear modulus corresponding to shear strain level ε_{max}, determined by test curve.

Calculated results of maximum acceleration of some soil layers are shown in Fig. (4). Fig. (5) shows an example of a soil layer profile. There seems to have coincidence with the results obtained from mode superposition in determinante analyses.

CONCLUSION:

The analytical result shows that there is quite close coincidence between this result obtained in this paper and that of "exact" analysis.

The power spectral density eqation suggested by Kanai and Tajimi is widely used in the simulation of earthquake. Liu(1969) and others analyzed spectral density of a dozen or more strong earthquake records. The result reveals that there are two obvious peaks in some spectral curves, and suggests to express the transfer characteristics by single-degree-of-freedom or by two degrees of freedom. However, the question of how to determine adequately the soil layer transfer parameters of the site has not yet been satisfactoritily solved. The results provided in this paper may be used as reference for work in this respect.

Whitman has pointed out the limitation of mode superposition method. The procedure used in this article is based on mode superposition method, so it has unavoidably the same shortcomings as those of mode superposition method. Result of calculations by Liu(1978) shows that very soft subsoil layer has obvious effect on natural period of soil layer only when its thickness is greater than four meters.We deem it necessary to take this point into consideration when the method suggested in this paper is being used.

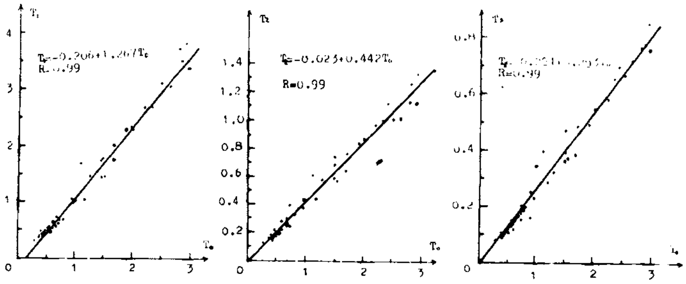


Fig.1. First three natural Period of Soil Layers

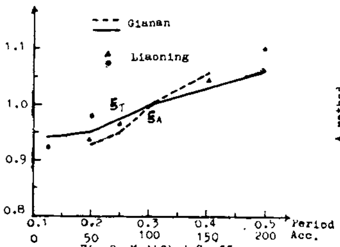
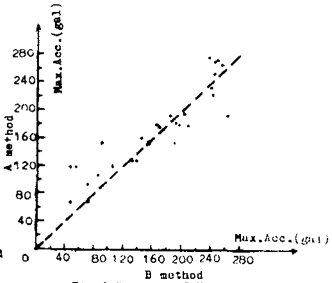


Fig.2. Modified Coeff.



B method
Fig.4.Compare of Max. Acc.

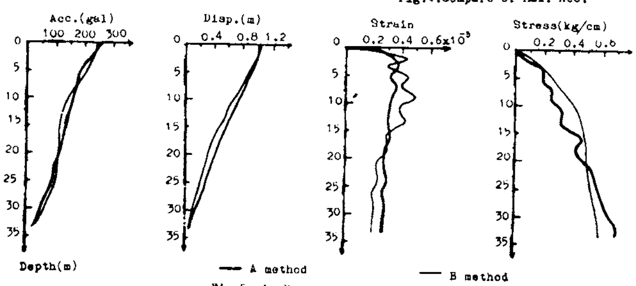


Fig.5. An Example of A Soil Layer Progile

Table 1. First Three Model and Standard Diviation

Deth Radio (Hi/Hmax)	1st model		2nd model		3rd model	
	$\varphi_1(n)$	σ_1*	$\varphi_2(n)$	σ_2	$\varphi_3(n)$	σ_3
0	1.481	0.12	-0.761	0.16	0.441	0.16
0.05	1.444	0.11	-0.652	0.14	0.290	0.08
0.10	1.362	0.06	-0.437	0.09	0.029	0.13
0.15	1.269	0.05	-0.227	0.13	-0.157	0.14
0.20	1.191	0.05	-0.073	0.14	-0.250	0.11
0.25	1.099	0.06	0.082	0.14	-0.283	0.06
0.30	1.013	0.08	0.207	0.13	-0.255	0.07
0.35	0.932	0.09	0.292	0.12	-0.192	0.08
0.40	0.852	0.11	0.354	0.11	-0.111	0.10
0.45	0.774	0.12	0.392	0.10	-0.021	0.11
0.50	0.690	0.12	0.413	0.09	0.073	0.11
0.55	0.618	0.13	0.414	0.07	0.142	0.10
0.60	0.548	0.13	0.402	0.06	0.193	0.09
0.65	0.478	0.12	0.378	0.05	0.229	0.07
0.70	0.402	0.11	0.338	0.05	0.243	0.06
0.75	0.330	0.11	0.293	0.05	0.237	0.06
0.80	0.270	0.09	0.245	0.05	0.214	0.06
0.85	0.200	0.07	0.187	0.05	0.173	0.05
0.90	0.129	0.05	0.125	0.04	0.120	0.04
0.95	0.067	0.04	0.065	0.03	0.063	0.03
1.00	0	0	0	0	0	0

* Standard Diviation

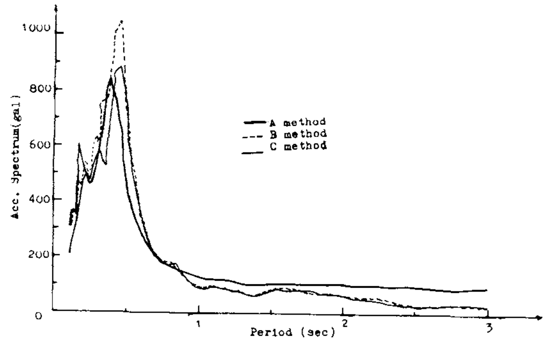


Fig.3.Acc. Response Spectrum

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