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DISTRIBUTED POWER CONTROL OF CELLULAR NETWORKS IN THE PRESENCE OF RAYLEIGH FADING CHANNEL[1]

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Abstract—**A novel distributed power control (DPC) scheme for cellular networks in the presence of radio channel uncertainties such as path loss, shadowing, and Rayleigh fading is presented. Since these uncertainties can attenuate the received signal strength and can cause variations in the received Signal-to-Interference ratio (SIR), the proposed DPC scheme maintains a target SIR at the receiver provided the uncertainty is slowly varying with time. The DPC estimates the time varying nature of the channel quickly and uses the information to arrive at a suitable transmitter power value. Further, the standard assumption of a constant interference during a link's power update used in other works in the literature is relaxed.**

A CDMA-based cellular network environment is used to compare the proposed scheme with earlier approaches. The results show that our DPC scheme can converge faster than others by adapting to the channel variations. The proposed DPC scheme can render outage probability of 5 to 30% in the presence of uncertainties compared with other schemes of 50 to 90% while consuming low power per active mobile user. In other words, the proposed DPC scheme allows significant increase in network capacity while consuming low power values even when the channel is uncertain.

Keywords—Wireless Cellular Networks, Fading Channels, Distributed Power Control, Signal-to-Interference Ratio (SIR), Outage Probability.

I. INTRODUCTION

The objectives of transmitter power control include minimizing power consumption while increasing the network capacity, and prolonging the battery life of mobile units, by managing mutual interference so that each mobile unit can meet its signal-to-interference ratio (SIR) and other quality of service (QoS) requirements. Early work on power control [1] [2] focused on balancing the signal-to-interference (SIR) ratios on all radio links using centralized power control. Later, distributed SIR–balancing schemes [3][4] were developed to maintain quality of service requirements of each link. In particular, Foschini and Miljanic [5] proposed a more general and realistic model in which a positive receiver noise and a user-specific target SIR were taken into account. This distributed algorithm was proven to converge either synchronously [5] or asynchronously [6] to a fixed point of a feasible system. Based on this, Grandhi and Zander suggested a distributed constrained power control (DCPC) algorithm [7], in which a ceiling was imposed on the transmit power of each user. Another distributed algorithm was proposed by Bambos et al. [8], which aimed at protecting the active links from performance degradation when new users try to access the channel. In [9] and [10], second-order power control (CSOPC), state space-based distributed power control design (SSCD) and its optimization are respectively proposed to efficiently manage the user interferences so that the network capacity can be increased.

However, the radio channel ultimately places fundamental limitations on wireless communication systems since the path between the transmitter and the receiver can vary from simple line-of-sight to one that is severely obstructed by buildings, mountains, and foliage. Unlike wired networks, radio channel uncertainties in a wireless network for instance path loss, shadowing, Rayleigh fading can attenuate the power of the transmitted signal at the receiver and thus cause variations in the received SIR and therefore degrading the performance of any distributed power control (DPC) scheme. Low SIR at the receiver will result in high bit error rate (BER), which is unsatisfactory for 3G systems, where voice and video will be typically transmitted.

Though the earlier DPC schemes may be accurate in arriving at a power value, they all neglect the changes observed in the

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radio channel. In fact, they all assume that: 1) path loss component is only present, 2) no other channel uncertainty exists and 3) the mutual interference among users is held constant during power update for each user. Moreover, the slower rate of convergence of these algorithms and the associated power updates is of an issue in a highly dynamic wireless environment as shown in this paper. The proposed work overcomes these limitations.

In this paper, we propose a novel DPC scheme for the next generation wireless networks in the presence of radio channel uncertainties. The rate of convergence of the proposed DPC scheme to attain a target SIR is faster compared with the existing schemes. The proposed algorithm estimates the time varying channel accurately, and uses this information in the power update so that a desired SIR is maintained at the receiver provided the channel variation is slow compared to its estimation. The algorithms being highly distributive in nature doesn't require inter-link communication, centralized computation, and reciprocity assumption as required in a centrally controlled wireless environment. In addition, the proposed DPC scheme converges to any target SIR value in the presence of channel uncertainties. As the necessity of inter-link communication is eliminated, network capacity increases significantly and easy controlled recovery from error events is possible. Also, a comparison of the proposed DPC with certain earlier algorithms is included.

The organization of the paper is as follows. Section II describes wireless channel with uncertainties. In Section III, a suite of DPC schemes is studied and a novel DPC scheme is proposed along with convergence proofs. Section IV presents the simulations whereas Section V carries the conclusions.

II. RADIO CHANNEL WITH UNCERTAINTIES

The radio channel places fundamental limitations on wireless communication systems. Unlike wired channels that are stationary and predictable, radio channels involve many uncertain factors such as path loss, the shadowing, and Rayleigh fading, so they are extremely random and do not offer easy analysis. These channel uncertainties can attenuate the power of the signal at the receiver and thus cause variations in the received SIR and therefore degrading the performance of any DPC scheme. Therefore we focus our effort on these main channel uncertainties before the DPC development. They are discussed next.

A. Path Loss

If only path loss is considered, the power attenuation is taken to follow the inverse fourth power law [11]:

$$g_{ij} = \frac{\bar{g}}{d_{ij}^{n}} \tag{1}$$

where $\bar{g}$ is a constant usually equal to 1 and d_{ij} is the distance between the transmitter of the j^{th} link to the receiver of the i^{th} link and n is the path loss exponent. A number of values for n have been proposed for different propagation environments, depending on the characteristics of the communication medium. A value of n=4 is taken in our simulations, which is commonly used to model path loss in an urban environment. Further, without user mobility, g_{ij} is a constant.

B. Shadowing

High building, mountains and other objects block the wireless signals. Blind area is often formed behind a high building or in the middle of two buildings. This is often seen especially in large urban areas. The term $10^{0.1\zeta}$ is often used to model the attenuation of the shadowing to the received power [12][20], where ζ is assumed to be a Gaussian random variable.

C. Rayleigh Fading

In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. The Rayleigh distribution has a probability density function (pdf) give by [11]

$$p(x) = \begin{cases} \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right) & (0 \le x \le \infty) \\ 0 & (x < 0) \end{cases} \tag{2}$$

where x is a random variable, and σ^2 is known as the fading envelope of the Rayleigh distribution.

Since the channel uncertainties can distort the transmitted signals, therefore, the effect of these uncertainties is represented via a channel loss (gain) factor typically multiples the transmitter power. Then, the channel gain or loss, g, can be expressed as [11,12,20]

$$g = f(d, n, X, \zeta) = d^{-n} \cdot 10^{0.1\zeta} \cdot X^2 \tag{3}$$

where d^{-n} is the effect of path loss, $10^{0.1\zeta}$ corresponds to the effect of shadowing. For Rayleigh fading, it is typical to model the power attenuation as X^2, where X is a random variable with Rayleigh distribution. Typically the channel gain, g, is a function of time.

III. DISTRIBUTED POWER CONTROL (DPC)

"Distributed" implies per individual link. Each receiver of the link measures the interference it is faced and communicates this information to its transmitter. Each link decides autonomously how to adjust its power based on information collected on it exclusively. Therefore, the decision-making is fully distributed at the link level. The overhead due to the feedback control is minimal in DPC compared with its counterpart when centralized operations are used.

The goal of DPC is to maintain a required SIR threshold for each network link while the transmitter power is adjusted so that the least possible power is consumed in the presence of channel uncertainties. Suppose there are $N \in Z_+$ links in the network. Let g_{ij} be the power loss (gain) from the transmitter of the j^{th} link to the receiver of the i^{th} link. It involves the free space loss, multi-path fading, shadowing, and other radio wave propagation effects, as well as the spreading/processing gain of CDMA transmissions. The power attenuation is considered to follow the relationship given in (3). In the presence of such uncertainties, our objective is to propose a novel DPC and to compare its performance with others.

The channel uncertainties will appear in the power loss (gain) coefficient of all transmitter receiver pairs. Calculation of SIR, $R_i(t)$, at the receiver of i^{th} link at the time instant 't' [13], is given by

$$R_i(t) = \frac{g_{ii}(t)P_i(t)}{I_i(t)} = \frac{g_{ii}(t)P_i(t)}{\sum_{j \neq i} g_{ij}(t)P_j(t) + \eta_i(t)} \quad (4)$$

where $i, j \in \{1,2,3,\ldots,n\}$, $I_i(t)$ is the interference, $P_i(t)$ is the link's transmitter power, $P_j(t)$ are the transmitter powers of all other nodes, and $\eta_i(t)>0$ is the thermal noise at its receiver node. For each link i there is a lower SIR threshold γ_i. Therefore, it is clear that

$$\gamma_i \leq R_i(t) \leq \gamma_i^*, \quad (5)$$

for every i = 1,2,3,...n . The lower threshold value for all links can be taken equal to γ for convenience, reflecting a certain QoS the link has to maintain in order to operate properly.

An upper SIR limit is also set, in order to decrease the interference due to its transmitter power at other receiver nodes. In the literature, several DPC schemes have been proposed and it is very difficult to present all of them. The most recent and important work include DPC with active link protection [8], CSOPC [9], SSCD and optimal [10][16][18] and they are discussed next.

A. DPC Scheme with Link Protection [8]

When equation (5) is not satisfied (i.e. $R_i(t) < \gamma_i$), each link independently increases its power if its current SIR is below its target γ_i, and decreases it otherwise using the power update [8][13]

$$P_i(l+1) = \frac{\gamma_i P_i(l)}{R_i(l)} \quad (6)$$

where l=(1,2,3...), $t = lT$ and T is the sampling interval. When $P_i(l+1) > P_{\max}$, the new link is not added. Otherwise when $P_i(l+1) < P_{\min}$ (the minimum power needed to form a link), then $P_i(l+1) = P_{\min}$. The DPC algorithm updates the transmitter powers in steps (time slots) indexed by l = *1, 2, 3*, and so on. This scheme converges to the target SIR for peer-to-peer networks. However, only path loss component is considered.

B. Constrained Second Order Power Control(CSOPC)

In [9], the SIR from equation (4) is expressed as a set of linear equations

$$AP = \mu \quad (7)$$

where A = I-H, P $=(p_i)$, and H = $[h_{ij}]$ is defined as a Q x Q matrix, such that $h_{ij} = \frac{\gamma_i g_{ij}}{g_{ii}}$ for $i \neq j$ and $h_{ij} = 0$ for $i=j$. In addition $\mu = (\gamma_i v_i / g_{ii})$ is a vector of length Q. In addition, since the transmitter power is limited by an upper limit, the following condition is set as

$$0 \leq P \leq \overline{P}, \quad (8)$$

where $\overline{P} = (p_{\max})$ denotes the maximum transmission power level of each mobile. The algorithm assumes that there exists a unique power vector P^*, which would satisfy equation (8). Thus by feasible system, the matrix A is nonsingular and $0 \leq P^* = A^{-1}\mu \leq \overline{P}$. Iterative methods can be executed with local measurements to find the power vector P^*. Through some manipulations, following second-order iterative scheme is obtained as

$$p_i^{(l+1)} = \min\left\{ \overline{p_i}, \max\left\{ 0, w^l \gamma_i p_i^l / R_i^l + (1-w^l)\, p_i^{l-1} \right\} \right\} \quad (9)$$

where

$$w^l = 1 + 1/1.5^n \quad (10)$$

The min/max operators in (9) guarantee the allowable range of transmitter power. Only path loss is considered.

C. State Space ControlDesign (SSCD)

In [10][18], using state space linear system theory [14], the SIR, $R_i(l)$, at the $(l+1)^{th}$ iteration is expressed as

$$R_i(l+1) = R_i(l) + v_i(l), \quad (11)$$

where by definition $R_i(l) = \frac{P_i(l)}{I_i(l)}$, and interference $I_i(l) = \left(\sum_{j \neq i}^{n} P_j(l) \frac{g_{ij}}{g_{ii}} + \frac{\eta_i}{g_{ii}} \right)$, with n is the number of active links. The feedback input $v_i(l)$ for each link should only depend upon the total interference produced by the other users. To maintain the SIR of each link above a desired target γ_i and to eliminate any steady-state errors, the error in SIR, $x_i(l) = R_i(l) - \gamma_i$, has to be minimal by appropriately selecting a power update for the i^{th} link. The closed-loop system is expressed as

$$x_i(l+1) = x_i(l) + v_i(l) \tag{12}$$

The closed-loop system (12) can now be used to select a DPC scheme. First of all, each transmitter-receiver pair now is represented as a closed-loop system (12), which is subsequently used to design a DPC scheme. Moreover, analytically, the convergence can be demonstrated unlike other power control schemes.

Theorem 1[10]: Given the closed-loop SIR system in (12), and if the feedback for the i^{th} transmitter is chosen as $v_i(l) = -k_1 x_i(l) + \mu_i$ with k_1 representing the feedback gain, μ_i is the protection margin, and the power is updated as $P_i(l+1) = P_i(l) + v_i(l) I_i(l)$, the closed loop SIR system for each link is stable, and the actual SIRs converge to their corresponding target values.

Proof: See [16]. ■

Remark: The transmission power is subject to the constraint $P_{\min} \le P_i \le P_{\max}$ where $P_{\min}$ is the minimum needed to transmit, $P_{\max}$ is the maximum allowed power and $P_i(l)$ is the transmission power of the mobile i.

D. Optimal Control Algorithm

Performance criterion, $\sum_{i=1}^{\infty} \left(x_i^T T_i x + v_i^T Q_i v_i \right)$, is used to arrive at an optimal DPC, which can be stated as follows:

Theorem 2 (Optimal Control)[10]: Given the hypothesis presented in the previous theorem for DPC, with the feedback selected as $v_i(l) = -k_i x_i(l) + \mu_i$, where the feedback gains are taken as

$$k_i = (H_i^T S_\infty H_i + T_i)^{-1} H_i^T S_\infty G_i \tag{13}$$

where S_i is the unique positive definite solution of the Algebraic Ricatti Equation (ARE)

$$S_i = G_i^T [S_i - S_i H_i (H_i^T S_i H_i + T_i)^{-1} H_i^T S_i] G + Q_i \tag{14}$$

Then the resulting time invariant closed loop SIR system described by

$$x_i(l+1) = (I_i - k_i) x_i(l) \tag{15}$$

is asymptotically stable.

Proof: See [10]. ■

Only path loss component is considered both in SSCD and optimal DPC schemes.

E. Proposed DPC Algorithm

As indicated earlier, in the previous DPC schemes, only path loss uncertainty is considered. Moreover, the DPC algorithms appear to be slow in convergence and the outage probability is quite high in the presence of channel uncertainties. Consequently, the performance of these DPC schemes fails to render satisfactory performance as shown in our simulations. The proposed work is aimed at demonstrating the performance in the presence of several channel uncertainties.

In the time domain, however, the channel is time-varying when channel uncertainties are considered and therefore $g_{ij}(t)$ is not a constant. In [15], a new DPC algorithm is presented where $g_{ii}(t)$ is treated as a time-varying function due to Raleigh fading by assuming that the interference $I_i(t)$ is held constant. Since this is a strong assumption, in this paper, a novel DPC scheme is given where both $g_{ii}(t)$ and the interference $I_i(t)$ are time-varying, and channel uncertainties are considered for all the mobile users. In other words, in all existing works [8][9][10][18], both $g_{ij}(t)$ and $P_j(t)$ are considered to be held constant, whereas in our work, this assumption is relaxed.

Considering SIR from (4) where the power attenuation $g_{ij}(t)$ is taken to follow the time-varying nature of the channel and differentiating (4) to get

$$R_i(t)' = \frac{(g_{ii}(t)P_i(t))' I(t) - (g_{ii}(t)P_i(t)) I(t)'}{I_i^2(t)} \tag{16}$$

where $R_i(t)'$ is the derivative of $R_i(t)$ and $I_i(t)'$ is the derivative of $I_i(t)$.

To transform the differential equation into the discrete time domain, $x'(t)$ is expressed using Euler's formula as $\frac{x(l+1) - x(l)}{T}$, where T is the sampling interval. Equation (16) can be expressed in discrete time as

$$R_i(l)' = \frac{(g_{ii}(l)P_i(l))' I(l) - (g_{ii}(l)P_i(l))I(l)'}{I_i^2(l)}$$

$$= \frac{1}{I_i^2(l)}[(g_{ii}'(l)P_i(l))I(l) + (g_{ii}(l)P_i'(l))I_i(l)$$

$$-(g_{ii}(l)P_i(l))\left(\sum_{j\neq i} g_{ij}(l)P_j(l) + \eta_i(t)\right)'] \qquad (17)$$

In other words,

$$\frac{R_i(l+1)-R_i(l)}{T}$$

$$= \frac{1}{I_i(l)}\frac{[g_{ii}(l+1)-g_{ii}(l)]}{T}P_i(l) + \frac{1}{I_i(l)}g_{ii}(l)\frac{[P_i(l+1)-P_i(l)]}{T}$$

$$-\frac{g_{ii}(l)P_i(l)}{I_i^2(l)}\sum_{j\neq i}\left(\frac{g_{ij}(l+1)-g_{ij}(l)}{T}P_j(l) + \frac{P_j(l+1)-P_j(l)}{T}g_{ij}(l)\right) \qquad (18)$$

Canceling T on both sides and combining to get

$$R_i(l+1) = \left[\frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\left\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) + [P_j(l+1)-P_j(l)]g_{ij}(l)\right\}}{I_i(l)}\right]$$

$$R_i(l) + g_{ii}(l)\frac{P_i(l+1)}{I_i(l)} \qquad (19)$$

Now, define

$$\alpha_i(l) =$$

$$\frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\left\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) + [P_j(l+1)-P_j(l)]g_{ij}(l)\right\}}{I_i(l)},$$

$$= \frac{\Delta g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\Delta g_{ij}(l)P_j(l) + \Delta P_j(l)g_{ij}(l)}{I_i(l)} \qquad (20)$$

where

$$\beta_i(l) = g_{ii}(l), \qquad (21)$$

and

$$v_i(l) = \frac{P_i(l+1)}{I_i(l)} \qquad (22)$$

Equation (19) can be expressed as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) \qquad (23)$$

with the inclusion of channel noise, equation (19) is written as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) + r_i(l)\omega_i(l) \qquad (24)$$

where $\omega_i(l)$ is the zero mean stationary stochastic channel noise with $r_i(l)$ is its coefficient.

The SIR of each link at time instant, l, is obtained using (24). Carefully observing (24), it is clear that the SIR at the time instant, $l+1$, is a function of channel variation from time instant l to $l+1$. The channel variation is not known beforehand and this makes the DPC scheme development difficult and challenging. Since α is not known, it has to be estimated for DPC development. Note available DPC scheme [1-13,15-19] ignore the channel variations and therefore they render unsatisfactory performance.

Now define $y_i(k) = R_i(k)$, then equation (24) can be expressed as

$$y_i(l+1) = \alpha_i(l)y_i(l) + \beta_i(l)v_i(l) + r_i(l)\omega_i(l) \qquad (25)$$

The DPC development is given in two scenarios.

Case 1: α_i, β_i and r_i are known. In this scenario, one can select feedback as

$$v_i(l) = \beta_i^{-1}(l)[-\alpha_i(l)y_i(l) - r_i(l)\omega_i(l) + \gamma + k_v e_i(l)] \qquad (26)$$

where the error in SIR is defined as $e_i(l) = R_i(l) - \gamma$. This results in

$$e_i(l+1) = k_v e_i(l) \qquad (27)$$

By appropriately selecting k_v via placing the eigenvalues within a unit circle, it is easy to show that the closed-loop SIR system is asymptotically stable [21] in the mean or asymptotically stable, $\lim_{l\to\infty} E\{e_i(l)\} = 0$. This implies that $y_i(l) \to \gamma$.

Case 2: α_i, β_i and r_i are unknown. In this scenario, equation (25) can be expressed as

$$y_i(l+1) = [\alpha_i(l) \quad r_i(l)]\begin{bmatrix} y_i(l) \\ \omega_i(l)\end{bmatrix} + \beta_i(l)v_i(l) \qquad (28)$$

$$= \theta_i^T(l)\psi_i(l) + \beta_i(l)v_i(l)$$

where $\theta_i(l) = [\alpha_i(l) \quad r_i(l)]$ is a vector of unknown parameters, and $\psi_i(l) = \begin{bmatrix} y_i(l) \\ \omega_i(l)\end{bmatrix}$ is the regression vector. Now selecting feedback for DPC as

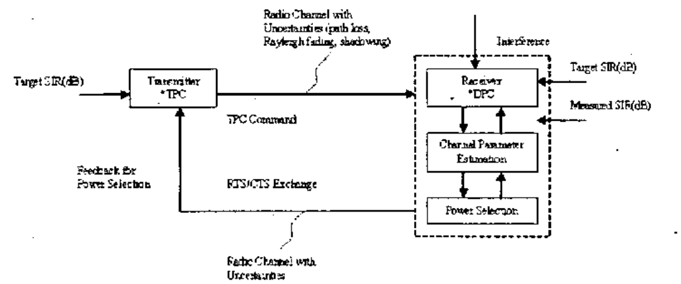
$$v_i(l) = \beta_i^{-1}(l)\left[-\hat{\theta}_i(l)\psi_i(l) + \gamma + k_v e_i(l)\right] \qquad (29)$$

where $\hat{\theta}_i(l)$ is the estimate of $\theta_i(l)$, then the SIR error system is expressed as

$$e_i(l+1) = k_v e_i(l) + \theta_i^T(l)\psi_i(l) - \hat{\theta}_i^T(l)\psi_i(l)$$

$$= k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l) \qquad (30)$$

where $\tilde{\theta}_i(l) = \theta_i(l) - \hat{\theta}_i(l)$ is the error in estimation. From (30), it is clear that the closed-loop SIR error system is driven by channel estimation error. If the channel uncertainties are properly estimated and the transmitter power is selected to overcome these uncertainties, then estimation error tends to zero. In this case, equation (30) becomes (27). In the presence of error in estimation, only a bound on the error in SIR can be shown. In other words, the power control scheme will ensure that the actual SIR is close to its target or bounded. In other words, the actual SIR is within the protective margin provided the channel uncertainties are properly estimated. Figure 1 illustrates the block diagram representation of the proposed DPC where channel state estimation and power selection are part of the receiver. To proceed further, the Assumption 1 is required.



*TPC: Transmitter power control

Figure 1. Block diagram representation of DPC.

Assumption 1: The channel changes slowly compared to the parameters updates.

Remark: The channel variation has to be slower than the speed of estimation. In fact, the estimation scheme is computationally simple that it can be executed extremely fast.

Theorem 3: Given the DPC scheme above with channel uncertainties (Path loss, Shadowing and Rayleigh fading) are estimated accurately (no estimation error), if the feedback from DPC scheme is selected as (29), then the mean channel estimation error along with the mean SIR error converges to zero asymptotically, if the parameter updates are taken as

$$\hat{\theta}_i(l+1) = \hat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \tag{31}$$

provided

$$\sigma\|\psi_i(l)\|^2 < 1 \tag{32}$$

$$k_{v\max} < \frac{1}{\sqrt{\delta}} \tag{33}$$

where $\delta = \frac{1}{1-\sigma\|\psi_i(l)\|^2}$, $k_{v\max}$ is the maximum singular value of k_v and σ is the adaptation gain.

Proof: See Appendix. ■

Consider now the closed-loop SIR error system with channel estimation error, $\varepsilon(l)$, as

$$e_i(l+1) = k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l) + \varepsilon(l) \tag{34}$$

using the proposed DPC.

Theorem 4: Assume the hypothesis as given in Theorem 3, with the channel uncertainty (Path loss, Shadowing and Rayleigh fading) is now estimated by

$$\hat{\theta}_i(l+1) = \hat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \tag{35}$$

with $\varepsilon(l)$ is the error in estimation which is considered bounded above $\|\varepsilon(l)\| \le \varepsilon_N$, with ε_N a known constant. Then the mean error in SIR and the estimated parameters are bounded provided (32) and (33) hold.

Proof: See Appendix. ■

IV. SIMULATION RESULTS

In the simulations, all the mobiles in the network have to achieve a desired target SIR value γ_i. The SIR value is a measure of quality of received signal, and can be used to determine the control action that needs to be taken. The SIR, γ_i can be expressed as

$$\gamma_i = ((E_b/N_0)/(W/R)) \tag{36}$$

where E_b is the energy per bit of the received signal in watts, N_0 is the interference power in watts per Hertz, R is the bit rate in bits per second, and W is the radio channel bandwidth in Hertz.

The cellular network is considered to be divided into 7 hexagonal cells covering an area of 10km x 10km (Figure 9). Each cell is serviced via a base station, which is located at the center of the hexagonal cell. Mobile users in each cell are placed using a normal distribution. It is assumed that the power of each mobile user is updated asynchronously. Consequently, the powers of all other mobile users do not change when link i^{th}'s power is updated. The receiver noise in the system η_i is taken as 10^{-12}. The threshold SIR, γ, which each cell tries to achieve is 0.04 (-13.9794dB). The ratio of energy per bit to interference power per hertz $\frac{E_b}{N_0}$ is 5.12 dB. Bit rate R_b, is chosen at 9600 bits/second. Radio channel bandwidth B_c is considered to be 1.2288 MHz. The maximum power for each mobile $P_{\max}$ is selected as 1mw. Two cases are considered: channel changing sharply at certain time instants and channel changing smoothly.

The system is simulated with different DPC algorithms with 100 users.

A. Performance Metrics

Outage probability and total power consumption are chosen as metrics to evaluate the performance of the power control scheme. Outage probability is defined as the probability of failing to achieve adequate reception of the signal due to co-channel interference. It is defined as the ratio of the number of disconnected or handed over users to that of the total number of users in the system. The total power consumed by the mobiles will be the sum of all the powers of mobiles in the network. Given these metrics, the power per active mobile can be derived.

B. Results

In the first few simulations, the users in each cell are placed at random and they are considered stationary for convenience to evaluate the proposed DPC. Later, the users are mobile.

Stationary Users: Case I: Constant and Abruptly Changing Channel

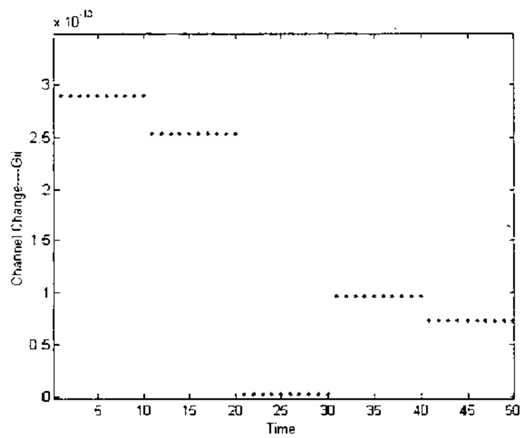


Figure 2. Channel fluctuations.

In this scenario, we select the parameters as $k_v = 0.01$ and $\sigma = 0.01$. Figure 2 shows the change of g_{ii} with time as a result of channel fluctuation, which obeys the path loss, Rayleigh fading and shadowing behaviors. Though channel changes sharply at every ten time units, g_{ii} is changed once in every 10 time units and it is held constant otherwise or between changes.

Figure 3 illustrates the actual SIR of a randomly selected mobile user. From this figure it is clear that the proposed DPC scheme is the only scheme that maintains the target SIR in the presence of channel variations, while the others [8,9,10,18] fail to meet the target. Moreover, using the proposed scheme the actual SIR converges faster (about 2 time units) compared to CSOPC and optimal (10 time units). Though the DPC in [8] converges similar to the proposed, it is unable to maintain the actual SIR to its target.

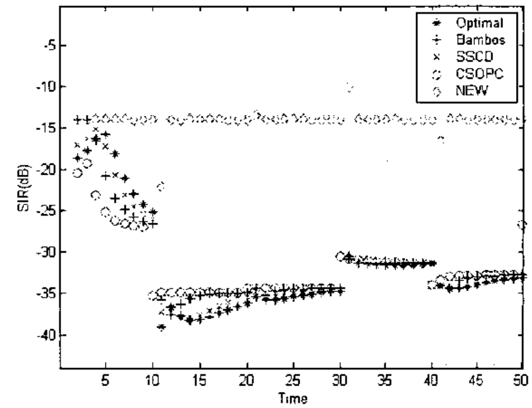


Figure 3. SIR of a randomly selected user.

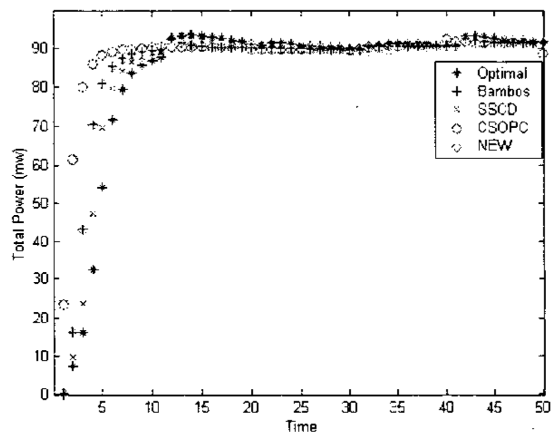


Figure 4. Total power consumed.

Figure 4 presents the total power consumed by all the mobiles in the network. The result shows that all the schemes consume about 90mw. From Figure 5 the outage probability using the proposed DPC scheme is significantly less (near zero) when compared to all the other schemes (about 85%), i.e., our approach can accommodate more number of mobile users rendering high channel utilization or capacity in the presence of channel variations. This together with similar total power consumption suggests that the average power consumed per active user is less using the proposed DPC compared to other schemes.

The incorporation of channel state estimator with the proposed DPC scheme allows the mobiles to consume low power. The channel state estimator was able to accurately obtain an assessment of the prevailing channel state and its variation with background noise, which is specified in terms of $\hat{\theta}_i(k)$. The proposed DPC uses the channel state estimate and the error in the SIR to arrive at a more appropriate transmitter power value for the next time instant (or for the packet) whereas the other DPC schemes do not. Consequently, the user interference experienced by each mobile is higher when other DPC schemes are employed (due to large power values for each mobile) whereas it is low with the proposed DPC.

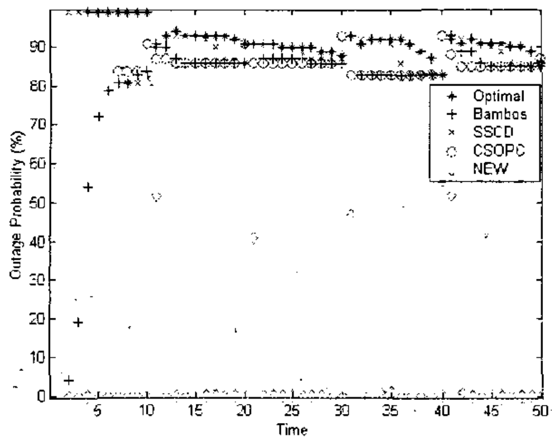


Figure 5. Outage probability.

Higher transmitter power of a mobile user can cause large interference to others resulting in an increase in transmitter powers of all mobiles until an equilibrium state is reached for the entire system. With the proposed DPC, an equilibrium state where the transmitter powers of all mobiles will be lower than when other DPC schemes are used, will be achieved. Therefore, a good channel estimator working in conjunction with the DPC is critical. Moreover, as per Assumption 1, when the channel condition changes suddenly at time instants 10, 20, 30 and 40 of Figure 2, the channel state estimator is unable to keep up. Though this is a disadvantage, but for most situations, the proposed DPC with the channel state estimator is able to perform superior than other available DPC schemes.

Case II: Slowly Varying Channel

In this scenario, $k_v = 0.01$ and $\sigma = 3$. In this case, though the channel changes every ten-time units and satisfies the path loss, Rayleigh fading and shadowing behavior, the channel variation is considered as a linear function between the changes (see Figure 6 of a randomly selected mobile). In this case also, the proposed DPC scheme renders a low outage probability of about 30% as observed in Figure 9 while consuming less power per active mobile and converging faster since the proposed DPC scheme maintains the SIR of each link closer to its target compared to others as displayed in Figure 7. Other schemes attain an outage probability of 85%. The low outage probability for the proposed DPC scheme is the result of faster convergence, better estimation of the channel state and using this information to select an appropriate transmitter power to maintain a low SIR error.

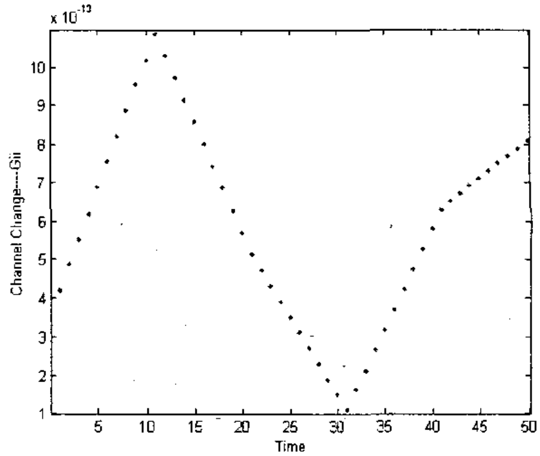


Figure 6. Channel variation with time.

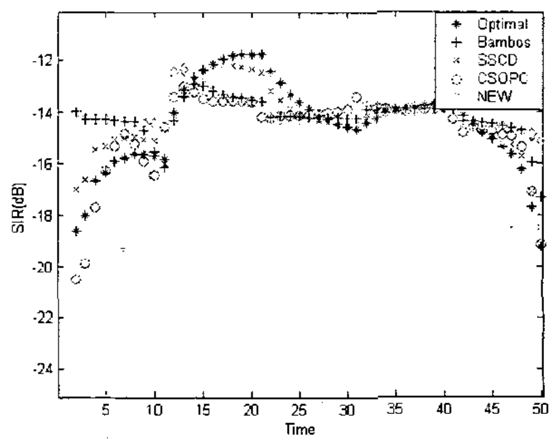


Figure 7. SIR of a randomly selected user.

The performance of the channel state estimator can be easily observed by comparing the outage probability plots for the cases when the channel changes abruptly but remains constant between changes and the case where it changes smoothly. In the case of channel changing abruptly, the estimator was unable to obtain an accurate assessment at the instant when the channel condition changes whereas in other instants, it was able to estimate the channel condition accurately and subsequently the proposed DPC was able to select a suitable transmitter power as presented in Figure 5.

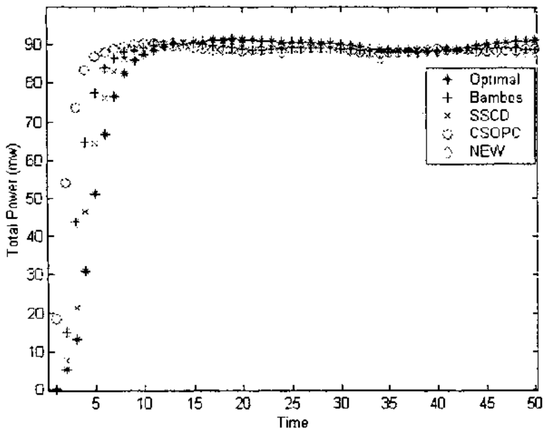


Figure 8: Total power consumed.

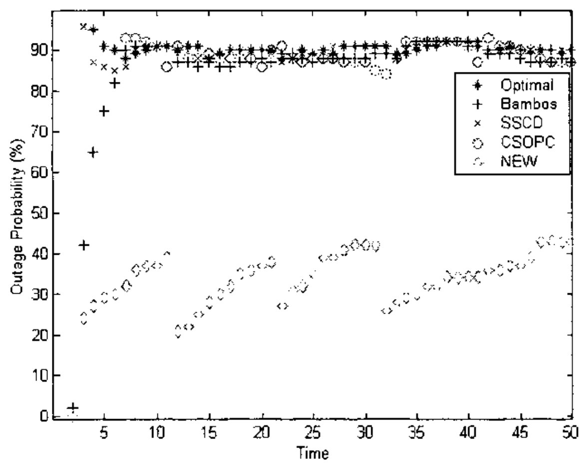


Figure 9. Outage probability.

On the other hand, as expected in the case of smooth channel variation, the outage probability changes with the variation in the channel state (See Figure 9). This is due to the delay in the channel estimation and the selection of the transmitter power value using this estimate (the channel state changes by the time the power is calculated and feedback, which results in a mismatch). Also, this is reflected in the error in SIR plot shown in Figure 7 for a randomly selected mobile. In any case, the outage probability presented in Figure 9 is significantly lower for the proposed DPC when compared with the other schemes due to the inclusion of the channel state estimator. Note also that the performance of the channel estimator depends upon the rate of change of the channel state. The change in the channel condition illustrated in Figure 6 is slightly higher (slope of the curve in Figure 6) than the scenario presented in Assumption 1.

For much slower change in the channel conditions (results not shown), the estimator performance is much superior when compared to the case in Figure 6. Consequently a much lower outage probability was observed. This clearly demonstrates that the addition of a channel state estimator is critical for any DPC scheme even though there could be feedback delays.

Performance Evaluation with Number of Users

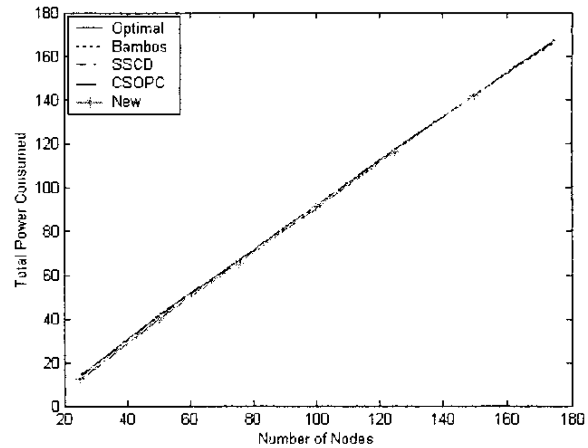


Figure 10. Total power consumed.

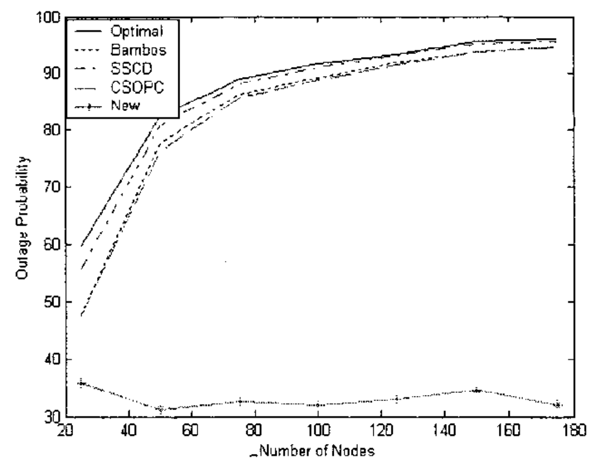


Figure 11. Outage probability.

When the total number of mobile users in the cellular network varies, the total power consumed and the resulting outage probability was compared. In this simulation scenario, the number of mobile users trying to gain admission increases by 25 from the previous value. Figures 10 and11 display the performance of the DPC in terms of total power consumed and outage probability respectively when the channel changes slowly. As expected, the proposed scheme renders significantly low outage probability while ensuring low power per active mobile user compared to other schemes.

As expected, a linear increase in power consumption with users is observed with all the DPC schemes though. the proposed DPC maintains a low outage probability consistently under 40% when compared with other schemes. Similar total power consumption together with low and consistent outage probability indicates that the proposed DPC was able to select a suitable transmitter power value thus lowering mutual interference between users even in the presence of channel uncertainties.

Mobile User Scenario

Figure 12 illustrates the initial location of 100 mobile users whereas Figure 13 presents their final location. Users in the cellular network try to move in any one of the 8 pre-defined directions chosen at random at the beginning of the simulation. A user can move a maximum of 0.01 km per time unit. Since the time unit is small, the 0.01 km is a considerable amount of distance for any mobile user. The channel state is also changed slowly for each mobile user. Figures 14 and 15 illustrate the total power consumed and the corresponding outage probability.

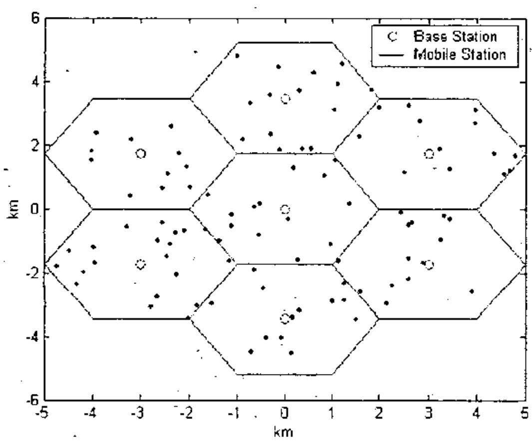


Figure 12. Initial state of mobile placement.

As evident from the result, the proposed DPC renders a lower outage probability (average of about 30%) compared to others (about 90%) while consuming low power per active mobile even in the presence of user mobility. The variation in outage probability displayed in Figure 15 with respect to time units is explained in the case of stationary user scenario with slowly varying channel.

As explained earlier, the mobility can be expressed as part of the uncertainty. In other words, the channel state now is a function of user mobility and can be included as part of the parameter estimation process. As a result, the proposed DPC was able to assess the accurate transmitter power for each mobile without the knowledge of its location based on the channel state thus lowering the mutual interference. This results in lower outage probability as displayed in Figure 15.

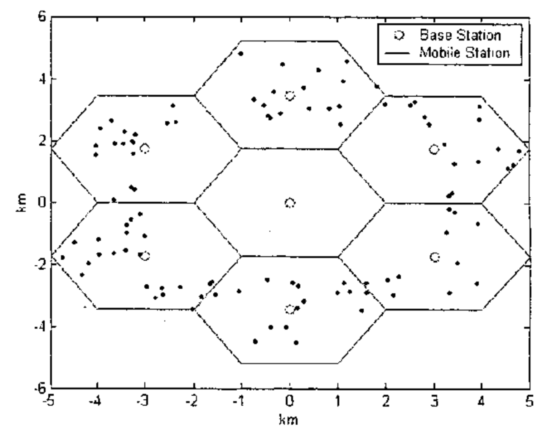


Figure 13. Final state of mobile placement.

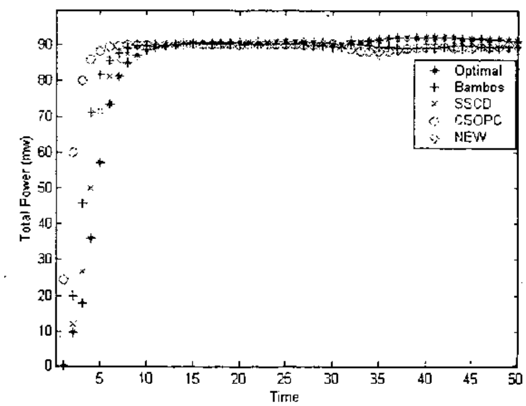


Figure 14. Total power consumed.

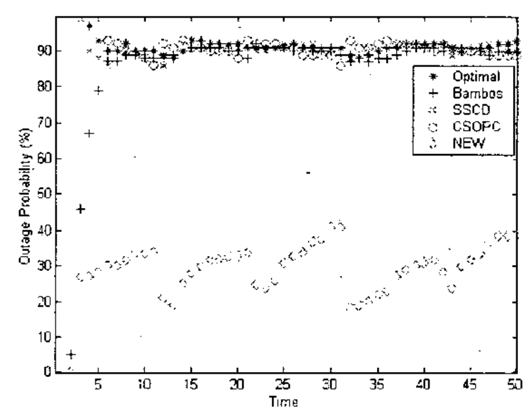


Figure 15. Outage probability.

V. CONCLUSIONS

In this paper, a novel DPC schemes is presented for cellular and can be potentially extended to ad hoc networks capturing the essential dynamics of power control. It was observed that the proposed DPC scheme allows fully distributed power and has rendered better performance in the presence of radio channel uncertainties. The inclusion of channel state estimator is very critical in the design of the DPC. The simulation results show that the proposed scheme converges faster than others, maintains a desired target SIR value for each link and can adapt to the radio channel variations better. In the presence of channel uncertainties, the proposed scheme can render lower outage probability (an improvement of about 50%), using significantly less transmitter power per active user compared to other DPC schemes. As a result, the proposed DPC scheme offers a superior performance in terms of convergence and it maximizes the network capacity compared to the available ones in the literature. Simulation results justify theoretical conclusion.

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APPENDIX

Proof of Theorem 3: Define the Lyapunov function candidate

$$J_i = e_i^T(l)e_i(l) + \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right] \tag{A.1}$$

whose first difference is

$$\begin{aligned} \Delta J &= \Delta J_1 + \Delta J_2 \\ &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &\quad + \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l+1)\tilde{\theta}_i(l+1) - \tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right] \end{aligned} \tag{A.2}$$

Consider ΔJ_1 from (35) and substituting (30) to get

$$\begin{aligned} \Delta J_1 &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &= \left(k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right)^T\left(k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right) - e_i^T(l)e_i(l) \end{aligned} \tag{A.3}$$

Taking the second term of the first difference from (A.2) and substituting (31) yields

$$\begin{aligned} \Delta J_2 &= \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l+1)\tilde{\theta}_i(l+1) - \tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right] \\ &= -2[k_v e_i(l)]^T\,\tilde{\theta}_i^T(l)\psi_i(l) - 2\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] \\ &\quad + \sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right] \end{aligned} \tag{A.4}$$

Combining (A.3) and (A.4) to get

$$\Delta J = -e_i^T(l)\left[I - \left(1+\sigma\psi_i^T(l)\psi_i(l)k_v^T k_v\right)\right]e_i(l)$$
$$+ 2\sigma\psi_i^T(l)\psi_i(l)[k_v e_i(l)]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]$$
$$-\left(1-\sigma\psi_i^T(l)\psi_i(l)\right)\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] \quad \text{(A.5)}$$
$$\le -\left(1-\delta k_{v\max}^2\right)\left\|e_i(l)\right\|^2 - \left(1-\sigma\left\|\psi_i(l)\right\|^2\right)$$
$$\left\|\tilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\left\|\psi_i(l)\right\|^2}{1-\sigma\left\|\psi_i(l)\right\|^2}k_v e_i(l)\right\|^2$$

where δ is given after (33). Taking now expectations on both sides yields

$$E(\Delta J) \le -E(\left(1-\delta k_{v\max}^2\right)\left\|e_i(l)\right\|^2 - \left(1-\sigma\left\|\psi_i(l)\right\|^2\right)$$
$$\left\|\tilde{\theta}_i^T(l)\psi_i(l) + \frac{\sigma\left\|\psi_i(l)\right\|^2}{1-\sigma\left\|\psi_i(l)\right\|^2}k_v e_i(l)\right\|^2) \quad \text{(A.6)}$$

Since $E(J)>0$ and $E(\Delta J)\le 0$, this shows the stability in the mean via sense of Lyapunov provided the conditions (32) and (33) hold, so $E[e_i(l)]$ and $E\left[\tilde{\theta}_i(l)\right]$ (and hence $E\left[\hat{\theta}_i(l)\right]$) are bounded in the mean if $E[e_i(l_0)]$ and $E\left[\tilde{\theta}_i(l_0)\right]$ are bounded in a mean. Sum both sides of (A.6) and take limits $\lim_{l\to\infty} E(\Delta J)$, the SIR error $E\left[\left\|e_i(l)\right\|\right]\to 0$. ■

Proof for Theorem 4: Define a Lyapunov function candidate as in (A.1) whose first difference is given by (A.2). The first term ΔJ_1 and the second term ΔJ_2 can be obtained respectively as

$$\Delta J_1 = e_i^T(l)k_v^T k_v e_i(l) + 2[k_v e_i(l)]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] +$$
$$\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i(l)\psi_i(l)\right] + \varepsilon^T(l)\varepsilon(l) \quad \text{(A.7)}$$
$$+ 2[k_v e_i(l)]^T\varepsilon(l) + 2\varepsilon^T(l)e_i(l) - e_i(l)^T e_i(l)^T$$

$$\Delta J_2 = -2[k_v e_i(l)]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] - 2\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]$$
$$+\sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right]$$
$$-2\left[1-\sigma\psi_i^T(l)\psi_i(l)\right]e_i^T(l)\varepsilon(l) + 2\sigma\psi_i^T(l)\psi_i(l)[k_v e_i(l)]^T\varepsilon(l)$$
$$+\sigma\psi_i^T(l)\psi_i(l)\varepsilon^T(l)\varepsilon(l)$$
(A.8)

Using (A.8) and completing the squares for $\tilde{\theta}_i^T(l)\psi_i(l)$ yields

$$\Delta J \le -\left(1-\delta k_{v\max}^2\right)\left(\left\|e_i(l)\right\|^2 - \frac{2\sigma k_{v\max}\left\|\psi_i(l)\right\|^2}{1-\delta k_{v\max}^2}\varepsilon_N\left\|e_i(l)\right\| - \frac{\delta}{1-\delta k_{v\max}^2}\varepsilon_N^2\right)$$
$$-\left(1-\sigma\left\|\psi_i(l)\right\|^2\right)\left\|\tilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\left\|\psi_i(l)\right\|^2}{1-\sigma\left\|\psi_i(l)\right\|^2}(k_v e_i(l)+\varepsilon(l))\right\|^2$$
(A.9)

with δ is given after (33). Taking expectations on both sides to get

$$E(\Delta J) \le -E(\left(1-\delta k_{v\max}^2\right)$$
$$\left(\left\|e_i(l)\right\|^2 - \frac{2\sigma k_{v\max}\left\|\psi_i(l)\right\|^2}{1-\delta k_{v\max}^2}\varepsilon_N\left\|e_i(l)\right\| - \frac{\delta}{1-\delta k_{v\max}^2}\varepsilon_N^2\right)$$
$$+\left(1-\sigma\left\|\psi_i(l)\right\|^2\right)\left\|\tilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\left\|\psi_i(l)\right\|^2}{1-\sigma\left\|\psi_i(l)\right\|^2}(k_v e_i(l)+\varepsilon(l))\right\|)$$
(A.10)

as long as (32) and (33) hold, and

$$E\left[\left\|e_i(l)\right\|\right] > \frac{1}{\left(1-\sigma k_{v\max}^2\right)}\varepsilon_N\left(\sigma k_{v\max} + \sqrt{\sigma}\right) \quad \text{(A.11)}$$

This demonstrates that $E(\Delta J)$ is negative outside a compact set U. According to a standard Lyapunov extension, the SIR error $E[e_i(l)]$ is bounded for all $l\ge 0$. It is required to show that $\hat{\theta}_i(l)$ or equivalently $\tilde{\theta}_i(l)$ is bounded. The dynamics in error in the parameters are

$$\tilde{\theta}_i(l+1) = \left[I - \sigma\psi_i^T(l)\psi_i(l)\right]\tilde{\theta}_i(l) - \sigma\psi_i(l)[k_v e_i(l) + \varepsilon(l)]^T \quad \text{(A.12)}$$

where SIR error, $e_i(l)$, is bounded and estimation error, $\varepsilon(l)$, is bounded. Applying the persistency of excitation $\varepsilon(l)$[21], one can show that $\tilde{\theta}_i(l)$ is bounded. ■