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IMPORTANCE OF SOIL ANISOTROPY ON FOUNDATION DISPLACEMENT FUNCTIONS

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SYNOPSIS Consideration is given to the effect of soil cross-anisotropy on the dynamic displacement functions of a rigid strip surface foundation subjected to vertical or horizontal forces and moments that vary harmonically with time and are distributed uniformly across the longitudinal axis so that plane strain conditions prevail. The results are obtained using an analytical-numerical formulation which models realistically the rough soil-foundation interface, properly accounts for phenomena associated with propagation of waves emanating from the foundation and considers linear hysteretic material damping in the soil. Particular emphasis is accorded to the sensitivity of the calculated frequency-dependent foundation displacements to the assumed values of the anisotropic soil constants, appropriate for *drained* loading conditions.

INTRODUCTION

Although conventional methods of analysis of the dynamic response of foundations idealize the supporting soil as a homogeneous isotropic elastic continuum (halfspace or layer), it frequently occurs in practice that the supporting strata are *transversely isotropic* with a vertical axis of material symmetry. This is generally true of several deposits of overconsolidated clays, such as the London Clay (e.g. Atkinson, 1975), of normally consolidated naturally cemented clays, such as the Leda Clay (e.g. Yong et al, 1979), of sendimentary sands (e.g. Oda, 1972) and of varved clays (Gazetas, 1981a). Therefore, it is natural to enquire to what extent static and dynamic displacements of foundations are affected by the transversely isotropic nature of soil.

Regarding the static problem, the progression from simple isotropy to transverse isotropy* is computationally rather straightforward. Numerous studies have been published dealing with various aspects of the problem; reference is made to Gibson (1974), Burland et al (1977) and Gazetas (1981c,d) for comprehensive lists of related publications, and to Gerrard & Harrison (1970 a,b) for complete tabulated solutions in terms of stresses and displacements arising from several types of normal, shear and moment loads acting on circular and strip foundations.

The solution of the dynamic problem, on the other hand, presents a special difficulty due to the fact that the differential equations of motion do not, in general, uncouple into classical dilatational and shear wave equations, as in the case of isotropic materials. Thus, few dynamic soil-foundation interaction studies have modeled the soil as a transversely isotropic material. Recently, by utilizing an experimentally verified relationship among the anisotropic soil constants (Eq. 1) through which the wave equations uncouple in closed-form.

*the terms 'cross-anisotropy' and 'hexagonal anisotropy' are also used in the literature in place of 'transverse isotropy'.

Kirkner (1979) and Gazetas (1981b) obtained the dynamic displacement (compliance) functions of rigid circular and strip foundations on viscoelastic halfspace. Gazetas (1981 c) extended the above formulation (1981 b) to a soil deposit consisting of an arbitrary number of cross-anisotropic uniform layers and presented a detailed study of the significance of soil anisotropy in determining *undrained* static and dynamic response of foundations resting on a halfspace or on a soil layer-on-bedrock.

This paper parametrically investigates the effect on foundation displacement functions of *drained* cross-anisotropic soil constants. The results have been obtained with the formulation of Gazetas (1981 b,c) and are portrayed in the non-dimensional, generally applicable, form of normalized displacement-load or rotation-moment ratios as functions of a dimensionless frequency factor, for a wide range of the material parameters E_H/E_V, ν_{VH} and ν_{HH}, defined in the following section. It is shown that neglecting soil anisotropy may lead to appreciable errors when estimating the static and low-frequency foundation displacements under drained conditions; at higher frequencies of oscillation, unsafe conclusions may be reached regarding the possibility and consequences of resonance phenomena.

TRANSVERSE SOIL ISOTROPY AND MATERIAL CONSTANTS

Natural sendimentary soils, deposited in approximately horizontal layers and then subjected to one-dimensional loading and unloading, acquire a structure characterized by a preferred horizontal orientation of particles. As a result, such soils exhibit a particular type of anisotropic deformational behavior: their vertical axis is an axis of material symmetry and all horizontal planes are planes of isotropy. This type of anisotropy is referred to as cross-anisotropy or, equivalently, transverse isotropy.

It appears that in sands this structural anisotropy arises primarily from the effect of gravity: sands in a gravitational field tend to rest in a preferred, more stable, position with the interparticle contact normal stresses being greatest in the vertical direction of deposition and least in the horizontal direction (Oda, 1972). This is true even for sands consisting of rounded particles (Ladd et al, 1977). In fact, Graton et al (1935) showed that for a random packing of equal spheres most of the tangent planes of the contact will have small-angle dips. Moreover, elongated or flat shaped sandy particles have a strong tendency to adopt preferential orientation with the maximum dimension(s) aligned in a horizontal plane; thus, the number of contacts per unit horizontal area is smaller than the number of contacts per unit vertical area (Rowe, 1962; Lafaber & Willoughby, 1971). Such an anisotropic structure leads to cross-anisotropic deformation-strength characteristics: the largest modulus and strength occurs for loading in the same direction as deposition and the lowest for loading perpendicular to this direction. Although variation in drained strength is usually rather modest, changes in modulus by a factor of two are rather typical. However, in extreme cases, differences by a factor of about five are possible (Lafaber & Willoughby, 1971).

In clays, fabric anisotropy takes primarily the form of preferred orientation in a horizontal arrangement of the plate-shaped particles and/or particle clusters (Yong & Warkentin, 1975). Experiments with clays consolidated from a slurry state under the action of a stress state having an n-fold axis of symmetry showed that the resulting fabric and mechanical properties are also characterized by an n-fold axis of symmetry, i.e. they are cross-anisotropic (Gerrard, 1977; Kirkpatric & Rennie, 1973). Overconsolidation, through removal of overburden or disiccation, has a major effect on clay fabric since it tends to produce a strong pattern of fissures parallel to the bedding direction; an extreme example is the eventual formation of strongly anisotropic laminated shales, slates and mudstones. Thus, in most cases, overconsolidated clays are characterized by larger horizontal than vertical moduli; a well known example is the heavily overconsodiated London Clay, in which horizontal-to-vertical moduli ratios of up to 2.5 have been measured (Atkinson, 1975; Hooper, 1975).

Besides particle orientation, fabric anisotropy in clays may be the result of anisotropic electrochemical bonds between clay particles. A perhaps extreme case of such anisotropy has been observed in the Canadian Leda Clay (Mitchell, 1970) whose particles are randomly orientated but, yet, it exhibits a strongly cross-anisotropic deformational behavior with a modulus in the vertical direction larger than the modulus in the horizontal direction by a factor of about 2.

To mathematically describe the deformational behavior of cross-anisotropic soils, anisotropic elasticity is used (Lekhnitskii, 1963; Pickering, 1970). It is well known that with the small-amplitude dynamic strains developing beneath machine foundations soils behave approximately as linear elastic materials. Also, the local soil nonlinearities arising (mainly under the foundation edges) during strong earthquake or ocean wave excitation have a rather secondary influence on the overall foundation response.

The following five independent elastic constants describe the deformational cross-anisotropy of a material:

E_V : Young's modulus in the vertical direction, i.e. along the axis of material symmetry,

E_H : Young's modulus in all horizontal directions, i.e. on the plane of isotropy,

ν_{VH}: Poisson's ratio for transverse horizontal due to imposed vertical strain,

ν_{HH}: Poisson's ratio for transverse horizontal due to imposed horizontal strain (i.e. deformation on the plane of isotropy), and

G_{VH}: shear modulus describing the deformation of a square element that has two vertical and two horizontal sides.

For many types of soils the following relationship has been found to approximately hold between shear modulus G_{VH} and the other elastic constants:

$$G_{VH} = \frac{D_{11} D_{33} - D_{13}^2}{D_{11} + 2D_{13} + D_{33}} \qquad (1)$$

in which

$$\left.\begin{aligned} D_{11} &= E_H(1-n\nu_{VH}^2)/a & D_{13} &= E_H\nu_{VH}(1+\nu_{HH})/a \\ D_{33} &= E_V(1-\nu_{HH}^2)/a & a &= (1+\nu_{HH})(1-2\nu_{HH}-2n\nu_{VH}^2) \end{aligned}\right\} \qquad (1a)$$

$$\text{and} \quad n = E_H/E_V$$

The experimental corroboration of Eq. 1 has been documented by the Author in two other publications (Gazetas, 1981 a,c). The implication of this equation is that only four elastic constants are independent. Moreover, these constants cannot assume arbitrary values since they are restricted by the requirement of non-negative strain energy function to satisfy the following inequalities (Pikering, 1970) :

$$E_V,\ E_H \geq 0\ ; \quad -1 < \nu_{HH} < 1-2n\ \nu_{VH}^2 \qquad (2)$$

The limiting values of Poisson's ratios are shown graphically in Fig. 1 for a broad range of $n=E_H/E_V$ values. Note that isotropy reduces from cross-anisotropy if n=1 *and* $\nu_{VH}=\nu_{HH}=\nu$; the permissible values of ν are confined to the line joining the points $\nu= -1$ and $\nu= 0.5$, in Fig. 1.

In addition to the preceding elastic constants, a (single) frequency independent damping constant, ξ, is needed to properly account for the internal dissipation of energy in the medium. Herein, the soil is assumed to be a constant hysteretic solid and in the formulation the material constants, e.g. E_V, are replaced by complex moduli of the form $E_V(1+2i\xi)$. The reported results are based on $\xi= 0.05$. The effect of different ξ values on the dynamics of undrained cross-anisotropic soils has been investigated by Gazetas (1981 b,c).

PARAMETER STUDY: DRAINED SOIL RESPONSE

Results are presented for sliding, rocking and vertical oscillations of a rigid massless strip footing adhesively connected to a *compressible* cross-anisotropic linearly hysteretic elastic halfspace. The reader is referred to Gazetas (1981 b,c) for parallel studies of the response of a footing on an *incompressible* cross-

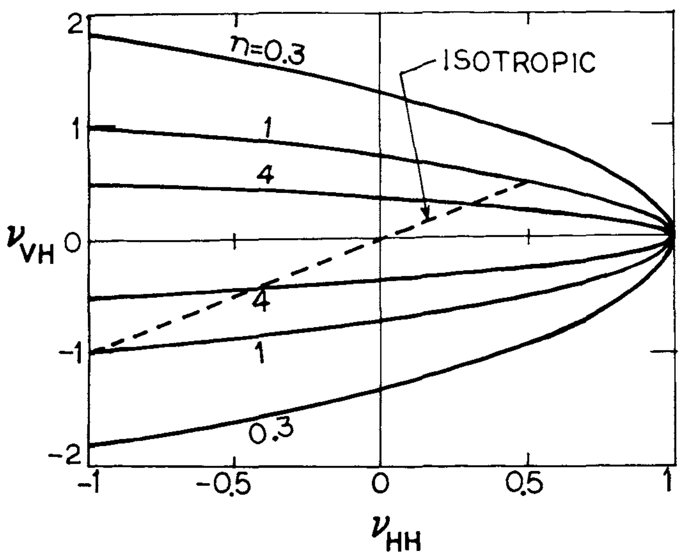

Fig. 1. Limiting values of Poisson's ratios for cross-anisotropic materials

anisotropic halfspace and on a uniform layer on rigid bedrock. The latter studies are of interest to the geotechnical engineer involved in determining displacements of foundations on saturated clayey or silty deposits where undrained conditions prevail. The results offered herein are applicable in cases of dry or partially saturated soils as well as for fully saturated medium to coarse sandy deposits, i.e. when volumetric changes do take place in the soil.

Figs. 2-5 portray the variation of the dimensionless displacement matrix S versus the frequency factor $A_V = \omega B \sqrt{\rho/E_V}$, for a wide range of values of the independent dimensionless material constants, ν_{VH}, ν_{HH} and n. The displacement matrix is composed of the following elements:

$$S_{HH} = E_V\, d_{HH}/P_H \tag{2a}$$

$$S_{MM} = E_V\, B^2\, \phi_o/M_o \tag{2b}$$

$$S_{HM} = E_V\, B\, d_{HM}/M_o \tag{2c}$$

$$S_V = E_V\, d_V/P_V \tag{2d}$$

in which d_{HH}, ϕ_o and d_V denote the amplitudes of the harmonically varying with time horizontal displacement, angle of rotation and vertical displacement, due to harmonic forces of amplitude P_H, M_o and P_V, respectively. $d_{HM} = d_{MH}$ denotes the amplitude of the horizontal displacement due to a moment M_o. Each displacement consists of two components, one *in phase* and the other 90^o *out of phase* with the applied harmonic load; they are shown on the left and right hand half of each figure, respectively.

The results shown correspond to four different sets of Poisson's ratios, ν_{VH} and ν_{HH}, covering a wide range of values that may be encountered in practice. They range from $\nu_{VH} = \nu_{HH} = 0.33$ (Fig. 2), (typical of *sandy soils*) to $\nu_{VH} = 0.10$ & $\nu_{HH} = 0.90$ (Fig. 5), which may represent *dilatant* soils if, also, the moduli ratio $n = E_H/E_V$ is larger than one. Indeed, a material with these Poisson's ratio and, say, n = 2, when subjected to hydrostatic compression p undergoes lateral extension, since

$$\varepsilon_H = \frac{p}{E_H}(1-\nu_{HH} - n\nu_{VH}) = -0.10\ p/E_H \tag{3}$$

which implies that ε_H and p are of opposite sign (i.e. "scalar" dilatancy, according to Gerrard, 1977). Note furthermore, that the set $\nu_{VH} = 0.25$ & $\nu_{HH} = -0.33$ (Fig. 3) may represent material somewhat similar to the overconsolidated *London Clay* under undrained conditions (Hooper, 1975, Gerrard, 1977). Finally, the set $\nu_{VH}=0.33$ & $\nu_{HH} = -0.98$ (Fig. 4) may approximately simulate the behavior of *very loose* soils that undergo large volume decreases when subjected to uniaxial compression.

It is evident from Figs. 2-5 that the moduli ratio n and the two Poisson's ratios have an important influence on all dynamic displacement functions(compliances). In particular, for a given set of Poisson's ratios, as the ratio n increases the halfspace becomes stiffer and the low-frequency compliances appreciably decrease. However, this is not necessarily true at higher frequency factors; in fact, for $A_V \geq 1.5 - 2.0$ foundations on soils with large n values may experience larger (dynamic) displacements and rotations than on soils with small n.

Figs. 2-5 can be utilized to determine by standard procedures the steady-state of any (long) foundation with mass and of any linear structure superposed on such a foundation.

CONCLUSION

The dynamic, frequency-dependent displacement functions of rigid foundations subjected to harmonic horizontal, moment and vertical loading under fully drained conditions are significantly influenced by the assumed values of the cross-anisotropic soil constants E_H, E_V, ν_{VH} and ν_{HH}. Therefore, assigning realistic values to these elastic constants is a major, and perhaps the most difficult, task in studying the behavior of foundations and structures in dynamic loading environments.

The results of this study complement the findings of two parallel investigations by the author (Gazetas, 1981 b,c) on the effect on displacements of the cross-anisotropic elastic constants under undrained dynamic loading conditions.

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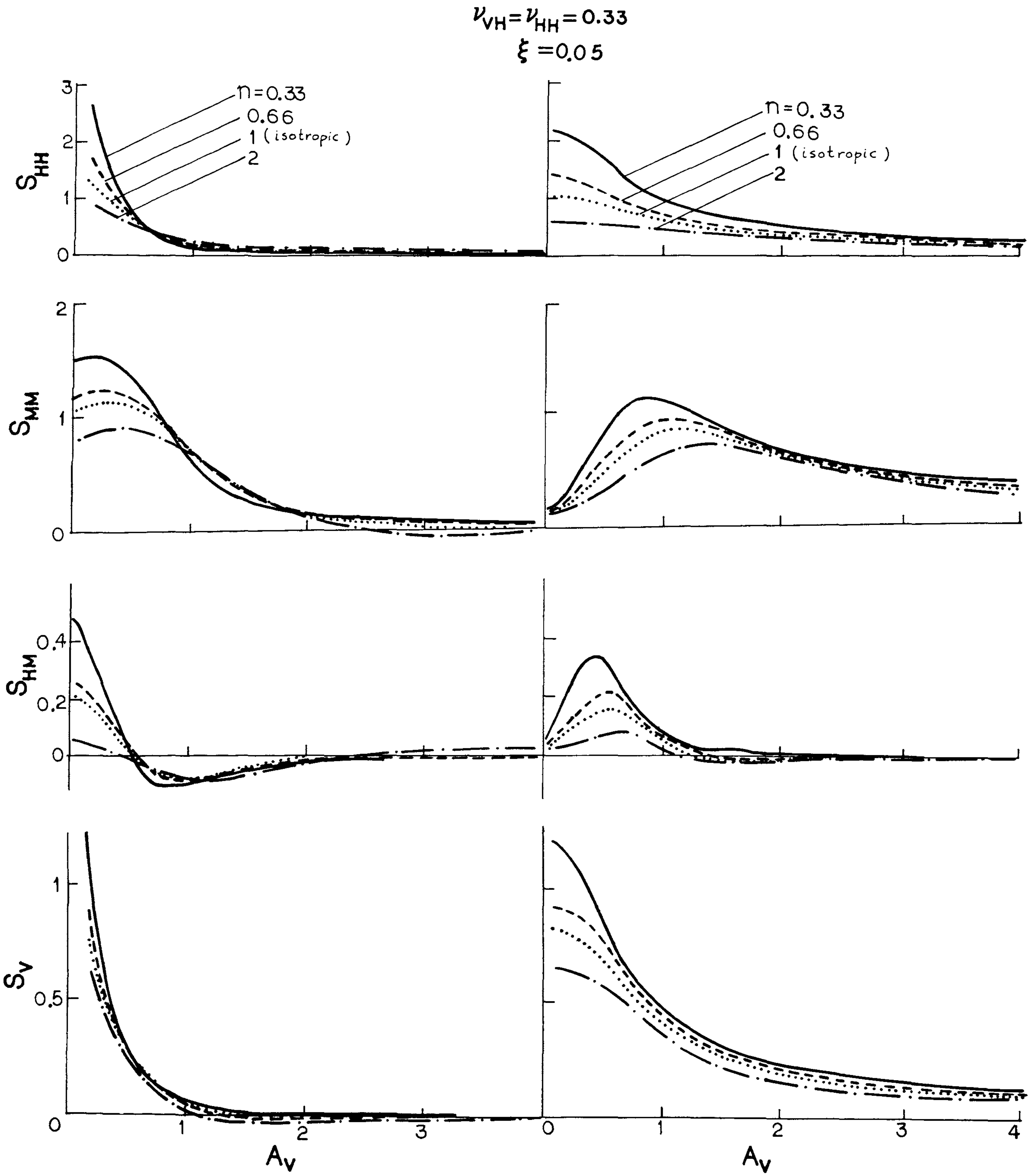

Fig. 2. In-phase (left) and 90°-out-of-phase (right) components of dynamic displacement functions for $\nu_{VH}=\nu_{HH}=0.33$

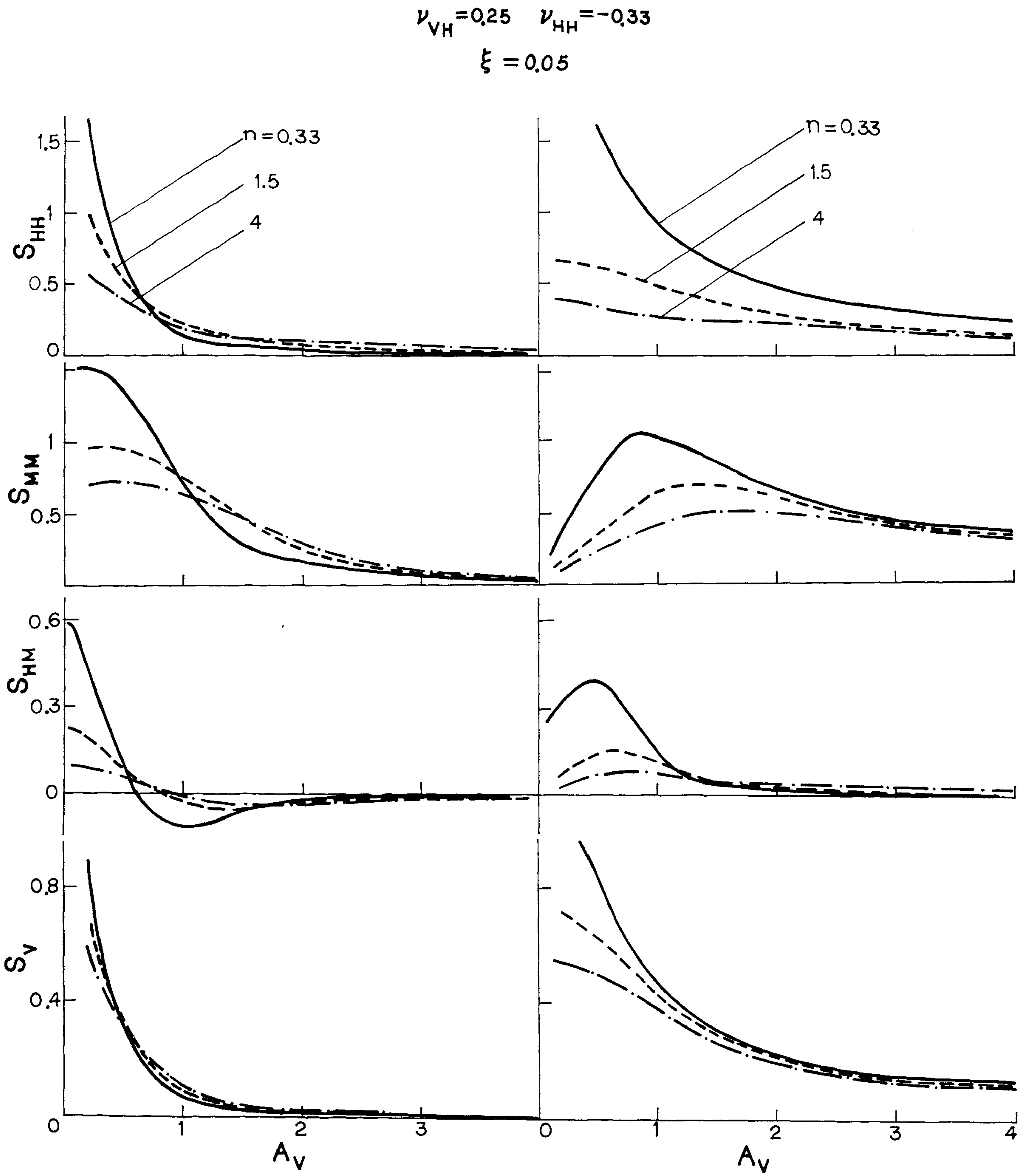

Fig. 3. In-phase (left) and 90°-out-of-phase (right) components of dynamic displacement functions for $\nu_{VH}=0.25, \nu_{HH}=-0.33$

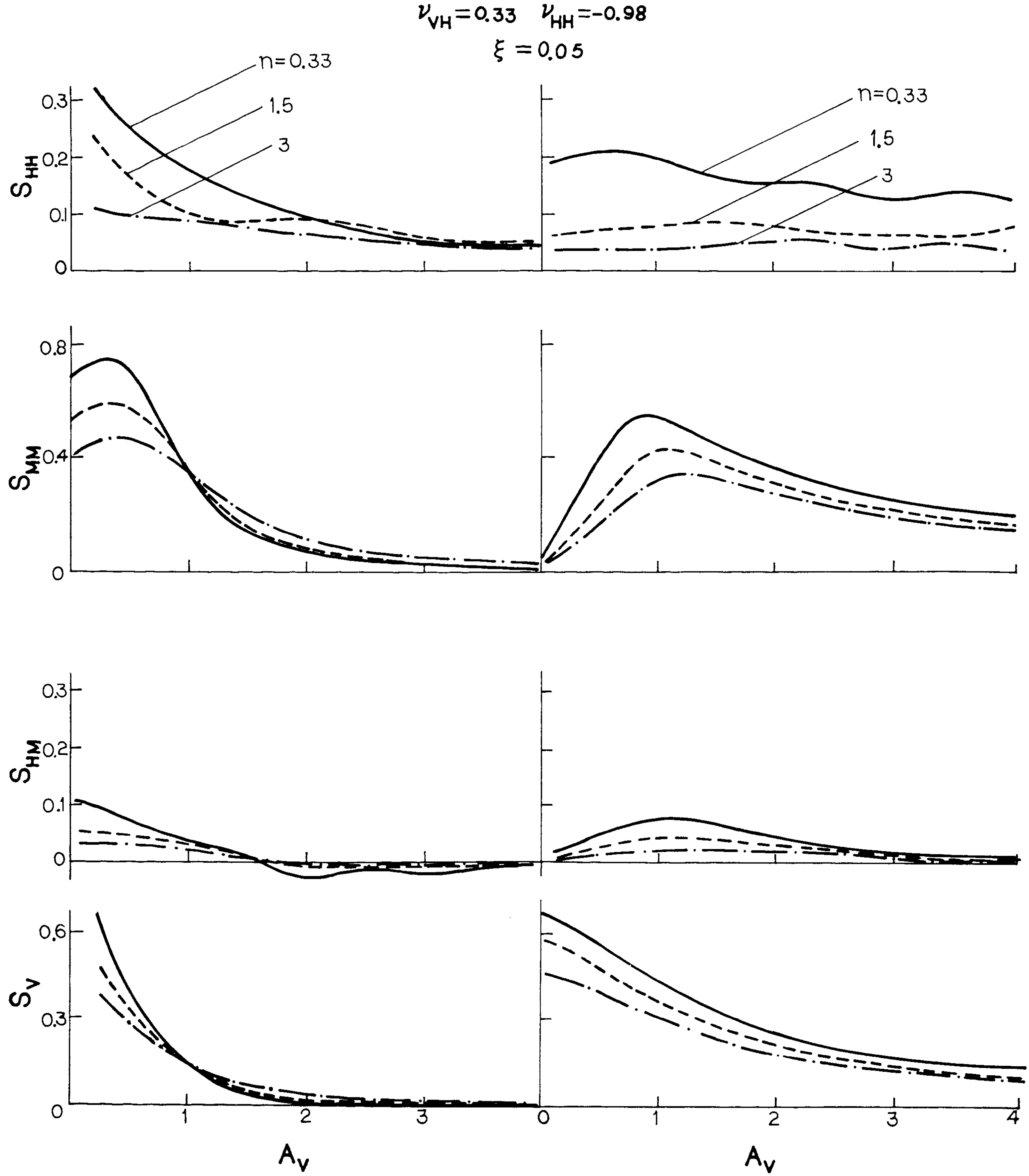

Fig. 4. In-phase (left) and 90°-out-of-phase (right) components of dynamic displacement functions for $\nu_{VH}=0.33, \nu_{HH}=-0.98$

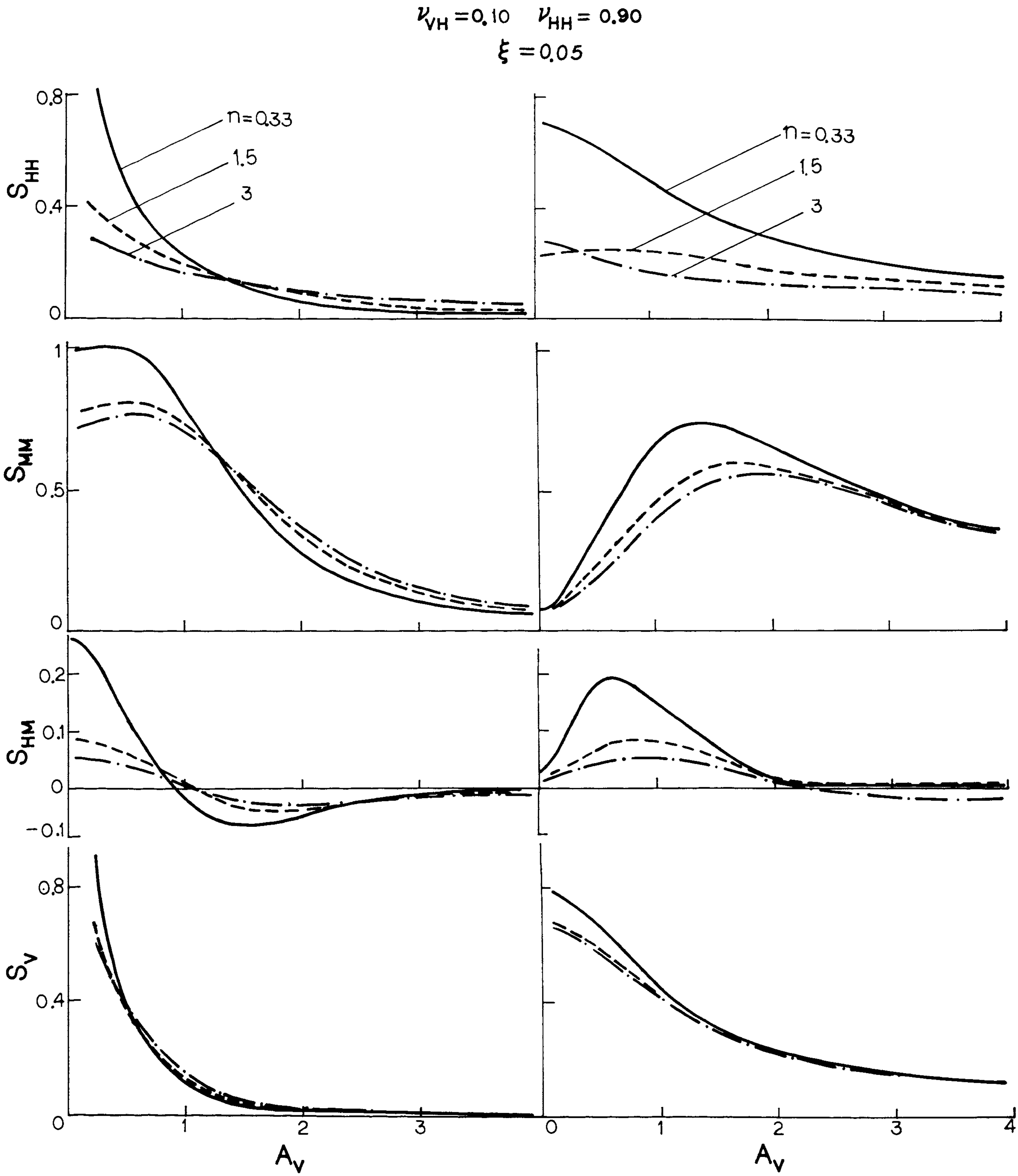

Fig. 5. In-phase (left) and 90°-out-of-phase (right) components of dynamic displacement functions for $\nu_{VH}=0.10, \nu_{HH}=0.90$

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