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SYNOPSIS In this investigation, a mathematical hybrid model developed previously is employed to study soil-structure interaction of embedded structure. In the analysis, the near field including the embedded structure and its surrounding foundation soil is modelled with a coventional finite element mesh, and the far field is modelled as a semi-infinite medium with a hemi-spherical pit. The impedance functions at the nodes around the special element, which have been determined analytically, can represent the behavior of outgoing propagation of waves. A concept of superposition is proposed to analyze the response of an embedded structure excited by an incoming SH-wave. The governing equations of the whole system will be formulated by enforcing the compatibility and equalibrium conditions at the nodes of the finite mesh. Basing on these equations, the response of the embedded structure and its surrounding ground can be determined accordingly. Numerical results have been obtained, and correlations with available solutions using continuum approaches are studied. The effects of the embedment on the responses are also shown and discussed.

INTRODUCTION

Dynamic soil-structure interaction has received considerable attention in connection with the design of massive structures. The foundations of the structures are usually embedded at a considerable depth in a soil deposite. In this situation, an embedment plays an important role in the interaction between the structure and its surrounding soil medium. This problem is usually analyzed by either a continuum approach or by a finite element method. By the first approach, M. Novak and Y. Beredugo (1970 & 1972), M. Novak and K. Sachs (1973), H. Tajimi (1969) obtained approximate impedance functions, which accounts for radiation damping in the semi-infinite medium but only for foundations of simple geometry in an elastic medium. The finite element method, which is employed by J. Lysmer and R.L. Kuhlemeyer (1969 & 1973), J. Lysmer, et al (1974), has been used extensively because of its ability to handle ir-regular shapes, complex constitutive relationships and structures with flexible foundations. However, in this approach, a semi-infinite medium is modelled by a finite mesh having rigid boundaries, which may cause the energy radiating away from the foundation to be trapped. Thus no matter how large a mesh size is chosen, there is a condition of resonance which does not really exist.

Then a hybrid model is developed which takes advantage of the good features of each method described above while at the same time minimizing their undesired features. In this model, the structure including the soil medium is divided into two fields, a far field and a near field. The far field is considered as a half-space with a hemi-spherical pit along which the impedance functions of the nodes are constructed analytically basing on the dynamic theory of elasticity. In the near field, the structure with foundation and part of its surrounding medium is modelled by a conventional finite element mesh. The equations of motions for the whole system are obtained by enforcing the compatibility and equilibrium conditions at the nodes of interface of two fields. This approach has been applied by C.S. Yeh, T.W. Lin and J. Penzien (1979) to investigate the dynamic response of an embedded structure under a dynamic torque, and has been developed and discussed more extensively by S. Gupta, T.W. Lin, J. Penzien and C.S. Yeh (1980).

The study of the dynamic interaction between structure and ground motion was started by J.E. Luco (1969), who treated an infinite shear wall on a rigid semi-circular foundation under the excitation of a plane harmonic SH-wave. Later, M.D. Trifunac (1972) found that the motion of this structure is independent of the angle of incidence. U. Gamer and Y.H. Pao studied this problem by considering an elastic foundation with a rigid mass on the top of the shear wall and showed numerically considerable differences with the results of the rigid foundation case. The investigations cited above are the two dimensional cases. For the three dimensional case, J.E. Luco (1976a,b) studied the torsional responses of structures with rigid hemispherical foundation and with rigid disk-type foundation to SH-waves. He obtained the results through analytic methods. However, the solutions for arbitrary shapes foundation may be very difficult to obtain analytically.

In this investigation, a concept of superposition combining with application of the hybrid model developed previously is proposed to analyze the response of an embedded structure subjecting to an incoming SH-wave. Correlations with Luco's results are studied first and the proposed method is intended to study the response of a structure with a rigid circular cylindrical embedded foundation. The numerical results are shown and discussed.

FREE-FIELD MOTION

The free-field motion is genereted by the incident wave and the reflected wave on the free-surface. For an incident plane SH-wave whose nonzero component is directed the y-axis can be expressed as

$$u_{yo}=u_{go}\cos\{\omega z\cos(\frac{\bar{\theta}}{c})\}\exp[-i\omega\{t+x\sin(\frac{\bar{\theta}}{c})\}] \qquad (1)$$

where

- u_{go} = amplitude of plane SH-wave at ground surface (z=0)
- ω = frequency of excitation
- c = shear wave speed of semi-infinite elastic medium
- $\bar{\theta}$ = angle between z-axis and the direction of incident plane SH-wave

With expression in spherical coordinates as shown in Fig.1, Eq.(1) may be written as (Luco 1976a)

$$u_{yo}=u_{go}\exp(-i\omega t)[j_o(kr) + \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}(-1)^n\{(-1)^m+(-1)^n\}\frac{(2n+1)!\,(n-m)!}{(n+m)!} \cdot j_n(kr)P_n^m(\cos\bar{\theta})P_n^m(\cos\theta)\cos m\phi] \qquad (2)$$

where $j_n(\cdot)$ denotes spherical Bessel function of n^{th} order, and k is the wave number.

Since only the torsional response is investigated in this study, it may be sufficient to consider the twisting angle corresponding u_{yo} in an average sense as defined as follows,

$$\bar{\phi}_o(r,\theta)=\frac{1}{2\pi r\sin\theta}\int_0^{2\pi}u_y\cos\phi d\phi = \frac{-u_{go}}{r\sin\theta}\sum_{n=1,3}^{\infty}(-1)^n\frac{(2n+1)}{n(n+1)}j_n(kr)P_n^1(\cos\bar{\theta}) \cdot P_n^1(\cos\theta)\exp(-i\omega t) \qquad (3)$$

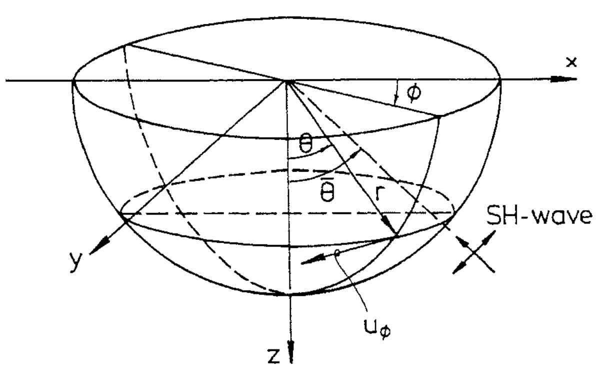

Fig.1 Semi-infinite Medium With a Hemi-spherical Pit

The torque about z-axis, $T_o(r,\theta)$ developed by shear stresses $\tau_{r\theta}$ along a ring with unit width at (r,θ) is

$$T_o(r,\theta)=\int_0^{2\pi}\tau_{r\theta}r^2\sin^2\theta d\phi = -2\pi G u_{go} r\sin^2\theta\sum_{n=1,3}^{\infty}(-1)^n\frac{(2n+1)}{n(n+1)} \cdot\{(n-1)j_n(kr)-(kr)j_{n+1}(kr)\} \cdot P_n^1(\cos\bar{\theta})P_n^1(\cos\theta)\exp(-i\omega t) \qquad (4)$$

where G denotes the shear modulus of soil.

ANALYSIS

In order to analyze the response of a structure during the pasage of seismic waves, we propose a concept of superposition and its application on the analysis of embedded structure.

Basic Principle

The equation of motion for a real structural system (RS system) under external actions may be written as

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=\{f\} \qquad (5)$$

where $[M]$, $[C]$, and $[K]$ denote the mass matrix, damping matrix and stiffness matrix of the RS system, respectively, $\{y\}$, $\{\dot{y}\}$ and $\{\ddot{y}\}$ denote the displacement vector, velocity vector and acceleration vector of the same system excited by an external action (or force) $\{f\}$, respectively.

Consider an auxiliary structural system (AS system) whose equation of motion is written as

$$[M_o]\{\ddot{y}_o\}+[C_o]\{\dot{y}_o\}+[K_o]\{y_o\}=\{f\}+\{\tilde{f}\} \qquad (6)$$

where $[M_o]$, $[C_o]$, $[K_o]$ denote the mass matrix, damping matrix and stiffness matrix of the AS system, respectively, $\{y_o\}$, $\{\dot{y}_o\}$ and $\{\ddot{y}_o\}$ are the given displacement vector, velocity vector and acceleration vector of the same system, respectively, and $\{\tilde{f}\}$ denotes the auxiliary force vector which is associated with $[M_o]$, $[C_o]$, $[K_o]$, $[y_o]$, $[\dot{y}_o]$ and $\{\ddot{y}_o\}$ and can be determined in advance.

By substracting Eq.(6) from Eq.(5), we have

$$[M]\{\ddot{y}_1\}+[C]\{\dot{y}_1\}+[K]\{y_1\}=\{f_1\} \qquad (7)$$

where

$$\{f_1\}=-[M_1]\{\ddot{y}_o\}-[C_1]\{\dot{y}_o\}-[K_1]\{y_o\}-\{\tilde{f}\} \qquad (8)$$

$$\{y_1\}=\{y\}-\{y_o\} \tag{9}$$

$$[M_1]=[M]-[M_o] \tag{10}$$

$$[C_1]=[C]-[C_o] \tag{11}$$

$$[K_1]=[K]-[K_o] \tag{12}$$

It is observed that in Eq.(7), the only unknown $\{y_1\}$ may be determined through the standard procedure in structural dynamics and then the response of the structural system $\{y\}$ can be calculated using the relation $\{y\} = \{y_1\}+\{y_o\}$.

Application on Embedded Structure

In what follows, we shall describe the procedure to analyze the response of an embedded structure subjecting to the seismic waves by applying the concept described above. For sake of brevity, $[C]$ and $[C_o]$ are assumed to be $[0]$.

Case (A) Flexible embedded structure

Consider the semi-infinite space without structure as an AS system which is indicated as region n'. The RS system is divided into three regions, namely l region, m region and n region, respectively. The degrees of freedom at nodes are classified as a, b, c and d groups. As shown in Fig.2, n region denotes the semi-infinite space, m region denotes the structural elements connecting with semi-infinite space, and l region indicates the rest elements of structure. The group a nodes are inside the region l, group b nodes locate at intersection of region l and region m, group c nodes locate at intersection of region m and region n, and group d nodes are inside the region n. Thus the matrices in Eqs. (5) and (6) can be written as

$$[M_o]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & M^{n'}_{cc} & 0 \\ 0 & 0 & 0 & M^{n'}_{dd} \end{bmatrix}, \quad [K_o]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & K^{n'}_{cc} & K^{n'}_{cd} \\ 0 & 0 & K^{n'}_{dc} & K^{n'}_{dd} \end{bmatrix} \tag{13}$$

$$[M]=\begin{bmatrix} M^{l}_{aa} & 0 & 0 & 0 \\ 0 & M^{lm}_{bb} & 0 & 0 \\ 0 & 0 & M^{mn}_{cc} & 0 \\ 0 & 0 & 0 & M^{n}_{dd} \end{bmatrix}, \quad [K]=\begin{bmatrix} K^{l}_{aa} & K^{l}_{ab} & 0 & 0 \\ K^{l}_{ba} & K^{lm}_{bb} & K^{m}_{bc} & 0 \\ 0 & K^{m}_{cb} & K^{mn}_{cc} & K^{n}_{cd} \\ 0 & 0 & K^{n}_{dc} & K^{n}_{dd} \end{bmatrix} \tag{14}$$

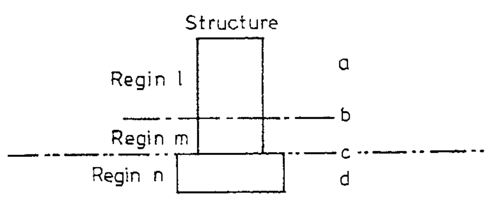

Fig.2 Model for Flexible Structure

where the superscript denotes the region and the subscript denotes the group of nodes. Note that M's and K's in region n has included the consideration of the existence of the embedded structure.

Let $\{y_{ao}\}=\{0\}$, $\{y_{bo}\}=\{0\}$

and $\{y_{co}\}$ and $\{y_{do}\}$ be displacements in free field motion. Thus $\{f\}=\{0\}$ and $\{f_1\}$ in Eq.(8) can be determined accordingly. Finally, the difference of displacement $\{y_1\}$ can be obtained through Eq.(7) and $\{y\}$ can be computed since $\{y\} = \{y_1\}+\{y_o\}$.

Case (B) Embedded structure including part of rigid body

In this case, the K's for a rigid part of the structure become infinite and the procedure described above may not be applicable. In order to overcome this difficulty, the auxiliary structural system should include all part of rigid body. In the analysis shown below, the real structural system will be chosen as the auxiliary structural system. The whole system as in case (A) is also divided l, m, n regions and the degrees of freedom are also classified as four groups, i.e., group a, b, c and d, respectively and are defined as in case (A). As shown in Fig.3, the region l includes the structure, the region n includes the field without containing the structure, which means the region l and n should not have common nodes, and the region m indicates the part between the regions l and n. It should be noted that there are no nodes existing inside the region m.

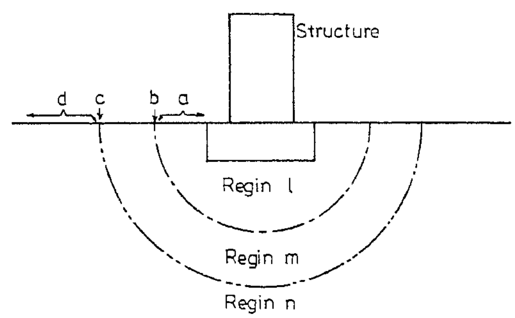

Fig.3 Model for Structure with Rigid Parts

Thus $[M_o]=[M]$, $[K_o]=[K]$, $\{y_{ao}\}=\{0\}$, $\{y_{bo}\}=\{0\}$, $\{y_{co}\}$ and $\{y_{do}\}$ are also the displacements in free field motion. The components of $\{\tilde{f}\}$ at nodes of groups a and d are zero, i.e., $\{\tilde{f}_a\}=0$ and $\{\tilde{f}_d\}=\{0\}$. However, the components of $\{\tilde{f}\}$ at nodes of groups b and c can not vanish and can be determined by the following equation,

$$\begin{Bmatrix} \tilde{f}_b \\ \tilde{f}_c \end{Bmatrix} = \begin{bmatrix} 0 \\ M^{m}_{cc} \end{bmatrix}\{\ddot{y}_{co}\}+\begin{bmatrix} K^{m}_{bc} \\ K^{m}_{cc} \end{bmatrix}\{y_{co}\}+\begin{Bmatrix} 0 \\ \tilde{f}_{co} \end{Bmatrix} \tag{15}$$

where $\{\tilde{f}_{co}\}$ is the stress on the intersectional surface between region m and n generated by free field motion. The remained procedure is the same as in case (A).

NUMERICAL EXAMPLES AND DISCUSSIONS

For the purpose of checking the applicability of the analysis procedure proposed, the torsional responses of circular column with rigid hemi-spherical foundation and with rigid disk-type foundation subjecting to SH-wave are calculated by employing the proposed method first, and comparisons are made to Luco's results (Luco,1976a, b). Then the same method is adopted to analyze the torsional response of circular column with rigid circular cylindrical embedded foundation subjecting to SH-wave.

The frequency of column with height H, torsional rigidity per unit length GJ and mass polar moment of inertia about its axis I_b may be expressed as (Luco, 1976a)

$$\omega_n = (2n-1)/(2H\sqrt{GJH/I_b}) \;;\quad n = 1,2,\cdots \tag{16}$$

The harmonic torque applied at the bottom of column in order to generate a twisting angle $\alpha(0)\exp(-i\omega t)$ is

$$T(0) = (-\omega^2 I_b \frac{\tan k_b H}{k_b H})\,\alpha(0)\exp(-i\omega t) \tag{17}$$

and the associated twisting angle at top of column is

$$\alpha(H) = \alpha(0)\sec k_b H \exp(-i\omega t) \tag{18}$$

where ω is the frequency of applied torque, and

$$k_b H = \frac{\pi}{2}(\frac{\omega}{\omega_1})/\sqrt{1+2\xi i} \tag{19}$$

in which ξ denotes the hysteretic damping coefficient of the column.

It should be mentioned that in the analysis, the displacement field $\{y_o\}$ is considered as $\{\bar{\phi}_o\}$, which can be calculated with Eq.(3), and the associated force field $\{\tilde{f}\}$ is regarded as $\{T_o\}$ which can be determined with Eq.(4) accordingly.

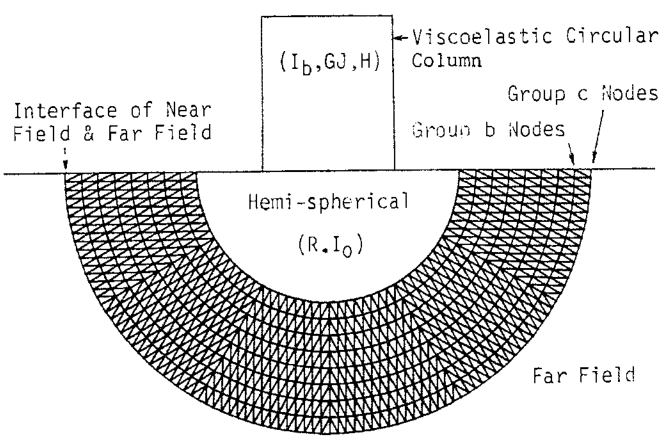

Fig.4 Analysis Model for Circular Column with Rigid Hemi-spherical Foundation

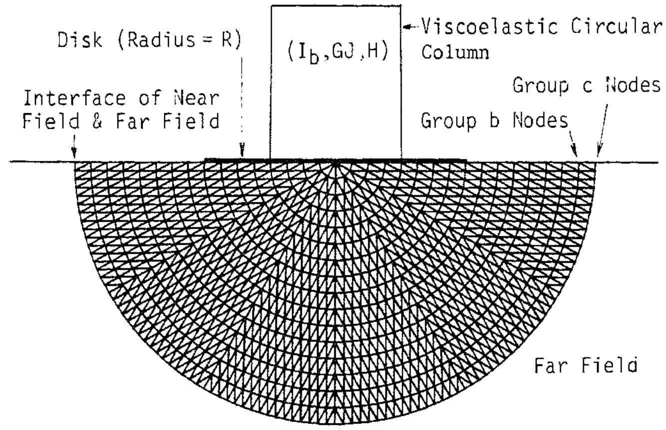

Fig.5 Analysis Model for Circular Column with Rigid Disk-type Foundation

The models for analysis of rigid hemi-spherical foundation and rigid disk-type foundation are shown in Fig.4 and Fig.5, respectively. The parameters are chosen as $I_b/I_s=1.2$, $I_o/I_s=0.3$, $\xi=0.01$ and $\bar{\theta}=90^o$, where $I_s = (4\pi/15)\rho_s R^5$ denotes the mass moment of inertia of hemi-spherical medium with mass density ρ_s, and I_o denotes the mass moment of inertia of the foundation about z-axis.

The responses of these two cases are shown in Fig.6 and Fig.7, respectively, where the Luco's analytic results are also shown for the purpose of comparison. It is observed that the results calculated by the proposed method agree very well with the analytic results, and the applicability of the proposed method may be confirmed accordingly.

Thus the proposed method is extended to analyze the response of a case of rigid circular cylindrical embedded foundation in order to show the feasibility of the method. The analytic solution of this problem is difficult to obtain.

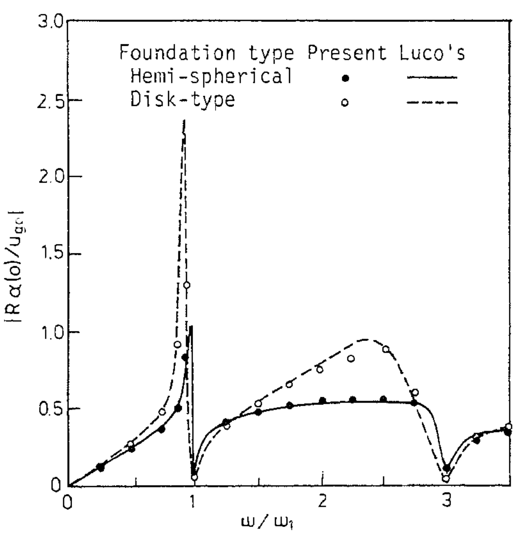

Fig.6 Normalized Response at Foundation

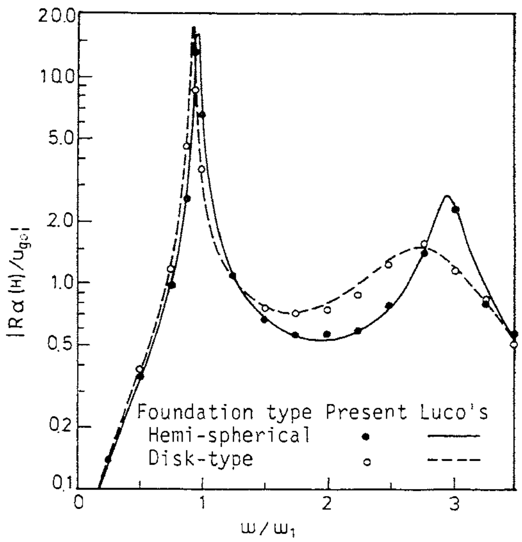

Fig.7 Normalized Response at The Top

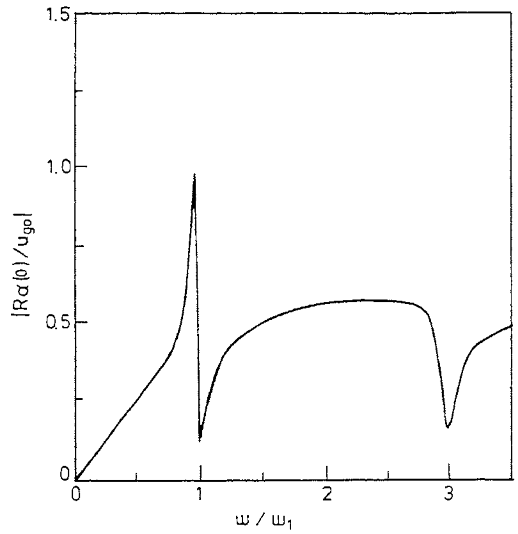

Fig.9 Normalized Response at Rigid Circular Cylindrical Embedded Foundation

The model for analysis is shown in Fig.8, and the results are shown in Fig.9 and Fig.10. According to the results obtained, it should be pointed out that the response at the top of the column is closer to that of case of rigid hemispherical foundation.

As shown in Figs. 6 and 9, the responses of foundation corresponding to the vicinities of $\omega/\omega_1 = 1$ and 3, are quite small. This may be caused by the large dynamic stiffness at the bottom of super-structure when ω equals to its natural frequencies of first mode and second mode, respectively.

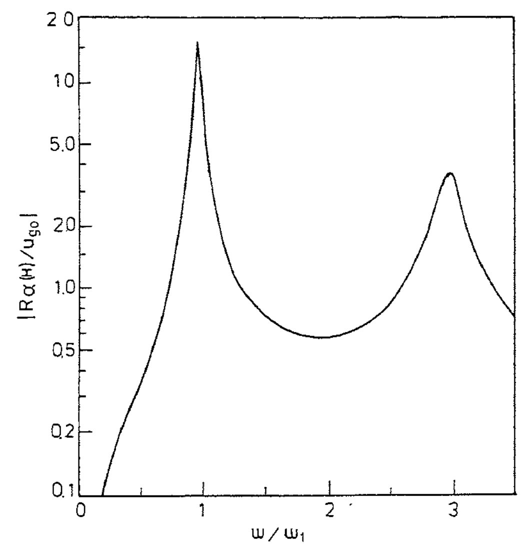

Fig.10 Normalized Response at the Top of Column with Rigid Circular Cylindrical Embedded Foundation

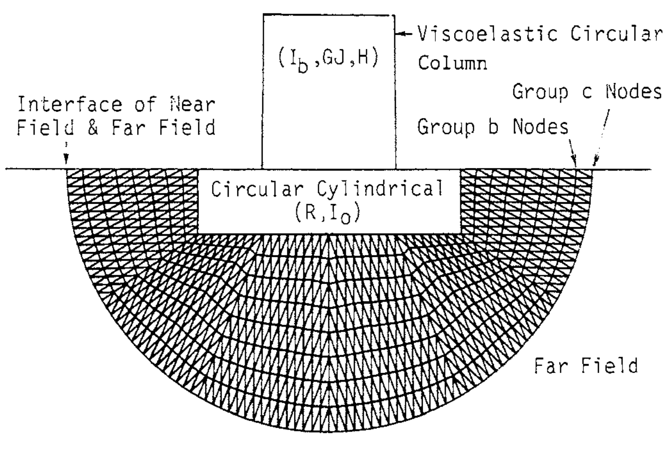

Fig.8 Analysis Model for Circular Column with Rigid Circular Cylindrical Foundation

CONCLUDING REMARK

A proposed concept of superposition combining with the use of hybrid model may be employed to study the dynamic interaction of soil and atructure with ir-regular embedded foundation subjecting to incoming seismic waves. Although the results shown in this paper are only for the torsional responses of structures with rigid foundation, the method has the potential of being extended to study the more general cases, for example, vertical, horizontal and rocking responses of the embedded structures.

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