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Free Response of Shells on Flexible Foundation

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SYNOPSIS Free response of axisymmetric shells resting on flexible foundation in their asymmetric modes is studied using Finite Elements based on harmonic formulation. Influence of various parameters, such as subgrade modulus, proximity of boundary, density ratio, shear wave velocity, embedment of structure and depth of flexible foundation, on free response is studied. Axisymmetric interface element with harmonic functions is developed and introduced between the raft slab and foundation media.

INTRODUCTION

Free response of axisymmetric structures in their asymmetric modes is greatly affected by foundation flexibility. Works of Chandrasekaran, et al (1977) and Agarwal, et al (1973) are notable in this regard. Solution of this problem using finite element formulation based on harmonic analysis is attempted here. An 8-noded isoparametric element is used for modelling structure and foundation media. The interface between the structure and foundation is modelled by extending concept of interface element to harmonic analysis.

Parameters associated with foundation flexibility are considered for studying their influence on frequencies and mode shapes.

FORMULATION

The displacement functions, Fig.1(a), for m^{th} symmetric harmonic are

$$u=\sum_{i=1}^{\infty} u_m \cos m\theta;\quad v=\sum_{i=1}^{\infty} v_m \cos m\theta;\quad w=\sum_{i=1}^{\infty} w_m \sin m\theta \qquad (1)$$

Using these displacement functions for an 8-noded isoparametric element and finite element discretization in meridional direction, the variation of displacements in r, z and θ directions in the element take the form

$$u_m=\sum_{i=1}^{8} N_i u_{mi};\quad v_m=\sum_{i=1}^{8} N_i v_{mi};\quad w_m=\sum_{i=1}^{8} N_i w_{mi} \qquad (2)$$

where N_i are the usual quadratic shape functions.

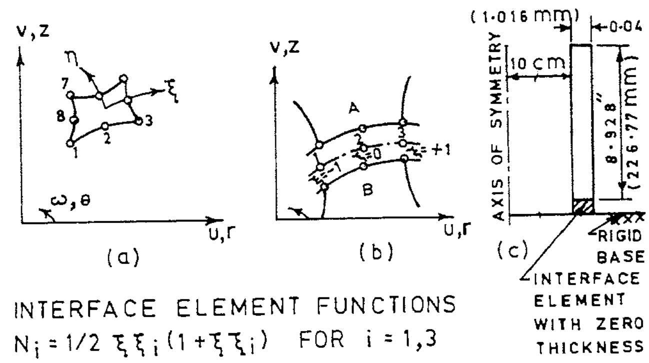
For interface element the relative displacements (strains), Fig. 1(b), are expressed as

$$\Delta u_m=\sum_{i=1}^{3} N_i \Delta u_{mi}\cos m\theta;\quad \Delta v_m=\sum_{i=1}^{3} N_i \Delta v_{mi}\cos m\theta$$

$$\Delta w_m=\sum_{i=1}^{3} N_i \Delta w_{mi}\sin m\theta \qquad (3)$$

where $\Delta u_{mi}=u_{mBi}-u_{mAi};\quad \Delta v_{mi}=v_{mBi}-v_{mAi}$

$$\Delta w_{mi}=w_{mBi}-w_{mAi} \qquad (4)$$



INTERFACE ELEMENT FUNCTIONS
$N_i = 1/2\ \xi\xi_i(1+\xi\xi_i)$ FOR $i = 1,3$
$N_i = (1-\xi^2)$ FOR $i = 2$

FIG.1 VARIOUS FINITE ELEMENTS USED

Substituting equation (3) into (4), one obtains

$$\underset{\sim}{\varepsilon_e} = \underline{B}\ \underset{\sim}{\delta_e} \qquad (5)$$

where underlined quantities represent rectangular matrices and wavy line denotes column matrix.

The strains in global coordinates are related to local coordinates as

$$\underline{T} = \frac{1}{S}\begin{bmatrix} \frac{dr}{d\xi} & \frac{dz}{d\xi} & 0 \\ -\frac{dz}{d\xi} & \frac{dr}{d\xi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

$$\text{where } S = \sqrt{(dr/d\xi)^2 + (dz/d\xi)^2} \qquad (7)$$

The tangential and normal stresses corresponding to strains are obtained using relation

$$\underset{\sim}{\sigma_e} = \underline{D_e} \; \underset{\sim}{\varepsilon_e} \; ; \; \underline{D_e} = \text{diag}\,[k_s \; k_n \; k_\theta] \qquad (8)$$

in which diagonal elements k_s, k_n and k_θ are the tangential, normal and rotational interface stiffness parameters.

Element Stiffness and Mass Matrices

The element stiffness matrix for 8-noded element is arrived at following usual procedure, Vyas (1981),

$$\underline{K_e} = \lambda \int_{-1}^{+1} \int_{-1}^{+1} \underline{B}^T . \underline{D} . \underline{B} . r . |\det| . d\xi . d\eta \qquad (9)$$

where $\lambda = \pi$ for any harmonic $m \geq 1$, $\lambda = 2\pi$ for $m = 0$, $|\det|$=determinent of Jacobian matrix, $\underline{D}$ = elasticity matrix and r is radius.

The element stiffness matrix for interface element is obtained as

$$\underline{K_e} = \lambda \int_{-1}^{+1} \underline{B}^T . \underline{T}^T . \underline{D_e} . T . B . |\det| r . d\xi \qquad (10)$$

where $|\det|$ is the length of the element and λ is same as mentioned in equation (9).

The element consistent mass matrix is obtained using the relation

$$\underline{M} = \lambda \int_{-1}^{+1} \int_{-1}^{+1} (\underline{N}^T \underline{N}) . \rho . r . |\det| . d\xi . d\eta \qquad (11)$$

where $\underline{N}$ is the shape function matrix and ρ is the mass density.

NUMERICAL STUDIES

Free response of cylinder with raft slab resting on flexible foundation involving interface element is studied for various parameters associated with foundation flexibility. Results are presented in Tables I through VII and Figs. 2 through 6. Free response of cylinder on rigid foundation involving interface element is studied, for influence of interface stiffness parameters, and results for various harmonics are given in Tables VIII and IX, and in Fig. 7.

TABLE I. Variation of Frequencies with Harmonic Number

Harmonic No.	Frequencies in Hz Mode 1	2	3	4	5
1	0.3864	0.5363	0.6259	0.6700	0.8200
2	0.4551	0.7078	0.7878	0.8384	0.9623
3	0.5452	0.8395	0.9376	0.9709	1.0662

Elastic modulus for cylinder = 2.5×10^6 T/M^2,
Poisson's ratio = $\mu = 0.2$,
$\rho = 0.2446$ T-sec^2/M^4

TABLE II. Foundation Material Density Variation

Mass Density (T-sec^2/M^4)	Frequencies in Hz Mode 1	2	3	4	5
0.1650	0.3864	0.5363	0.6259	0.6700	0.8201
0.1800	0.3699	0.5139	0.6015	0.6209	0.7856
0.2000	0.3509	0.4879	0.5709	0.6087	0.7458

TABLE III. Variation of Subgrade Modulus

Subgrade Modulus (T/M^2)	Frequencies in Hz Mode 1	2	3	4	5
6000	0.3864	0.5363	0.6259	0.6700	0.8201
11000	0.5231	0.7261	0.8490	0.9072	1.1110
30000	0.8637	1.1991	1.4003	1.4981	1.8332

TABLE IV. Variation of Proximity of Boundary

Radial Distance of Boundary Nodes (m)	Frequencies in Hz Mode 1	2	3	4	5
150	0.3864	0.5363	0.6259	0. 6700	0.8200
125	0.4056	0.5937	0.7427	0. 7589	0.8632
85	0.4681	0.7577	1.0317	0. 1760	1.1049

TABLE V. Variation of Depth of Foundation

Depth of Foundation (m)	Frequencies in Hz Mode 1	2	3	4	5
150	0.3864	0.5363	0.6259	0.6700	0.8200
90	0.4851	0.6719	0.6985	0.7687	1.0390
60	0.6221	0.7877	0.9460	1.1060	1.1480

TABLE VI. Variation in Shear Wave Velocity

Shear Wave Velocity (m/sec)	Frequencies in Hz Mode 1	2	3	4	5
110	0.3864	0.5363	0.6259	0.6700	0.8200
150	0.4984	0.6472	0.8159	0.8812	1.0070

TABLE VII. Variation of Embedment of Structure

Depth of Embed-ment (m)	Frequencies in Hz				
	Mode				
	1	2	3	4	5
0.00	0.3864	0.5363	0.6279	0.6700	0.8200
16.67	0.3583	0.5179	0.6100	0.6628	0.7941

DISCUSSION AND CONCLUSIONS

The study of influence of Poisson's ratio on frequencies showed negligible variation, whereas increase in density of foundation strata results into decrease in frequencies of all the modes, Table II. A reduction of 9.18 percent in frequency of first mode is observed for about 18 percent increase in density. Higher mode frequencies are also affected to the same extent. The increase in density alters the deformation pattern as seen in Fig. 3.

Subgrade modulus varied from 6000 T/M^2 to a value of nearly rigid foundation as seen from Table III, has significant influence on frequencies of all the modes while mode shape amplitudes are least affected.

A range of 85 m to 150 m is considered for studying influence of proximity of boundary on the frequencies and mode shapes as presented in Table IV and Fig. 4. An increase of 4.96 percent in first mode frequency is observed for shifting the boundary nodes from 150 m to 125 m, while for 150 m to 85 m shift the corresponding value is increased by 21 percent. This is consistent with the increase in rigidity of the system with boundary nodes coming close to the structure. The vibration pattern also is greatly affected as observed from Figs. 2 and 4, in which the vertical displacement components are significantly different.

As seen in Table V, the variation in depth of flexible foundation significantly affects the frequencies. An increase of 25 percent in first mode frequency is observed for reducing the depth of flexible foundation layer from 150 to 90 m. The corresponding value is 69 percent for reducing the depth from 150 m to 60 m. Unlike the influence of proximity of horizontal boundary, in this case horizontal components of displacement in first mode are affected to a greater extent. The location of occurrence of maximum radial displacement amplitude also has changed, Fig. 6.

TABLE VIII. Effect of Interface Stiffness on Frequencies - Fourth Harmonic

Interface Stiffnesses			Frequencies in Hz		
k_s	k_n (T/M^2)	k_θ	Mode 1	Mode 2	Mode 3
$7.08x10^5$	$7.08x10^5$	$7.08x10^5$	345.0	1280.0	2820.0
$7.08x10^6$	$7.08x10^6$	$7.08x10^6$	399.0	1400.0	3060.0
$7.08x10^7$	$7.08x10^7$	$7.08x10^7$	454.0	1540.0	3255.0
$7.08x10^8$	$7.08x10^8$	$7.08x10^8$	466.0	1600.0	3380.0
$7.08x10^8$	$7.08x10^8$	0.00	466.0	1540.0	3190.0

Elastic Modulus for cylinder = $2.11x10^7$ T/M^2; $\rho = 0.795$ $T\text{-}sec^2/M^4$; $\mu = 0.3$

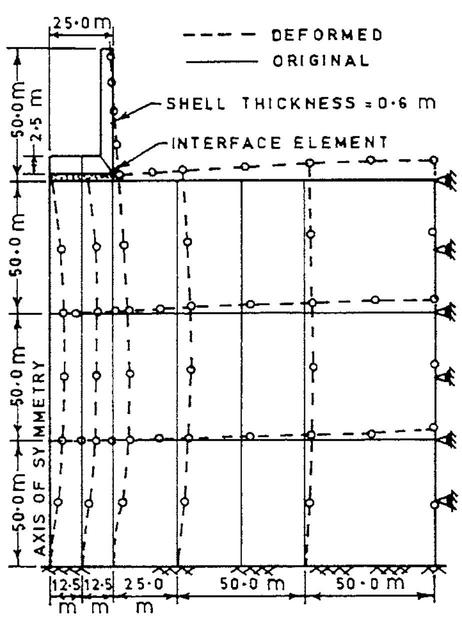


FIG.2 OPEN CYLINDER ON ELASTIC FOUNDATION - FIRST HARMONIC MODE SHAPES

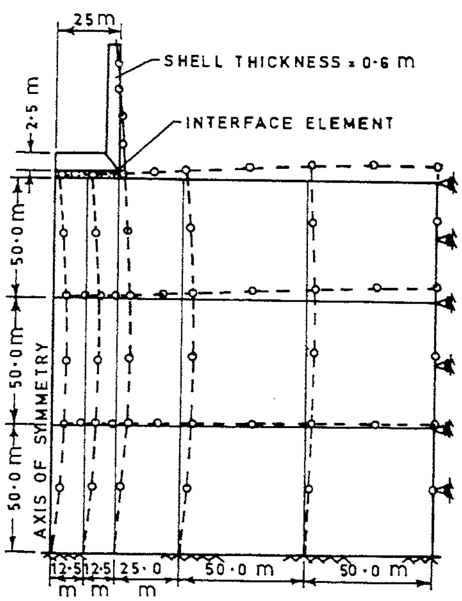


FIG.3 OPEN CYLINDER ON ELASTIC FOUNDATION FIRST HARMONIC MODE SHAPE - DENSITY VARIATION

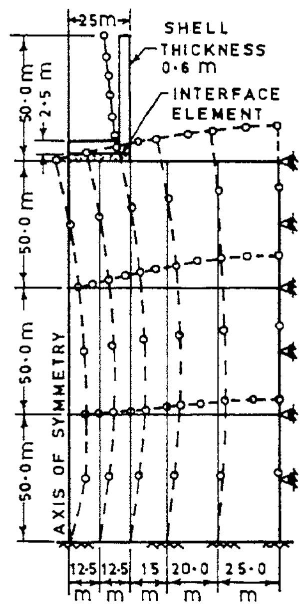


FIG.4 OPEN CYLINDER ON ELASTIC FOUNDATION FIRST HARMONIC PROXIMITY VARIATION

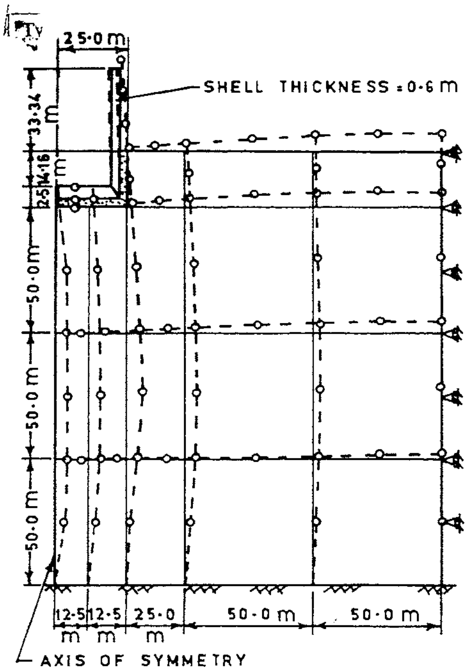


FIG.5 CYLINDER ON ELASTIC BASE WITH INTERFACE ELEMENT FIRST HARMONIC MODE SHAPES WITH EMBEDMENT

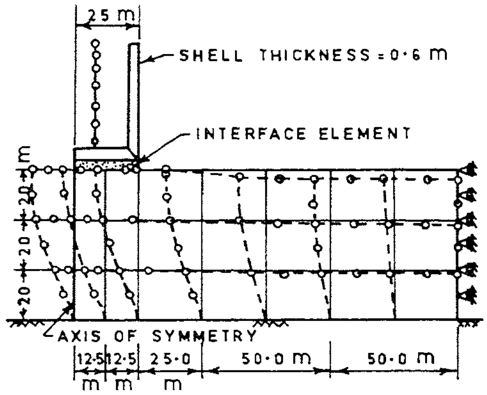


FIG.6 OPEN CYLINDER ON ELASTIC FOUNDATION-FIRST HARMONIC MODE SHAPES-FOUNDATION DEPTH VARIATION

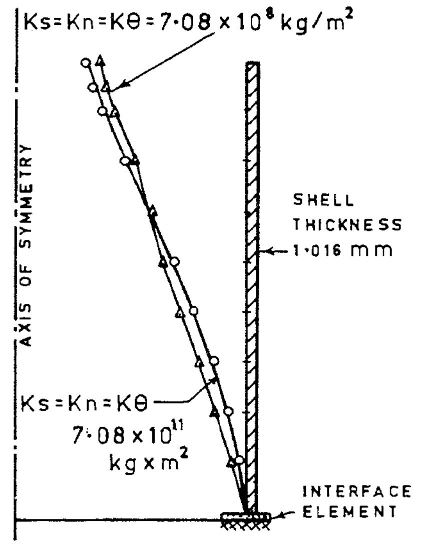


FIG.7 CYLINDER ON RIGID BASE WITH INTERFACE ELEMENT FOURTH HARMONIC MODE SHAPES

TABLE IX. Frequencies (Hz) of Cylinder Fixed at Base - Role of Interface Element

Har monic No.	With Interface Mode 1	2	3	Without Interface Mode 1	2	3
1	1624.0	4660.0	6582.0	2008.0	5440.0	7028.0
2	736.0	3050.0	5408.0	970.0	3470.0	5870.0
3	441.0	2000.0	4025.0	515.0	2330.0	4550.0
4	399.0	1400.0	3060.0	423.0	1680.0	3530.0
5	538.0	1140.0	2436.0	-	-	-
6	858.0	1180.0	2110.0	873.0	1250.0	-

Value of $k_s = k_n = k_\theta = 7 \cdot 08 \times 10^6$ is assumed

The shear wave velocity, which is a measure of subgrade stiffness and mass, is varied from 110 m/sec. to 150 m/sec. The increase in the shear wave velocity results in increase in frequencies of all the modes as seen in the Table VI.

Embedment of structure in flexible media results in nonproportional increase in the mass of the system, thereby reducing the frequencies of the system. A reduction of 7.38 percent in first mode frequency is observed for embedment of one-third height of the structure, Table VII. The horizontal components of modal displacements, Fig. 5, are significantly influenced.

The influence of extent of bonding between the base of the structure and the foundation media, Fig. 1(c), in various harmonics studied through stiffness parameters of interface element, on frequencies of structural system is presented in Table VIII and Fig. 6. It is observed that the circumferential stiffness has negligible effect on first five mode frequencies predicted using fourth harmonic. Modal amplitudes are also affected, with variation in the interface stiffness parameters k_s, k_n and k_θ, as seen in Fig. 7.

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