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T. Kobori
Kyoto University, Japan

R. Minai
Kyoto University, Japan

K. Baba
Osaka University, Japan

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T. Kobori, Professor
Architectural Engineering, Faculty of Engineering, Kyoto University, Japan

R. Minai, Professor
Disaster Prevention Research Institute, Kyoto University, Japan

K. Baba, Research Assistant
Architectural Engineering, Faculty of Engineering, Osaka University, Japan

SYNOPSIS In order to obtain a theoretical prediction on the seismic response of the soil-pile-structure systems, the frequency response of the soil-pile system excited by ground incident waves may be necessary in addition to the response due to the excitation at the pile head.

This paper is concerned with a theoretical analysis based on the three-dimensional wave propagation theory to find dynamical interaction characteristics of an elastic pile embedded in the viscoelastic soil stratum on a rigid bed rock, subjected to a concentrated external force or forced displacement at the pile head and to uniformly distributed bed rock motion. In dealing with this complicated boundary configuration and exciting condition, the technique of superposition principle associated with the auxiliary subproblems is effectively used. And, the governing equations in frequency domain reduce to the Fredholm type integral equations of the second kind, whose solutions are expressed in terms of multiple summations and integrals.

INTRODUCTION

There has recently been an increasing interest in the dynamical behavior of soil-pile-structure systems during wind and earthquake excitations. As far as analytical studies based on the wave propagation theory are concerned, this problem has been explored by H. Tajimi (1966, 1976) and M. Novak (1974,1976,1977). The authors have also studied on the dynamical interaction of a pile embedded in a viscoelastic half space when subjected to a concentrated force excitation at the pile head (1975, 1976, 1977).

The objective of this paper is to present a method of theoretical analysis of an elastic pile and the surrounding viscoelstic soil stratum on a rigid bed rock when subjected to the concentrated external excitation at the pile head and to uniformly distributed bed rock motion.

General approaches to such dynamical interaction problems based on the three-dimensional wave propagation theory are to be related to solve a class of mixed boundary value problems with complex boundary configurations and exciting conditions.

In dealing with these complicated boundary value problems, the total soil-pile interaction field is separated into the free-field motion without a pile and the interacted field due to the presence of the pile. And, the latter field is further separated into the two sets of fields, corresponding to the following subproblems;

(I) one related to a viscoelastic soil stratum enlarged symmetrically with respect to the free-surface and with an elastic pile inclusion,

(II) the other is the original viscoelastic soil stratum rested on a rigid bed rock. Then, the respective stress and displacement components of the auxiliary problems are combined to satisfy the original boundary conditions.

By applying the integral transforms and series expansions with respect to time and spatial variables, the mixed equations composed of the Fredholm type simultaneous series and integral equations are derived in the domain of frequency and wave numbers. By making use of iterated kernel method in solving the above equations, the solutions in frequency domain are expressed in terms of multiple summations and integrals. Finally, numerical results are presented for some physical properties of a soil-pile interaction system.

FORMULATION OF THE PROBLEM

The displacement vector u along the pile, consisting of the components of horizontal translation and rotation about the horizontal axis, is represented as the following forms;

$$u = Su_G + Bu_{ro}$$
$$u_{ro} = [(I-K_p^{-1}K_s)^{-1}-I]S_o u_G \qquad (1)$$

where u_G is the harmonic displacement uniformly distributed on the bed rock, while u_{ro} is the relative displacement vector associated with the harmonic excitation at the pile head, and K_p, K_s are the stiffness matrices of a pile and a super-structure. S, S_o are the transfer vectors of the displacement along a pile and at the pile head to u_G, B is the transfer matrix of the displacement along a pile associated with u_{ro}, and I is the unit matrix. Determination of the stiffness and transfer matrix of the pile requires the solution of the following mixed boundary value problem in the frequency domain,

$$\begin{aligned} L(u) &= 0 && x \in V \\ \beta_1(u) &= 0 && x \in \Gamma_1 \\ \beta_2(u) &= 0 && x \in \Gamma_2 \\ \beta_3(u) &= f && x \in \Gamma_3 \end{aligned} \qquad (2)$$

in which x and u are the position vector and the displacement vector in a viscoelastic soil stratum on a rigid bed rock, and (i) the three-dimensional wave equation of a viscoelastic stratum V given by the vector differential operator L, (ii) the stress-free condition associated with the operator β_1 at the surface Γ_1 of the soil stratum outside the region of a pile, (iii) the welded contact condition associated with the operator β_2 at the interface Γ_2 of the stratum and the rigid bed rock, (iv) the condition of equilibrium of the pile, the boundary conditions at the tips of pile and non-deformability with respect to the circular section of the pile Γ_3, as well as the welded contact condition between a pile and its surrounding soil stratum, associated with the operator β_3. In addition, the radiation condition in the infinitely far field is required to be satisfied.

In the other hand, for the determination of the transfer vector S of the pile for the harmonic bed rock motion, it is convenient to write the soil-pile interaction field in the absolute coordinate system as follows;

$$u = u_i + u_r \tag{3}$$

where u_i is the free-field motion of the soil stratum without a pile and u_r is the interacted field due to the presence of the pile foundation. It is required that the free-field motion satisfies

$$\begin{aligned} L(u_i) &= 0 && x \in V \\ \beta_1(u_i) &= 0 && x \in \Gamma_1 \\ \beta_2(u_i) &= u_G && x \in \Gamma_2 \end{aligned} \tag{4}$$

in which the displacement field u_i is obtained in the following form,

$$u_i = u_G \sec\left(\frac{\omega H}{c^*}\right)\cos\left(\frac{\omega z}{c^*}\right) \tag{5}$$

where ω is the frequency of the harmonic excitation, c^* and H are the phase velocity and thickness of the viscoelastic soil stratum.

The equations requested for the interacted field u_r have the same form as in the case of the field excited at the pile head in absence of the bed rock excitation, namely, eq.(2), but the inhomogeneous term f in this case is determined by using the free-field motion, whereas the one due to the excitation at the pile head is given in term of the external excitation.

The displacement vector field u presented in cylindrical polar coordinates (r,θ,z), as shown in Fig. 1, in which the pile-axis coincides with the z-direction, can be expressed in terms of potentials of dilatational and distortional components as follows;

$$u = \nabla\phi + \nabla\times(\psi k) + \nabla\times\nabla\times(\chi k) \tag{6}$$

where ∇ and k denote the gradient operator and the unit base vector along the z-axis, ϕ, ψ and χ are particular solutions of the associated scalar Helmholtz equations;

$$[\nabla^2 + (\frac{\omega}{c_1^*})^2]\phi = [\nabla^2 + (\frac{\omega}{c_2^*})^2]\psi = [\nabla^2 + (\frac{\omega}{c_2^*})^2]\chi = 0 \tag{7}$$

in which c_1^* and c_2^* are the phase velocities of dilatational and distortional waves.

This field is separated into the two sets of displacement fields u_I and u_{II}, which correspond to the subproblems mentioned previously, and given in the following potential forms;

$$\begin{aligned} \left.\begin{matrix}\phi_I \\ \psi_I \\ \chi_I\end{matrix}\right\} &= \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \left\{\begin{matrix} A_{mn}K_m(\alpha_n r) \\ B_{mn}K_m(\beta_n r) \\ A_{mn}K_m(\beta_n r)\end{matrix}\right\} \exp i(p_n z + m\theta) \\ \left.\begin{matrix}\phi_{II} \\ \psi_{II} \\ \chi_{II}\end{matrix}\right\} &= \sum_{m=-\infty}^{\infty}\int_0^{\infty} dq \left\{\begin{matrix} [E_{mn1}\exp(-\alpha z)+E_{mn2}\exp(\alpha z)] \\ [F_{mn1}\exp(-\beta z)+F_{mn2}\exp(\beta z)] \\ [G_{mn1}\exp(-\beta z)+G_{mn2}\exp(\beta z)]\end{matrix}\right\} \times J_m(qr)\exp(im\theta) \end{aligned} \tag{8}$$

where p_n and q are the parameters of wave number, and their associated parameters are;

$$\alpha_n = \sqrt{p_n{}^2 - h^2},\quad \beta_n = \sqrt{p_n{}^2 - k^2},\quad \alpha = \sqrt{q^2 - h^2},$$

$$\beta = \sqrt{q^2 - k^2},\quad h = \omega/c_1^*,\quad k = \omega/c_2^*,$$

$$p_n = (2n-1)\pi/2H,\quad n = 1,2,3\cdots\cdots$$

and $J_m(x)$, $K_m(x)$ are the Bessel and modified Bessel functions of integer order m, both of which are zero at infinity.

The stress and displacement components derived from the potentials of the subproblems (I), (II) are superposed to satisfy the boundary conditions.

In order to derive the boundary equations in the domain of wave numbers, the Hankel transform with respect to r is applied to the potentials of the subproblem (I), which are multiplied by the cutoff operator $U(r-a)$ to avoid the singularity of the modified Bessel functions, for the boundary surfaces perpendicular to the z-axis. Similarily, for the boundary surface along the pile, the finite Fourier transform with respect to z is applied to the potentials of the subproblem (II) and the inhomogeneous terms, f, which are extended symmetrically or anti-symmetrically in the image domain of the soil stratum of the subproblem (II).

The similar procedures of the superposition and the integral transform techniques have been used by H. Tajimi (1974) and T. Kobori, et al. (1977). As regards to θ, the finite Fourier transform is operated by considering the periodicity condition.

In consequence, all terms of each boundary equation are arranged to have the same transform operator, and any spatial variable disappears, so that the mixed equations composed of the Fredholm type simultaneous series integral equations of the second kind determining the unknown coefficients of the potentials are obtained in the form,

$$\begin{aligned} G_{11}(\omega,p_n)X_1(\omega,p_n) + \int dq G_{12}(\omega,p_n,q)X_2(\omega,q) &= f(\omega,p_n) \\ G_{21}(\omega,q)X_2(\omega,q) + \sum_n G_{22}(\omega,q,p_n)X_1(\omega,p_n) &= 0 \end{aligned} \tag{9}$$

By making use of the reciprocal kernel method in solving the above series integral equations, any frequency response of this interaction system is expressed in terms of multiple summations and integrals as follows;

$$Y(\omega)=A_0(\omega)+\sum_n A_1(\omega,p_n)f(\omega,p_n)+\sum_n\int dq A_2(\omega,p_n,q)f(\omega,p_n)+\cdots \qquad (10)$$

Only in the case of bed rock excitation, the first term in the right hand side of this equation appears, showing the contribution of the field without a pile. The second term represents the frequency response of the pile bedded in the enlarged soil stratum, and the remaining terms may show the effects of the surface waves propagating along the free-surface. In this study, the following systems are analyzed; (1) the one subjected to the forced horizontal displacement u_0 and the forced rotation θ_0 about the horizontal axis at the pile head, (2) the other subjected to the uniformly distributed horizontal bed rock motion. For the brevity of expressions, the following dimensionless parameters and dimensionless components with superscript ($\bar{}$) are introduced, though the letter will be suppressed throughout the analysis, unless otherwise noted;

$$\bar{p}_n=p_nH,\ \bar{q}=qH,\ \bar{\alpha}_n=\alpha_nH,\ \bar{\beta}_n=\beta_nH,\ \bar{\alpha}=\alpha H,\ \bar{\beta}=\beta H,$$
$$\bar{k}=kH,\ \bar{h}=hH,\ \bar{k}_0=\mathrm{Re}(\bar{k}),\ d=a/H,\ \bar{z}=z/H,$$
$$\bar{\mu}=\frac{\mu^*}{\mathrm{Re}(\mu^*)},\ D=\frac{\mathrm{Im}(\lambda^*)}{\mathrm{Re}(\lambda^*)}=\frac{\mathrm{Im}(\mu^*)}{\mathrm{Re}(\mu^*)},\ \mu=\mathrm{Re}(\mu^*), \qquad (11)$$
$$\overline{E_pI_p}=\frac{E_pI_p}{\mu a^4},\ \bar{\rho}_p=\frac{\rho_p}{\rho},$$
$$\bar{u}=u/a,\ \bar{u}_G=u_G/a,\ \bar{\theta}=\theta,\ \bar{Q}=Q/\mu a^2,\ \bar{M}=M/\mu a^3.$$

where E_pI_p and a are the bending stiffness and radius of the pile, ρ_p and ρ are the densities of the pile and soil medium, respectively.
In order to constract the model of soil-pile-structure systems and to obtain the dynamical responses of the pile and its surrounding soil, the degrees of freedom of the pile head are to be at least two in translational and rotational directions.
For instance, in the case of the soil-pile systems subjected to both concentrated external excitations at the pile head and uniformly distributed bed rock motions in the horizontal direction, the stiffness matrix associated with the horizontal and rotational displacement, u_0 and θ_0 at the pile head, and the displacement transfer vector for the bed rock motion are expressed as follows;

$$\begin{Bmatrix} Q \\ M \end{Bmatrix}=\begin{bmatrix} K^{HH} & K^{HR} \\ K^{RH} & K^{RR} \end{bmatrix}\begin{Bmatrix} u \\ \theta \end{Bmatrix}$$
$$\begin{Bmatrix} u \\ \theta \end{Bmatrix}=\begin{Bmatrix} S^{H} \\ S^{R} \end{Bmatrix}u_G \qquad (12)$$

where K^{IJ} and S^I are the stiffnesses and amplification functions of the systems, respectively, and K^{HR} is equal to K^{RH} in consequence of reciprocity theorem. For instance, one of the elements K^{HH} of the dimensionless stiffness matrix of the soil-pile system can be analytically determined by using the following quantities;

$$\frac{u_0}{Q_0}=\sum_{n=1}^{\infty}U_0^1(k_0,p_n)+\sum_{n=1}^{\infty}\int_0^{\infty}dq U_0^2(k_0,p_n,q)+\cdots \qquad (13)$$

in which

$$U_0^1=\frac{2}{\pi d D(p_n)}[k^2Y_\beta+(1+\beta_n dY_\beta)(p_n^2Y_\beta-\alpha_n\beta_nY_\beta)],$$
$$U_0^2=\frac{2\beta_n q}{\pi d D(p_n)F(q)}(2+\beta_n dY_\beta)[qdJ_0(qd)-J_1(qd)]u_0^2(p_n,q),$$
$$D=(\frac{d^2}{\pi}\overline{E_pI_p}p_n^4-\overline{\rho_p}k_0^2)\times$$
$$\times[k^2Y_\beta+(1+\beta_n dY_\beta)(p_n^2Y_\beta-\alpha_n\beta_nY_\alpha)]$$
$$+\frac{\bar{\mu}\beta_n}{d}[k^2(4+\beta_n dY_\beta)+\beta_n d(p_n^2Y_\beta-\alpha_n\beta_nY_\alpha)],$$
$$F=4\alpha\beta q^2(2q^2-k^2)\mathrm{sech}(\alpha)\mathrm{sech}(\beta)$$
$$-\alpha\beta[(2q^2-k^2)^2+4q^4]$$
$$+q^2[(2q^2-k^2)^2+4\alpha^2\beta^2]\tanh(\alpha)\tanh(\beta),$$
$$u_0^2=(-1)^n k^2 p_n\beta(\xi_\beta-\xi_\alpha)[2\alpha\beta\,\mathrm{sech}(\alpha)\tanh(\beta)-(2q^2-k^2)\tanh(\alpha)\mathrm{sech}(\beta)]$$
$$+[(2\alpha_n^2+k^2)\xi_\alpha-2p_n^2\xi_\beta]\times$$
$$\times[\alpha\beta(4q^2-k^2)\{\mathrm{sech}(\alpha)\mathrm{sech}(\beta)-1\}$$
$$+\{q^2(2q^2-k^2)+2\alpha^2\beta^2\}\tanh(\alpha)\tanh(\beta)],$$
$$Y_\beta=\frac{K_0(\beta_n)}{K_1(\beta_n)},\qquad \xi_\beta=\frac{\beta_nJ_0(qd)Y_\beta+qJ_1(qd)}{q^2+\beta_n^2}.$$

The complete stiffness matrix is constructed by adding the relevant terms to the above basic elements to satisfy the elastic support condition for the rotation at the lower end of the pile.
It can be shown that the rectifying term to satisfy the elastic support condition of the lower end of the pile is determined through the similar procedure to that in this analysis.
As to the convergence of the solution associated with higher order differential operators with respect to spatial variables, the higher terms than the third may be necessary in the series integral representation of the solutions, as given by eq. (10).

NUMERICAL ANALYSIS AND CONCLUSIONS

In evaluating the basic dynamical characteristics of the soil-pile systems, it is assumed that the soil is composed of the linear hysteretic type viscoelastic medium, and its generalized Lame's constants are expressed by

$$\mu^*=\mu(1+iD)$$
$$\lambda^*=\lambda(1+iD) \qquad (14)$$

The numerical values of dimensionless system parameters for the pile and its surrounding soil stratum are chosen as;

$$\bar{\rho}_p = 1$$

$$\overline{E_p I_p} = 500,\ 1000,\ 1500 \qquad (15)$$

$$d = 0.025,\ 0.05,\ 0.075$$

$$D = 0.1,\ 0.2,\ 0.3$$

$$\nu = 0.25,\ 0.333,\ 0.45$$

In the numerical integration and summation to obtain the frequency responses according to eq. (10) or (13), there are no singular points such as poles and branch points as long as the real-valued wave number parameters are concerned, because of the presence of dissipative damping in the soil stratum. Therefore ordinary methods of computation can be applied while an appropriate interpolation technique is necessary in evaluating multiple integrals.

The following remarks can be made on the results of numerical analysis;

(1) As shown in Table 1, the convergence of the series expansions of the frequency response is rather rapid in the parameter ranges considered here.

(2) As found from Figs. 2 to 7, the complex-valued stiffness functions show the considerable reduction in the real parts, accompanied by a sudden increase in the imaginary part when the frequency parameter increases across the natural frequencies of the soil stratum. For zero material damping, the dimensionless natural frequencies of the soil stratum of horizontal vibration are given by

$$k_0 = (2n-1)\pi/2, \quad n=1,2,3\cdots\cdots$$

In the range below the first resonant frequency, the real part decreases with frequency whereas the imaginary part is very small mostly caused by material damping. In the frequency range between the first and second resonance, both real and imaginary parts of the stiffness function almost increase but the rotational components K^{RR} show relatively smooth variations.

(3) As for the effect of the slenderness ratio of a pile, the real part of the stiffness function shows somewhat complicated variations, whereas the imaginary part decreases significantly in the frequency range above the first resonance.

(4) As the stiffness ratio of pile to soil medium or Poisson's ratio of soil increases, the stiffness function becomes large particularly for the real part and for the imaginary part in the range above the first resonant frequency.

(5) The effect of material damping of soil is remarkable on the imaginary part almost all frequency range considered here, but not so significant for the real part.

(6) As shown in Figs. 8 to 9, the displacement amplification functions of the pile head to the horizontal bed rock motion show very marked variations around the resonant frequencies of soil stratum. The real parts change rapidly their sign and the imaginary parts have extremal points near the resonant frequencies of the soil stratum. As the order of the resonant frequency increases, the values near the resonant frequencies of the horizontal component of the amplification function remain to be in the same magnitude, whereas those of the rotational component show remarkable increase in its magnitude.

(7) In Figs. 10 and 11, the transfer functions of bending moment along the pile to the horizontal displacement excitation at the pile head and to the uniformly distributed bed rock motion are shown, respectively, at the dimensionless frequencies $k_0=1,2,3$.

It is noted, however, that the boundary condition associated with the rotation at the lower end of the pile are not yet completely satisfied in these figures. About the general trend of Fig. 10, the distribution characteristics of the bending moment are rather different in the frequency ranges below and above the resonant frequency of the soil stratum, especially for the imaginary part. In Fig. 11, it is shown that the general trend of the distribution characteristics do not change in the frequency range below the second resonance of the soil stratum.

In conclusion, it can be mentioned that the stiffness matrix and amplification vector associated with the pile head and the transfer functions of relevant outputs along the pile both to excitations at the pile head and on the bed rock as well as those for the superstructures are necessary to obtain the seismic responses of soil-pile-structure systems. And in determining the stiffness matrix and the transfer functions of soil-pile systems, the superposition and integral transform techniques are shown to be effective to the formulation as the Fredholm type simultaneous series integral equations.

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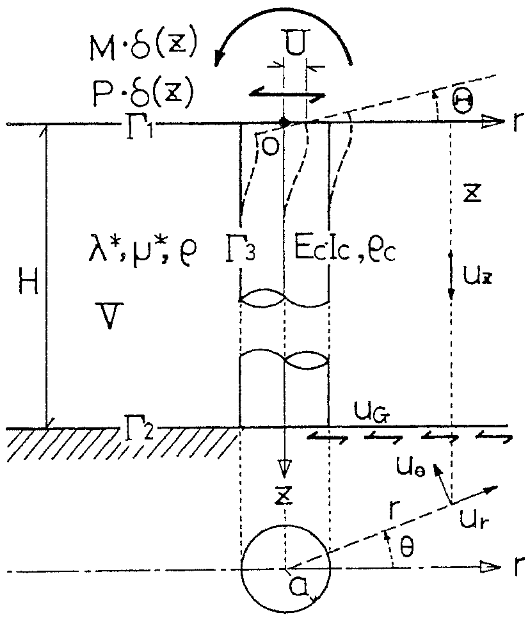

Fig. 1. Configuration of the soil-pile system

Table 1.

Convergence of series expanded solution in eq. (13)
($\overline{E_p I_p}$=1000, $\bar{\rho}_p$=1, d=0.05, ν=0.333, $\frac{Im(\mu^*)}{Re(\mu^*)}$=0.1)

$\bar{u}_0/\bar{Q}_0$		$\sum_{n=1}^{10} U_0^1$	$\sum_{n=1}^{10}\int_0^\infty dq\ U_0^2$
k_0=1.2	Re	0.08069	-0.00019
	Im	-0.00801	0.00044
2.4	Re	0.06343	0.00010
	Im	-0.03114	0.00012
3.6	Re	0.05820	-0.00008
	Im	-0.02782	-0.00018

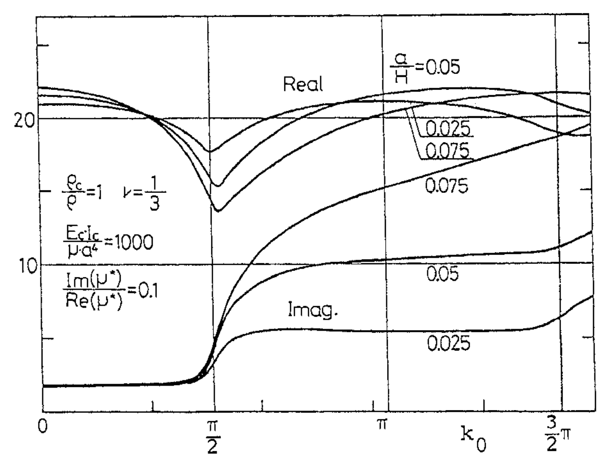

Fig. 2. Dimensionless stiffness function K^{HH}

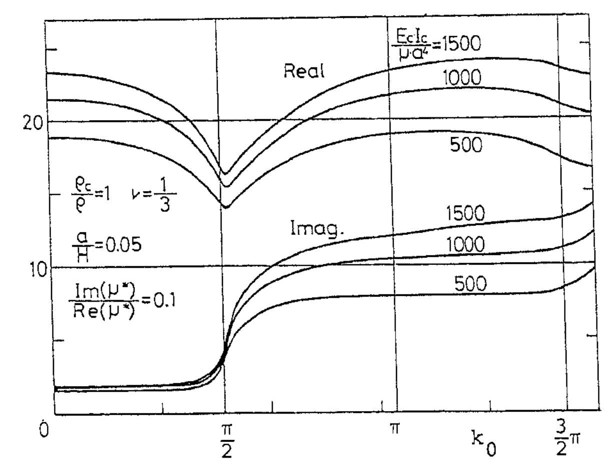

Fig. 3. Dimensionless stiffness function K^{HH}

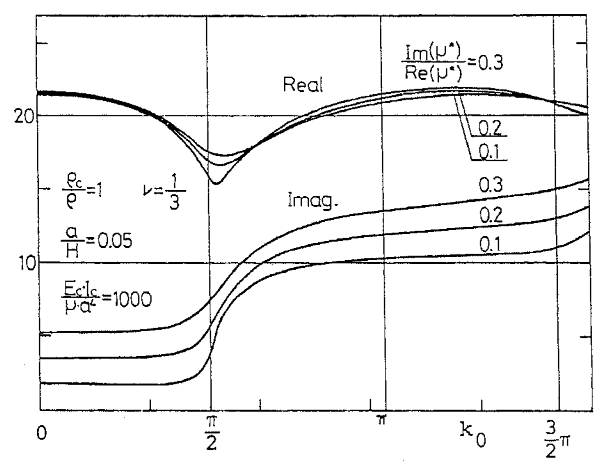

Fig. 4. Dimensionless stiffness function K^{HH}

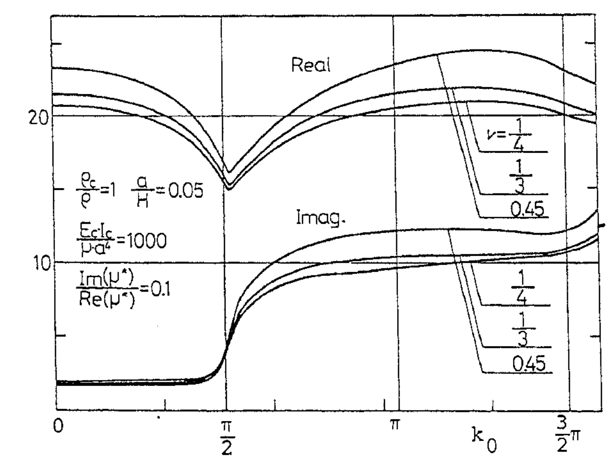

Fig. 5. Dimensionless sfiffness function K^{HH}

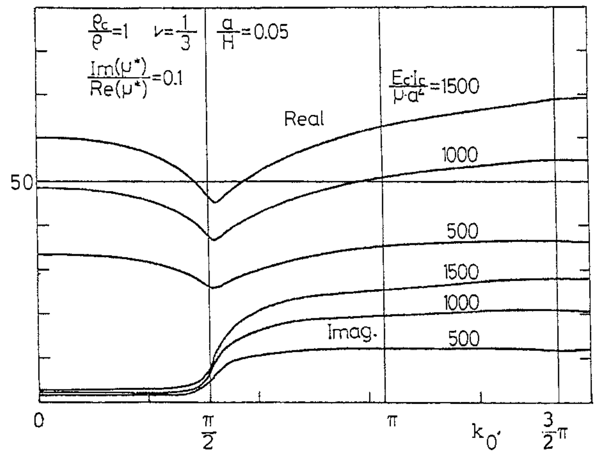

Fig. 6. Dimensionless stiffness function K^{RH}

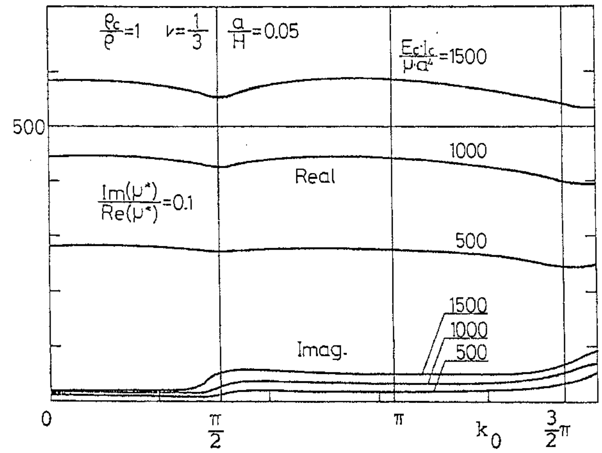

Fig. 7. Dimensionless stiffness function K^{RR}

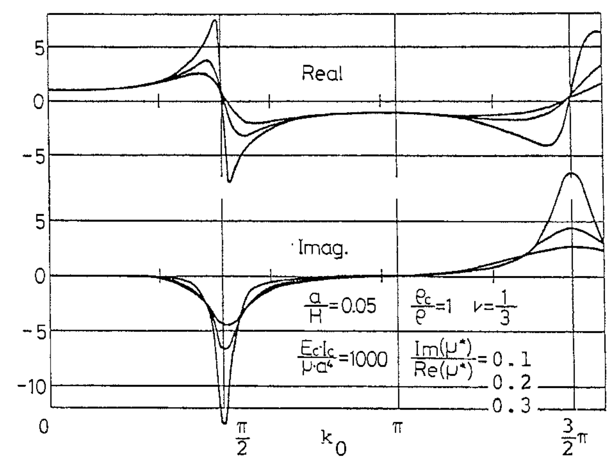

Fig. 8. Dimensionless amplification function S^H

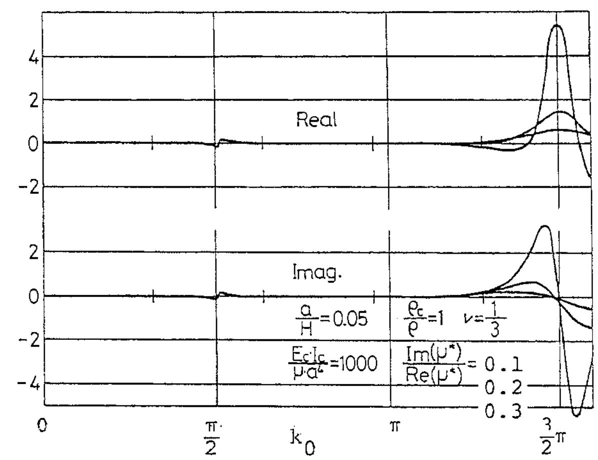

Fig. 9. Dimensionless amplification function S^R

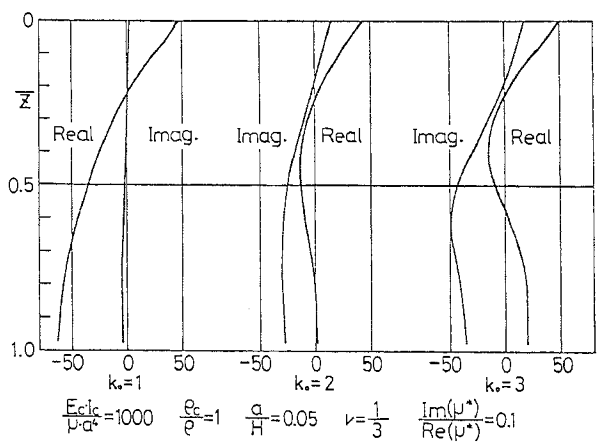

Fig. 10. Distribution of dimensionless bending moment M^H

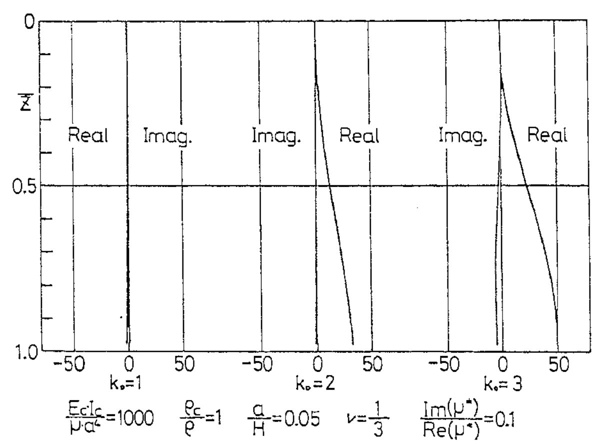

Fig. 11. Distribution of dimensionless bending moment M^G