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SYNOPSIS **The** dynamic soil-pile interaction problem is solved by the method of characteristics. The nonlinear, non-homogeneous problem was idealised as a piecewise linear problem. The numerical instability of semi-infinite soil column model has been reported, and a stable model, wherein the soil column below the pile tip is replaced by a single spring and a dashpot, has also been presented. The results obtained from the method of characteristics have been compared with those obtained by explicit finite difference scheme. The convergence and stability were studied numerically.

INTRODUCTION

The interaction between the soil and the pile is a complex phenomenon especially when the loading is dynamic. Most of the current methods of analysis of the interaction problems can be divided into either spring-dashpot-mass model or the one dimensional wave equation approach. The latter in particular is adopted for the prediction of the static load carrying capacity of the pile. Smith (1960, 1962) has solved the wave equation with the help of finite difference expansion along the length of the pile and over time as an initial value problem. Excellent reviews (Coyle et al., 1973, Forehand et al., 1964) and applications (Thompson, 1980) of wave equation approach are available. A number of solution procedures viz. CAPWAP, WEAP, PDA etc., have been developed and fairly close agreement (within ± 20 percent) between field observed and the predicted value have been shown (Appendino, 1980, and Thompson and Thompson, 1979). It is generally agreed that correct assessment of soil properties, e.g. quake factor, viscosity coefficient, distribution of load along pile etc., is extremely important for better prediction of the load carrying capacity.

The method of characteristics is a very powerful tool and an elegant method for solving hyperbolic partial differential equations. Using this approach Rakhmatulin (1966) and Cristescu (1967) present many solutions, and Freiburger (1952) studied the enlargement of a hole under dynamic loading for an elastic perfectly plastic material. Ginsburg (1964) applied this method for finding the soil response to blast in the air, and Streeter et al (1974) analysed the dynamic behaviour of saturated and unsaturated media. In this paper, the pile vibration problem has been formulated as a one dimensional wave propagation problem in a non-homogeneous medium, the pile itself resting on a soil column.

PILE-SOIL MODEL

Fig.1 shows a pile of length L, diameter d,

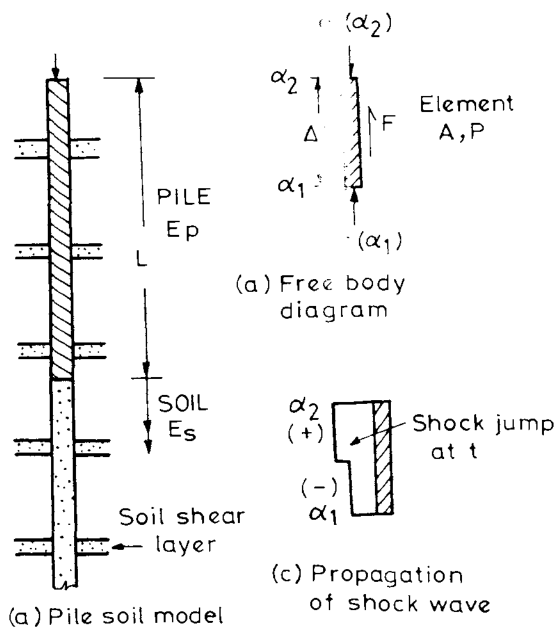

Fig.1. Definition Sketch

and with modulus of elasticity E_p. The pile material can take compressive or tensile forces but does not possess either viscous or plastic properties. The pile rests on a semi-infinite soil column whose deformation modulus is E_s. The soil takes only compressive forces and possesses plastic yielding and viscous resistance. The pile and soil columns are supported by the surrounding soil medium as a shear spring-dashpot system with plastic resistance.

FORMULATION

The formulation is based on Lagrangian co-ordinate system. Considering an element of length d α with cross sectional area A and perimenter P, the kinematic relationship is written as :

$$\frac{\delta\varepsilon}{\delta t} = \frac{\delta^2 u}{\delta t \delta\alpha} = \frac{\delta}{\delta\alpha}\left(\frac{\delta u}{\delta t}\right) = \frac{\delta v}{\delta\alpha} \tag{1}$$

where $\epsilon = \frac{\delta u}{\delta\alpha}$ - the engineering strain, t - time, u - the axial displacement, α - length variable, and v - velocity. The conservation of momentum relationship is derived as

$$\frac{\delta}{\delta t}\int_{\alpha_1}^{\alpha_2} m A v \, d\alpha = A\,\sigma(\alpha_2) - A\,\sigma(\alpha_1) - \int_{\alpha_1}^{\alpha_2} \tau\, P\, d\alpha \tag{2}$$

where m the mass density of the pile material, α_1 and α_2 distances to bottom and top of element (Fig.1b), σ - normal stresses, and τ - shear resistance offered by the shear layer. Eq. (2) can be simplified to

$$m\,\frac{\delta v}{\delta t} = \frac{\delta\sigma}{\delta\alpha} - \frac{P}{A}\,\tau \tag{3}$$

Neglecting heat energy losses stress-strain relation can be written as

$$\frac{\delta\epsilon}{\delta t} = f(\sigma,\varepsilon)\,\frac{\delta\sigma}{\delta t} + g(\sigma,\varepsilon) \tag{4}$$

where f (σ, ε) and g (σ, ε) are given functions.

When an impact is given to the top of the pile, a 'shock wave' takes place because of the abrupt change in the dependent variables, and their derivatives become infinite at the shock front. The jump in the values of dependent variables is denoted by [ ]. Fig. 1(c) depicts the propagation of shock front at time t. Denoting the magnitude of the dependent variables just above and and just below the shock front by + and - respectively, the conservation of momentum relation is written as:

$$\left(mv\,\frac{d\alpha}{dt}\right)_- + \int_{\alpha_1}^{\alpha(t)} \frac{\delta}{\delta t}(mv)\,d\alpha - \left(mv\,\frac{d\alpha}{dt}\right)_+ + \int_{\alpha(t)}^{\alpha_2} \frac{\delta}{\delta t}(mv)\,d\alpha = \sigma(\alpha_2) - \sigma(\alpha_1)$$

$$- \frac{P}{A}\left(\int_{\alpha_1}^{\alpha(t)} \tau_2\,d\alpha + \int_{\alpha(t)}^{\alpha_2} \tau_1\,d\alpha\right) \tag{5}$$

where $\alpha(t)$ is the location of the shock front. Neglecting the shock jump in the soil shear resistance, Eq.(5) simplifies to:

$$m\,\frac{d\alpha}{dt}(v)_- - m\,\frac{d\alpha}{dt}(v)_+ + (\sigma)_- - (\sigma)_+ = 0,$$

$$\text{or,}\; - m\,\frac{d\alpha}{dt}\,[v] = [\sigma] \tag{6}$$

Since the entire system of the above equations has to be solved simultaneously, an equation is formed as :

$$\mu_1(mv_t - \sigma_\alpha + F) + \mu_2(\varepsilon_t - v_\alpha) + \mu_3(\varepsilon_t - f\sigma_t - g) = 0 \tag{7}$$

where $F = \frac{P}{A}(k.u + S_j v)$, k and S_j - shear stiffness and viscous coefficients of soil, μ_1,μ_2 and μ_3 are arbitrary parameters selected to choose some direction α_t such that the dependent variables are most simply related in that directions, the subscript t or α denotes derivative with respect to t or α. Separating the variables of Eq.(7), one gets

$$\frac{dt}{d\alpha} = \frac{\mu_3 f}{\mu_1} = - \frac{\mu_1 m}{\mu_2} = \frac{-(\mu_2 + \mu_3)}{0} \tag{8}$$

These equalities yield 3 possible solutions, viz.

$$\mu_1 = \mu_2 = 0,\; \mu_3 \text{ is arbitrary} \tag{9a}$$

$$\mu_2 = -\mu_3, \quad \mu_1 = \mu_2 \sqrt{\frac{f}{m}} \tag{9b}$$

$$\mu_2 = \mu_3, \quad \mu_1 = -\mu_2 \sqrt{\frac{f}{m}} \tag{9c}$$

Eq.9(a) through (c) yield $d\alpha = 0$, and $\frac{d\alpha}{dt} = \pm \frac{1}{\sqrt{fm}} = \pm\sqrt{\frac{E}{m}} = \pm c$ where, $E = \frac{1}{f}$ is the tangent modulus and c is the wave propagation velocity of the medium. These are the desired characteristic directions. The corresponding characteristic equations are

Along $d\alpha = 0, \quad \varepsilon_t - f\,\sigma_t - g = 0$

Along $\frac{d\alpha}{dt} = -c, \; (\frac{1}{c}\, v_t - v_\alpha)$

$+ \frac{1}{mc}(\frac{1}{c}\, \sigma_t - \sigma_\alpha) + g_1 = 0$

Along $\frac{d\alpha}{dt} = c, \; -(\frac{1}{c}\, v_t + v_\alpha) +$ (10)

$+ \frac{1}{mc}(\frac{1}{c}\, \sigma_t + \sigma_\alpha) + g_2 = 0$

where $g_1 = (\frac{1}{mc}\, F + g)$ and

$g_2 = -(\frac{1}{mc}\, F - g)$

For mathematical simplicity, a coordinate transformation from $\alpha - t$ plane to $\varphi-\eta$ coordinate systems has been defined as:

$$\frac{\delta}{\delta\varphi} = \frac{1}{c}\frac{\delta}{\delta t} - \frac{\delta}{\delta\alpha} \quad \text{and}$$

$$\frac{\delta}{\delta\eta} = \frac{1}{c}\frac{\delta}{\delta t} + \frac{\delta}{\delta\alpha} \tag{11}$$

The above transformation changes Eq.(10) to

along $d\alpha = 0, \quad \varepsilon_t - f\sigma_t - g = 0$

along $d\eta = 0, \quad v_\varphi + \frac{1}{mc}\,\sigma_\varphi + g_1 = 0$

along $d\varphi = 0, - v_\eta + \frac{1}{mc}\,\sigma_\eta + g_2 = 0$ (12)

Each of these equations is related to one independent variable only. For the incremental analysis Eq. (12) become

$$d\,\varepsilon - f.\, d\sigma = g dt$$

$$d\,v + \frac{1}{mc}\, d\sigma = -g_1\, d\varphi$$

$$-dv + \frac{1}{mc}\, d\sigma = g_2\, d\eta \tag{13}$$

The solution proceeds from known initial conditions along some non-characteristic line. In case of pile subjected to an impact, the solution proceeds from the pile head to its tip and then to soil column below. The slope of the $\varphi - \eta$ characteristics for the pile length will be $\pm$ c for all values of t. The velocity of impact is assumed to be known and the stress rise is found out from the shock conditions ($\sigma = -$ m cv). Though this rise is instantaneous in nature, it is assumed to increase over a small time increment. The velocity and stress increase linearly from 0 at $t = 0$, to $v = v_{impact}$, and $\sigma = \sigma_{impact}$ at $t = t_{rise}$. For $t > t_{rise}$ the pile head has a stress free condition. For $t \leqslant t_{rise}$ both σ and v are known along a non-characteristic $\alpha = 0$, while for $t > t_{rise}$ only σ is known on $\alpha = 0$.

To evaluate the dependent variables in the $\alpha - t$ plane, the characteristic equations are expanded in their finite difference form along their corresponding characteristic directions. Fig. 2 (a) shows a schematic

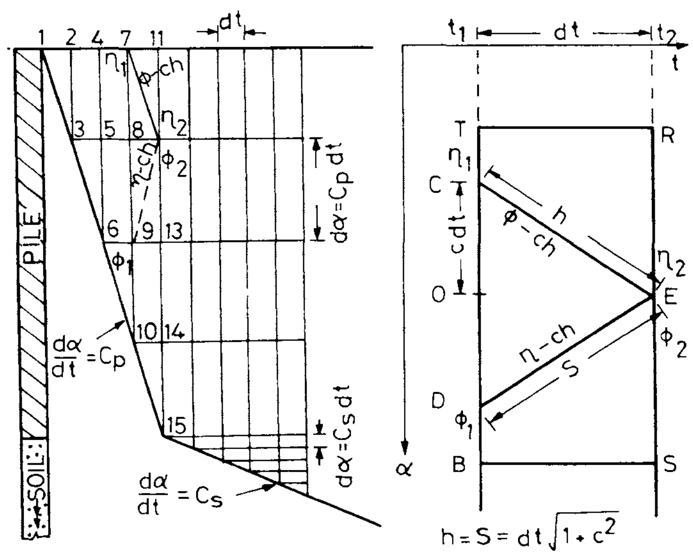

Fig. 2. Schematic Representation

representation of the hybrid scheme refering to Fig. 2(a), 12 is a typical node on line 11-15 on which the dependent variables are unknown. $\varphi_1 - \varphi_2$ is the η characteristic drawn from 12 which meets 8-9 at φ_1 and $\eta_1 - \eta_2$ is the φ characteristic drawn from 12 which meets 7-8 at η_1. Expanding Eq.(13)

with the aid of Taylor's series, over steps of lengths h and s along η and φ characteristics respectively,

$$v(\varphi_2) + \frac{1}{mc}\sigma(\varphi_2) = v(\varphi_1) + \frac{1}{mc}\sigma(\varphi_1) - \sum_{n=0}^{\infty} \frac{h^{n+1}}{(n+1)} \frac{\delta^n}{\delta_\varphi^n}\left(\frac{F}{mc} + g\right)$$

$$v(\eta_2) - \frac{1}{mc}\sigma(\eta_2) = v(\eta_1) - \frac{1}{mc}\sigma(\eta_1) - \sum_{n=0}^{\infty} \frac{s^{n+1}}{(n+1)} \frac{\delta n}{\delta_\eta n}\left(\frac{F}{mc} - g\right) \quad (14)$$

the summation terms appearing from Taylor's series expansions. However, $v(\varphi_2) = v(\eta_2)$ and $\sigma(\varphi_2) = \sigma(\eta_2)$ and Eq. (14) can be solved simultaneously for velocity and stresses at each node.

The stress-strain behaviour of the soil column is considered as bilinearly elastic during compressive loading and for compressive part of the unloading it is elastic, with higher tangent modulus than the previous loading, from the point of velocity reversal. Thus it allows plastic deformation of the soil column. During the propagation of wave in soil, whenever stress falls below zero, a very small value of elastic modulus is considered for the soil. This is a crude approximation to the real soil behaviour because soil cannot take tension and separation may take place under tensile stress conditions. A numerical experimentation carried out on the basis of the above assumptions indicated unstable conditions, the instability being initiated the instant the stress wave enters the soil column. The sudden change in the value of the soil modulus from compressive loading to tensile unloading caused a discontinuity on the wave front.

The column of soil was replaced by a spring dashpot system as is done in the conventional approaches. If the higher order terms are neglected, Eq. (14) simplifies to

$$v(\varphi_2) + \frac{1}{mc}\sigma(\varphi_2) = v(\varphi_1) + \frac{1}{mc}\sigma(\varphi_1) - \frac{h\,F(\varphi_1)}{mc}$$

$$v(\eta_2) - \frac{1}{mc}\sigma(\eta_2) = v(\eta_1) - \frac{1}{mc}\sigma(\eta_1) - \frac{s\,F(\eta_1)}{mc} \quad (15)$$

TRESBO (Fig. 2 b) is a typical grid. The dependent variables v and F are known at T, O, and B. E is the point on t_2 where the dependent variable are to be found out. CE and DE which represent φ and η characteristics respectively through E are at slope $\pm$ c and they intersect t_1 line at C and D respectively. The increment on the space axis is dL i.e. TO=OB = dL and the increment on time axis is dt.

Thus CE = DE $=\sqrt{(1+c^2)}\,dt$ implies $h = s$

$$OC = CD = c\,dt \quad (16)$$

The interpolated values of the dependent variables at C from the known values of dependent variables at T and O are

$$\sigma(\varphi_1) = \sigma_O + \frac{\sigma_T - \sigma_O}{dL}\cdot c\,dt$$

$$v(\varphi_1) = v_O + \frac{v_T - v_O}{dL}\cdot c\,dt$$

$$F(\varphi_1) = F_O + \frac{F_T - F_O}{dL}\cdot c\,dt \quad \text{etc.} \quad (17)$$

Eq. (15) and (17) combine to give

$$v_E = \frac{v_c + v_D}{2} + \frac{\sigma_c - \sigma_D}{2\,mc} - h\,\frac{F_c + F_D}{2\,mc}$$

$$\sigma_E = \frac{mc}{2}(v_c - v_D) + \frac{\sigma_c + \sigma_D}{2} - h\,\frac{F_c - F_D}{2} \quad (18)$$

$$\text{or, } v_E = v_O + \left(\frac{v_T - 2v_O + v_B}{2\,dL}\cdot c\,dt\right) + \frac{\sigma_T - \sigma_B}{2mc} - \frac{h}{mc}\left(F_O + \frac{F_T - 2F_O + F_B}{2\,dL}\,c\,dt\right)$$

$$\sigma_E = m\,c\,\frac{v_T - v_B}{2} + \left(\sigma_o + \frac{\sigma_T - 2\sigma_O + \sigma_B}{2\,dL}\cdot c\,dt\right) - h\,\frac{F_T - F_B}{2} \quad (19)$$

The dependent variables at C and D are evaluated by use of linear interpolation. The influence of the values of dependent variables at T, B and O in Eq. (17) depends on the ratio $c\,dt/dL$. By decreasing the space interval dL, or increasing the time interval dt, points C and D might shift toward T and B and the error due to interpolation reduced. However, reduction in d L will increase the amount of computation and error due to rounding off. On the other hand, increase in d t might reduce the number of computations but will increase the error of truncation of Taylor's series used in the finite difference expansion. Moreover, the nonlinear behaviour of soil shear resistance, which has been idealized as stepwise linear, does not permit a high value of time interval.

RESULTS

Numerical experimentation was conducted with this program to find the effect of time-step, length of the element of the pile and the effect of spring stiffness which introduces the nonlinearity in the wave equation through the shear resistance term. A concrete pile of 12'' Ø (30.48 cm) and of length 80 ft. (24.4 m) with a capblock and a ram was chosen for this experimentation. The stiffness of soil was selected as 40 lb/in^3. (1.1 kg/cm^3). It is observed that the time displacement curve for the method of characteristic always lies much above the explicit finite difference scheme (Smith's approach). For a time increment of 0.00025 sec and elemental length of 10 ft. (3.05 m) each, the maximum penetration in the explicit finite difference scheme was 1.38 in. (3.5 cm) and that for characteristic scheme was 0.96 in. (2.44 cm) and the rebound was 0.3 in in (7.6 cm) and 0.48 in (1.22 cm) respectively, for the assumed quake factor of 0.1 in. (0.25 cm). Values of time step (0.0001, 0.00025 and 0.00050 sec) do not change the time-displacement pattern appreciably, as also the length of the pile element considered 10 ft. (3.05 m) and 5 ft (1.52 m). But, when the element length is increased to 20 ft. (6.1 m) the time-displacement curves change significantly. It clearly indicates that $d\,t < d\,L/c_p$ cannot be the only criterion for convergence. Fig. 3(c) and (d) compare the time-displacement curves from both the methods for shear resistance of 120 lb/in^3 (3.3 kg/cm^3) and 200 lb/in^3 (5.5 kg/cm^3). Higher shear resistance introduces strong nonlinearities through the shear force term but it is observed that with time interval of 0.00025 sec and elemental length of 10 ft (3.05 m), a stable solution can be achieved.

Fig. 4 shows the distribution of displacement along the length at 2 selected elapsed times 0.01 sec. and 0.02 sec. The same 12'' Ø (30.48 cm) concrete pile of 80 ft (24.4 m) length and soil stiffness of 200 lb/in^3 (5.5 kg/cm^3) was selected for this study. It is observed that the length-displacement variations are smooth at the observed instants. Further study is needed to improve this method.

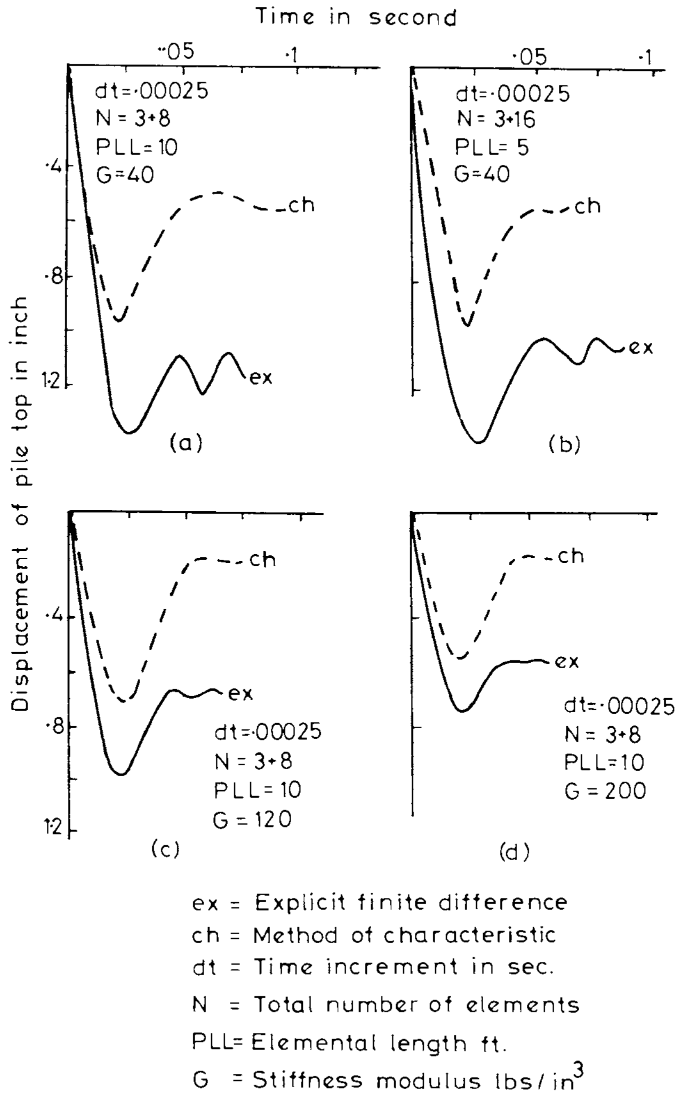

Fig. 3. Time Displacement Behaviour

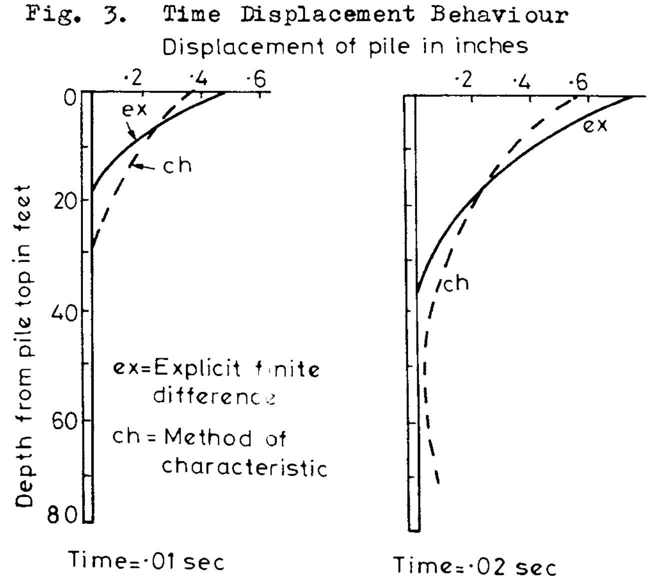

Fig. 4. Displacement Profile

CONCLUSIONS

The response of a pile to dynamic loading can be analysed by the method of characteristics. A formulation is presented for a pile subjected to an impact. The effect of shock front arising out of abrupt changes in dependent variables, is incorporated in the analysis. A numerical scheme to solve the equations is developed. The results obtained from this method are compared with those obtained from Smith's approach.

REFERENCES

Appendino, M. (1980) 'Prediction of Static Ultimate Resistance from Driving Data', Seminar on the Application of Stress-Wave Theory on Piles.

Coyle, H.M., R.E. Bartoskewitz and W.J. Berger (1973) 'Bearing Capacity Prediction by Wave Equation Analysis: State of the Art', Res. Rep. No.333, TRB.

Cristescu, N. (1967) 'Dynamic Plasticity, North-Holland Publishing Company.

Forehand, P.W., and Reese, J.L (1964) 'Prediction of Pile Capacity by the Wave Equation', Journal of the Soil Mechanics and Foundation Division.

Freiberger, W. (1952) 'A Problem in Dynamic Plasticity; The Enlargment of a Circular Hole in a Flat Sheet', Proc. Cambridge Philosophical Society.

Ginsburg, T. (1964) 'Propagation of Shock Waves in the Ground', Str.Div.Proc. ASCE.

Rakhmatulin, Khalil Akhmedovich and Yu.A. Demyanov (1966) 'Strength under High Transient Loads', Israel Programme for Scientific Translation.

Smith, E.A.L. (1960) 'Pile Driving Analysis by the Wave Equation', Journal of the Soil Mechanics and Foundation Division Proc. ASCE.

Smith, E.A.L. (1962) 'Pile Driving Analysis by the Wave Equation', Trans. AM. Soc. Civ. Engrs., 127, Part 1.

Streeter, V.L., Wylie, E.B., Richart, F. (1974) 'Soil Motion Computation by Characteristic Method', Journal of Geotechnical Engineering Division.

Thompson, C.D. and Thompson, D.E. (1979) 'Effects of Pile Driving System on Driveability and Capacity of Concrete Piles', ASCE.

Thompson, C.D. (1980) 'Evaluation of Ultimate Bearing Capacity of Different Piles by Wave Equation Analysis', Symposium on the Application of Stress Wave-theory on Piles, Royal Swedish Academy of Engineering Sciences, Stockholm, Sweden.