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Dynamic Analysis of Buried Structures

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SYNOPSIS The transient response of a circular cylindrical cavity in a linear elastic or viscoelastic infinite or semi-infinite medium under conditions of plane strain is examined. The method employed is the dynamic Boundary Integral Equation Method in conjunction with the Laplace Transform. The results obtained are compared with results stemming from analytical solutions, where available, and numerical solutions to assess the accuracy, efficiency and applicability of the method.

INTRODUCTION

The utilization of underground space is important because it facilitates the realization of national goals such as energy conservation, improved environment, industrial development, power generation, improved water resources and efficient transportation. Furthermore, the protective earth cover surrounding a buried structure ameliorates the effects of surface blasts while the shaking effect of seismic waves decreases with the depth of embedment.

The analysis of buried structures under dynamic loads was initiated in the early sixties by first considering the case of a circular cylindrical cavity in an infinitely extenting linear elastic medium. Analytic solutions were obtained by Baron and Matthews (1961), Baron and Parnes (1962), Pao (1962), Miklowitz (1964) and others. Analytical methods of solution, however, become extremely difficult, if not impossible, for arbitrary cavity geometries, for material behavior other than linear elastic and for semi-infinite media. Therefore, numerical methods of solution become imperative. Traditionally, the Finite Element Method (FEM) and the Finite Difference Method (FDM) have been used in the past for problems of this kind. These two methods, however, do not provide a reasonable solution for two reasons: i) the structure under consideration is infinitely extended and not finite as the models employed by both methods assume, and this may lead to errors due to wave reflections at the boundaries, unless one increases the size of the models considerably or constructs special non-reflecting boundaries and, ii) there can be singularities in the stress field around these structures that are badly simulated by both methods.

The proposed Boundary Integral Equation Method (BIEM) is a numerical technique well suited for problems of this kind since it can successfully simulate stress singularities and can easily handle infinite domains. The general formulation and solution of the transient elastodynamic problem by combining the BIEM and the Laplace transform technique was done by Cruse and Rizzo (1968), while Cruse (1968) applied this method to solve a simple half-plane wave propagation problem. The BIEM in conjunction with Fourier synthesis of appropriate steady-state solutions has been used by Niwa et al (1976) for problems involving cylindrical lined or unlined cavities in infinite media and the BIEM formulated in the time domain has been used by Cole et al (1978) for anti-plane strain transient problems. Applications of integral equation techniques to problems involving semi-infinite media have been limited so far to the anti-plane strain case. For instance, Wong and Jennings (1975) solved for the response of canyon-like discontinuities on the half-plane under earthquake loadings by Fourier synthesis and utilized the method of images to simulate the existence of the traction-free level surface.

FORMULATION AND SOLUTION

First the problem of determining the dynamic stress concentration around a circular cylindrical cavity embedded in a linear elastic infinite medium due to a biaxial system of suddenly applied compressive stresses under conditions of plane strain, as shown in Figure 1, is considered. The total stress distribution is obtained by superimposing the stress field produced by the pressure wave in the medium in the absence of the cavity to the stress field produced by the application of corrective tractions at the boundary of the cavity in order to render the boundary surface traction free. The first case is obviously trivial, so attention is focused to the second case, which is solved by the BIEM in conjunction with the Laplace transform along the lines of Cruse and Rizzo (1968).

The solution of the transient problem in linear elastodynamics by the BIEM is possible in the Laplace transformed domain as the hyperbolic partial differential equations of motion become elliptic partial differential equations (in the Laplace transform parameter s), permitting the construction of fundamental solutions to be used in an integral equation reformulation of the problem in the transformed domain. Taking the field point to lie on the boundary of the problem, one obtains constraint equations on the transformed boundary displacement and traction vectors. For specified boundary data, the constraint equations become sets of simultaneous singular integral equations with the unknown transformed boundary data as explicit unknowns. These equations are actually parametric in s, the Laplace transform parameter. The solution of these parametric integral equations is done numerically by using a constant property segment discretization for the boundary and by employing a Gaussian quadrature scheme for a sequence of valves of s. The time domain solution is finally obtained by a numerical inversion of the Laplace transformed solution.

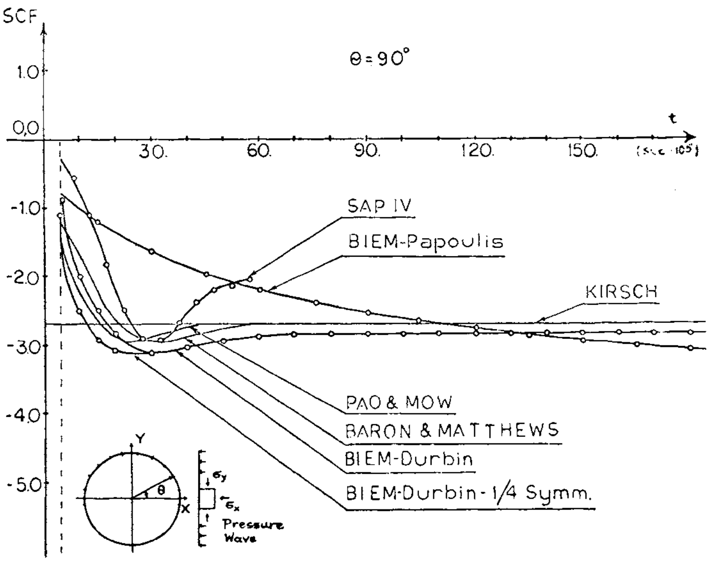

Fig. 1. Dynamic stress concentration factor for a circular cavity in an infinite medium under a suddenly applied pressure wave pulse.

Following the above procedure, the dynamic stress concentration factor for the case of the circular cylindrical cavity by considering only one-half of the hole due to symmetry was determined. Integration of the kernels along the boundary was done by employing a 7-point Gaussian quadrature scheme for non-singular cases, and a 20-point Gaussian quadrature scheme as well as information from an analytic mathematical treatment for singular cases. A singular case arises when the field point coincides with the source point. The computer program employed for the solution of the problem in the transformed domain was an extended and improved version of the one Cruse (1968) used for the solution of his half-plane wave propagation problem. As far as the numerical inversion of the Laplace transformed solution was concerned, two methods were used. The first method, due to Papoulis (1957), is based on trigonometric expansions and works with real data (i.e., real positive values of s), while the second method, due to Durbin (1974), is based on sine and cosine transforms and works with complex data. As shown in Beskos (1980), the second method is more time-consuming than the first one, but is very accurate for short as well as long time solutions, while the first one gives reasonable results for early time only. In Figure 1, the dynamic stress concentration factor, defined as the ratio of the resulting hoop stress at the perimeter of the cavity to the applied stress σ_x, is shown as a function of position and time. Very good agreement is observed with the results appearing in Baron and Matthews (1961) and Pao (1962) when Durbin's numerical Laplace inversion algorithm is used. Concurrently plotted are the results provided by the general purpose finite element code SAPIV as developed by Bathe et al (1973), which are not so good.

Next, the methodology developed was applied to the case of a circular cylindrical cavity, under the same pressure wave pulse mentioned above, embedded in a linear viscoelastic infinite plane. Solutions to problems involving linear viscoelastic materials can be readily obtained from the elastic solutions in the Laplace domain through use of the correspondence principle. Figure 2 presents the dynamic stress concentration factors for two linear viscoelastic materials, a Maxwell model and a Kelvin model, as well as the elastic solution. The symbol f is the ratio of the viscosity to the shear modulus. Both viscoelastic responses approach, in the limit, the elastic response for the appropriate f values.

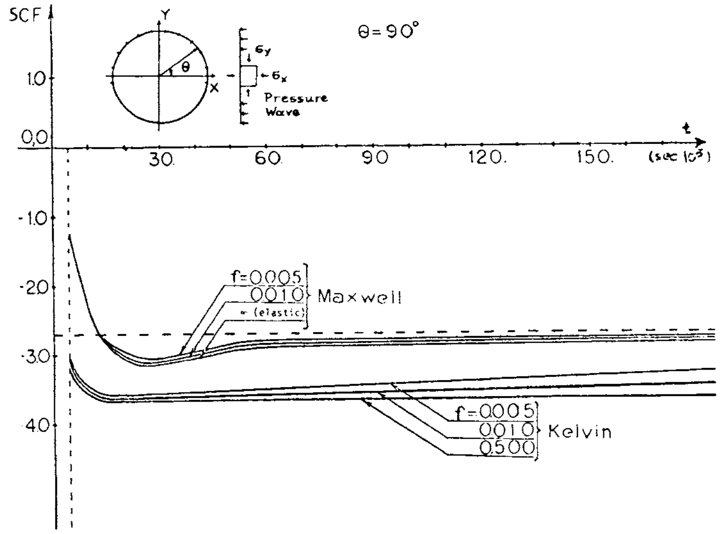

Fig. 2. Dynamic stress concentration factor for a circular cavity in an infinite viscoelastic medium under a suddenly applied pressure wave.

Finally, the response of the same cavity under the same transient load embedded in a linear elastic half-plane with material properties corresponding to granite is determined. The ratio of the depth of the center of the cavity from the traction-free level surface to the radius of the cavity is 1.50, which is sufficiently small to demonstrate the effect of the pressure wave reflections from the free surface on the dynamic stress concentration factor, as shown in Figure 3. Two paths were followed for simulating the presence of the free surface: i) The free surface was discretised, and ii) The Green's function entering the kernels of the integral equations was modified according to the method if images to automatically account for the traction free surface. The method of images employed resulted in an overspecification of the stress tensor at the level surface, but this overspecification is of minor importance from a computational point of view, as explained in Manolis (1980).

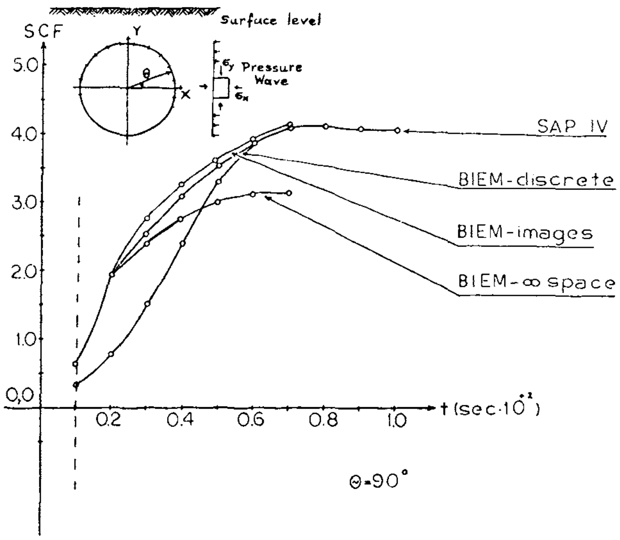

Fig. 3. Dynamic stress concentration factor for a circular cavity in a semi-infinite medium under a suddenly applied pressure wave.

CONCLUSIONS

A general numerical procedure has been presented, resulting from a combination of the BIEM and the Laplace transform, which is applicable to a wide range of problems involving underground structures in a dynamically active environment. Some sample cases were presented, but further cases can be considered with little additional effort, such as: i) the transient response of arbitrarily shaped cylindrical cavities, ii) plane stress conditions, iii) multiply-connected domains such as the lined cylindrical cavity, iv) the harmonic response of cylindrical cavities, and v) anisotropy of the infinite medium.

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