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Dynamic Response of Rigid Circular Footings

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SYNOPSIS In a simplified approach to the rigorous elastic half-space approach for the behaviour of a rigid circular surface footing acted upon by a dynamic excitation force, the elastic half-space model is replaced by an equivalent lumped parameter system. The equivalent spring constant (K) and the equivalent damping factor (D) are expressed in terms of dimensionless mass ratio (B). A change in the value of B thus affects the dynamic response of the footing. The paper explains a procedure to quantify this variation for vertical mode of vibration.

DYNAMIC RESPONSE OF FOOTINGS

A change in the radius of the footing or its mass or a change both in the mass and radius of the footing affects the value of B. The phenomenon is studied here by varying the radius and the mass of the footing, separately. Using the principle of superposition the approach could be used when the change occurs both in the mass and radius. Considering two footings, say 1 and 2 with radius and mass of r_1 and m_1 and r_2 and m_2 respectively, the following four cases have been investigated.

CASE-A: Constant force excitation and the mass of the system is constant ($r_1 \neq r_2$; $m_1=m_2$)

CASE-B: Rotating mass excitation and mass of the system is constant ($r_1 \neq r_2$; $m_1 = m_2$)

CASE-C: Constant force excitation and radius is constant ($r_1 = r_2$; $m_1 \neq m_2$)

CASE-D: Rotating mass excitation and radius is constant ($r_1 = r_2$; $m_1 \neq m_2$)

RESULTS

Expressions for ratio of amplitude of displacement and that of resonant frequency of footing 2 to that of 1 have been developed using appropriate results reported in literature (Richart et al, 1970). Table 1 presents the results. In the Equations 1 to 9 of Table 1,

A_z = amplitude of displacement at operating frequency

A_{zm} = amplitude of displacement at resonance

B_z = mass ratio of footing

ω_{mz} = resonant frequency of footing

ω_{nz} = natural frequency of footing

Subscript 1 or 2 indicates the footing (either 1 or 2) for which these quantities are given.

For CASE-A numerical evaluation of the variation of A_{zm2}/A_{zm1} with B_{z2}/B_{z1} (where $B_{z2} \geqslant B_{z1}$) using Equation 1 suggests the following generalised relationship for any combination of B_{z1} and B_{z2}.

$$A_{zm2}/A_{zm1} = 0.66\ (B_{z2}/B_{z1}) + 0.34 \qquad (10)$$

where $B_{z2}/B_{z1} \geqslant 1$

For CASE-A the variation of $\omega_{mz2}/\omega_{mz1}$ with B_{z2}/B_{z1} (where $B_{z2} \geqslant B_{z1}$) reduces to a single line. The effect of change in mass ratio on resonant frequency is not as much significant as it is in the case of resonant displacement. In Figures 1 and 2 are shown the comparison of the theoretically predicted ratios of resonant displacements and frequencies with those of measured values (Novak 1970, Ananda-Krishnan and Krishnaswamy 1973, Sridharan and Raman 1977). The theoretical predictions are good in the case of resonant displacements but not as much so in the case of resonant frequencies. Experiments show (Sridharan and Raman, 1977) little damping compared to Lysmer's theoretical value. Hence, direct use of elastic half-space model will predict higher displacements. This limitation can be overcome now by considering the ratio of displacements.

CONCLUSIONS

A set of mathematical relationships to quantify the variation in the dynamic response of footings have been developed. The theoretical quantification agrees very well in the case of resonant displacements than in the case of resonant frequencies.

TABLE 1

CASE-A

$$\frac{A_{zm2}}{A_{zm1}} = X^{4/3}\left(\frac{B_{z1} - 0.18}{B_{z2} - 0.18}\right)^{1/2} \qquad (1)$$

$$\frac{A_{z2}}{A_{z1}} = X^{1/3}\left(\frac{(1 - Y)^2 + 0.7225\,Y/B_{z1}}{(1-YX^{1/3})^2 + 0.7225\,YX^{1/3}/B_{z2}}\right)^{1/2} \qquad (2)$$

$$\frac{\omega_{mz2}}{\omega_{mz1}} = \frac{1}{X^{2/3}}\left(\frac{B_{z2} - 0.36125}{B_{z1} - 0.36125}\right)^{1/2} \qquad (3)$$

CASE-B

$$\frac{A_{zm2}}{A_{zm1}} = X\left(\frac{B_{z1} - 0.18}{B_{z2} - 0.18}\right)^{1/2} \qquad (4)$$

$\frac{A_{z2}}{A_{z1}}$ = Same as Equation 2

$$\frac{\omega_{mz2}}{\omega_{mz1}} = X^{1/3}\left(\frac{B_{z1} - 0.36125}{B_{z2} - 0.36125}\right)^{1/2} \qquad (5)$$

CASE-C

$\frac{A_{zm2}}{A_{zm1}}$ = Same as Equation 4

$$\frac{A_{z2}}{A_{z1}} = \left(\frac{(1 - Y)^2 + 0.7225\,Y/B_{z1}}{(1 - XY)^2 + 0.7225\,XY/B_{z2}}\right)^{1/2} \qquad (6)$$

$$\frac{\omega_{mz2}}{\omega_{mz1}} = \frac{1}{X}\left(\frac{B_{z2} - 0.36125}{B_{z1} - 0.36125}\right)^{1/2} \qquad (7)$$

CASE-D

$$\frac{A_{zm2}}{A_{zm1}} = \left(\frac{B_{z1} - 0.18}{B_{z2} - 0.18}\right)^{1/2} \qquad (8)$$

$\frac{A_{z2}}{A_{z1}}$ = Same as Equation 6

$$\frac{\omega_{mz2}}{\omega_{mz1}} = \left(\frac{B_{z1} - 0.36125}{B_{z2} - 0.36125}\right)^{1/2} \qquad (9)$$

Note: $X = B_{z2}/B_{z1}$: $Y = (\omega/\omega_{nz1})^2$

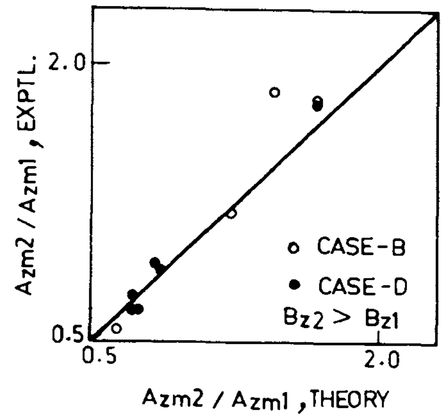

Figure 1

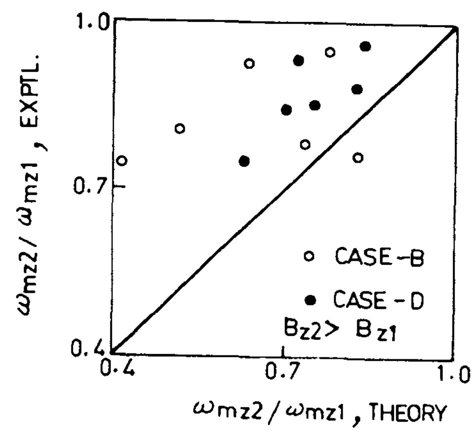

Figure 2

REFERENCES

Richart, F.E., Jr., Hall, J.R. and Woods, R.D. (1970), "Vibration of Soils and Foundations", Prentice-Hall, Englewood Cliffs, New Jersey, U.S.A.

Milos Novok (1970), "Prediction of Footing Vibrations", Journal of Soil Mechanics and Foundations Division, Proc. ASCE, Vol. 96, No. SM3, pp. 837-861.

M. Anandakrishnan and N.R. Krishnaswamy (1973), "Response of Embedded Footings to Vertical Vibrations", Journal of Soil Mechanics and Foundations Division, Proc. ASCE, Vol. 99, No. SM10, pp. 863-883.

A. Sridharan and J. Raman (1977), "Damping Characteristics of Soils", Technical Report No. 21, Central Board of Irrigation and Power, New Delhi, India.