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Dynamic Equations, Control Problems on Time Scales, and Chaotic Systems

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ABSTRACT The unification of integral and differential calculus with the calculus of finite differences has been rendered possible by providing a formal structure to study hybrid discrete-continuous dynamical systems besides offering applications in diverse fields that require simultaneous modeling of discrete and continuous data concerning dynamic equations on time scales. Therefore, the theory of time scales provides a unification between the calculus of the theory of difference equations with the theory of differential equations. In addition, it has become possible to examine diverse application problems more precisely by the use of dynamical systems on time scales whose calculus is made up of unification and extension as the two main features. In the meantime, chaos theory comes to the foreground as a concept that a small change can result in a significant change subsequently, and thus, it is suggested that nonlinear dynamical systems which are apparently random are actually deterministic from simpler equations. Consequently, diverse techniques have been devised for chaos control in physical systems that change across time-dependent spatial domains. Accordingly, this Editorial provides an overview of dynamic equations, time-variations of the system, difference and control problems which are bound by chaos theory that is capable of providing a new way of thinking based on measurements and time scales. Furthermore, providing models that can be employed for chaotic behaviors in chaotic systems is also attainable by considering the arising developments and advances in measurement techniques, which show that chaos can offer a renewed perspective to proceed with observational data by acting as a bridge between different domains.

KEYWORDS
Dynamic equations on time scales
Chaotic systems
Nonlinearity and chaos
Unification and extension
Control problems
Time-variations of systems

The theory of time scales, conceptualized and introduced by Stefan Hilger in 1988, makes a unification between the calculus of the theory of difference equations with the theory of differential equations. In other words, the unification of integral and differential calculus with the calculus of finite differences became possible by providing a formal structure to study hybrid discrete-continuous dynamical systems and offering applications in diverse fields which require simultaneous modeling of discrete and continuous data with regard to dynamic equations on time scales. It is also possible to investigate many application problems in a more precise way through the use of dynamical systems on time scales.

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Unification and extension make up the two main features of time scales calculus, with subject matters such as existence and uniqueness of solutions, periodicity, stability, Floquet theory, Cantor sets as well as boundedness, among many others, regarding solutions can be investigated in a more precise way and by and large by utilizing dynamical systems and differential (dynamic) equations on time scales. The study of dynamic equations on time scales enables one to avoid proving the related results twice: one time for differential equations and another time for difference equations (Bohner and Peterson 2001), (Bohner and Georgiev 2016).

The core concept is the proving of a result for a dynamic equation in which the unknown function's domain is a so-called time scale, which is, in fact, an arbitrary closed subset of the reals. As the time scale is chosen to be the set of real numbers, the general result generates a result pertaining to an ordinary differential equation as examined in a first course in differential equations. The same general result yields a result for difference equations by choosing the time scale to be the set of integers (Hilger 1990). A time scale, as a special case of a measure chain, refers to an arbitrary nonempty closed subset of real numbers such as, for example, $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{N}_0$, $[0,1] \cup [2,3]$, $[0,1] \cup \mathbb{N}$, and the Cantor set, whereas $\mathbb{Q}$, $\mathbb{R} \setminus \mathbb{Q}$, $\mathbb{C}$, $(0,1)$ are not time scales (Agarwal *et al.* 2002).

As chaotic systems can be characterized by a certain degree of spontaneous self-order, examining the interplay of nonlinearity and chaos can ensure a deep understanding of such systems, while the theory of calculus on time scales enables a sort of unification of the theories with respect to differential equations and difference equations, delay differential equations as well as population dynamics (Bohner *et al.* 2022b), outspreading the theories toward other types of dynamic equations. As a type of differential equation, delay differential equations, or time-delay systems, in mathematics hold that the derivative of the unknown function at a particular time is provided in terms of the function's values at previous times.

Delay differential equations often emerge as simple infinite-dimensional models in the highly complex scope of partial differential equations. Systems such as hereditary ones, equations that have deviating argument or differential-difference equations belong to the class of systems having functional state. Delay differential equations (Durga and Muthukumar 2019), (Bohner *et al.* 2022a) have aftereffect or dead-time, which is an applied problem since there is the emerging need of having models that behave more like the real process when the increasing expectations of dynamic performances arise. Many processes include aftereffect phenomena in their inherent dynamics besides the sensors, actuators and communication networks being involved in feedback control loops introducing the delays. Therefore, delay differential equations maintain their applicability in the areas of science, particularly in engineering fields related to control as voluntary introduction of delays can prove to be beneficial for the control system (Richard 2003), (Lavaei *et al.* 2010).

Time scales in different models that employ optimal control theory, with the extension of the calculus of variations as a mathematical optimization method, have significant applications to deal with finding a control for a particular dynamical system across a period of time so that an objective function affecting the dynamics can be optimized (Zacchia Lun *et al.* 2019). Dynamics being essentially nonautonomous (Wu *et al.* 2023) makes it compelling to verify the ingredients of chaos for unspecified time scales.

The paradigm of information processing by dynamical systems at the range of phase-space scales reflects the chaotic systems which show an opposite inclination, which is the phase-space expansion as a result of exponentially diverging trajectories. On the other hand, the forecasting of the final state necessitates more precise measurements related to the initial state as the separation of them over time goes up. At this point, chaos theory, as a mathematical field of study, seems as it is a concept which suggests that a small change can bring about a significant change afterwards.

Accordingly, it posits that nonlinear dynamical systems which are apparently random are actually deterministic from simpler equations (Devaney 2022). Control of chaos refers to the stabilization through as small system of perturbations and the result is to make an otherwise chaotic motion more predictable and also stable. Many techniques have been devised for chaos control for physical systems that change on time-dependent spatial domains. In these regards, small perturbations can change a system's behavior with the sensitivity serving to be beneficial for control purposes in chaos as has been implied.

Taken together, dynamic equations, time-variations of the system, difference and control problems are bound by chaos theory which can provide a novel way of thinking based on an innovative concept of measurements and time scales, enabling models to be used for chaotic behaviors. Based on the processing and comprehension of huge amounts of experimental data which can be analyzed by emerging developments and advances in measurement techniques, exploits that motivate mathematical developments can be modeled. As a matter of fact, chaos can offer a renewed perspective to proceed with observational data which may be erratic in natural phenomena by providing a bridge between different domains.

Conflicts of interest

The author declares that there is no conflict of interest regarding the publication of this paper.

Availability of data and material

Not applicable.

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