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SYNOPSIS The dynamic response of concrete pavement to a moving line load has been studied by idealizing the subgrade with different viscoelastic models. Complex Fourier Transformation has been used to solve the soil-structure interaction problem. The results are presented in non-dimensional form. The appropriate choice of models and the corresponding material constant values for different types of bases and/or subgrade generally used under concrete pavement have been discussed.

INTRODUCTION

Current procedures for designing and evaluating concrete pavements are still based on static loads and except for introducing equivalent static loadings, they do not account for the dynamic response of pavements to moving loads. Of all the components which play a part in vehicle-pavement interaction problem, the soil is the most variable and least understood. According to Scott (1962), "In practice, the hypothesis of a linearly elastic behavior without time effects (other than hydrodynamic process) for soils is used as a basis for calculations which extend the assumption far beyond reasonable limits". Fundamental understanding as well as analytical formulation of time-dependent uniaxial stress-strain behavior of soils, can be facilitated by means of idealized rheological models. The different viscoelastic models that are generally associated with soils are shown in Fig. 1. In this paper, the dynamic response of concrete pavement on subgrades idealized by different viscoelastic models has been studied and the appropriate choice of models and the corresponding material constant values for different types of bases and/or subgrade generally used under concrete pavement have been discussed.

ANALYSIS

In general, the governing differential equation describing the free transverse vibration of a free plate can be expressed as follows:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho H \frac{\partial^2 w}{\partial t^2} = q(x,y,t.) - p(x,y,t) \quad (1)$$

where D = flexural rigidity of the slab; w = mid-plane deflection of the slab (positive downward); x,y = fixed coordinates; ρ = density of the slab; H = slab thickness; q = surface loading; p = foundation reaction; and t = time.

It is assumed that the slab is supported by a standard solid model. The relationship between the deflection and the foundation reaction can then be written as:

$$p + \frac{\eta_1}{k_1}\frac{\partial p}{\partial t} = k_2 w + \eta_1 \left(\frac{k_1 + k_2}{k_1}\right)\frac{\partial w}{\partial t} \quad (2)$$

where k_1, k_2 = elastic subgrade constants; and η_1 = viscosity constant of the subgrade. The analysis problem can be simplified considerably by assuming the road width to be small, and solving the resultant narrow-road equation. Assuming that the deflection of the plate does not vary in the lateral (y axis) direction, Equation (1), for a constant cross section of pavement, becomes:

$$D \frac{\partial^4 w}{\partial x^4} + \rho H \frac{\partial^2 w}{\partial t^2} = F(x,t) - P(x,t) \quad (3)$$

where F(x,t) is the moving line load. Equations (2) and (3) govern the displacements of the elastic pavement on the viscoelastic foundation. If the applied load F(x,t) is a constant force F_0, which moves with constant velocity, v, over the pavement, it can be expressed as:

$$F(x,t) = F_0\delta(x - vt) \quad (4)$$

where $\delta(\ )$ is the Dirac delta function. To facilitate the solution of Equations (2) and (3) a transformation of variables is used that is suggested by physical considerations to describe the response of the plate in a moving coordinate system. This is accomplished by the change of variables

$$r = vt - x \quad (5)$$

which transforms Equations (3) and (2) into:

$$D \frac{d^4 w(4)}{dr^4} + \rho v^2 H \frac{d^2 w(r)}{dr^2} + p(r) = F_0\delta(r) \quad (6)$$

$$p + \frac{\eta_1 v}{k_1} = k_2 w(r) + \eta_1\left(\frac{k_1 + k_2}{k_1}\right) v\frac{dw(r)}{dr} \quad (7)$$

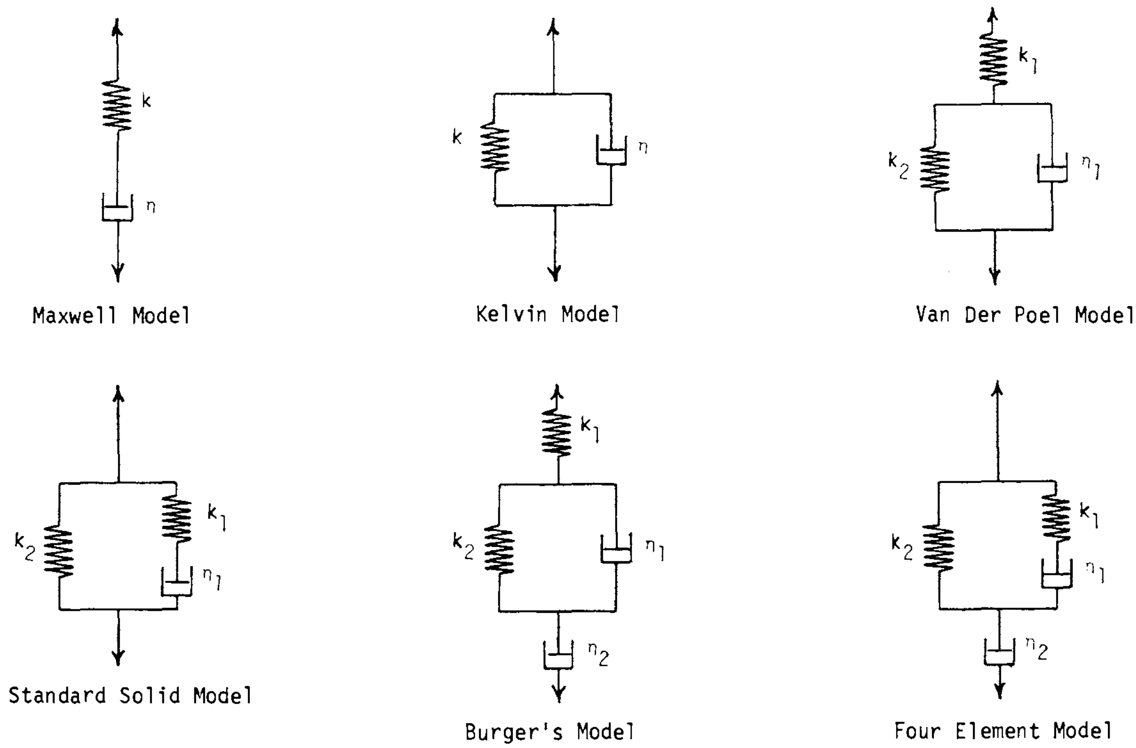

Fig. 1. Viscoelastic Models

Equation (5) defines a Galilean transformation which has been used to advantage in a number of recent studies. The change of variables may be given the following physical interpretations: an observer fixed with respect to the x-y coordinate system will see the line load F advance in the direction of the positive x-axis, and to him the deflection of the plate will appear to be dependent upon x, y, and t. However, an observer fixed with respect to the r, y coordinate system will move with the advancing load, and to him the deflection surface will appear stationary--that is, independent of t, and a function of r alone. It is noted that by neglecting the damped transients due to the starting of the motion, the implicit assumption that the load has been moving for a sufficiently long period has been made. It should also be noted that r is negative ahead of the load and positive behind the load.

Equations (6) and (7) are now put in dimensionless form by introducing the following dimensionless quantities:

$$W = \frac{w}{w_o}, \; R = \beta r, \; \theta = \frac{v}{v_{cr}}, \; P = \frac{2p}{F_o \beta},$$

$$m = \frac{K_1}{K_1 + K_2}, \; \zeta = \frac{\eta_1}{\sqrt{K_2 \rho h}}$$

where $w_o = \frac{F_o \beta}{2k_2}$; $\beta = \sqrt[4]{\frac{k_2}{4D}}$;

and $V_{cr} = \left[\frac{4k_2 D}{(\rho H)^2}\right]^{1/4}$;

The quantities w_o and v_{cr} both refer to the problem of a plate of unit width on an elastic foundation of spring constant k_2. The deflection w_o is the deflection at the point of application of a stationary load F_o. The velocity, V_{cr}, is the critical velocity of a transverse displacement wave along a freely vibrating, elastically supported plate of unit width with zero damping.

After the introduction of the dimensionless quantities, Equations (6) and (7) can be written as

$$\frac{d^4W(R)}{dR^4} + 4\theta^2 \frac{d^2W(R)}{dR^2} + 4P(R) = 8\delta(R) \tag{8}$$

$$P + \frac{\theta(1 - m)\zeta}{m} \frac{dP(R)}{dR} = W(R) + \frac{\theta\zeta}{m} \frac{dW(r)}{dR} \tag{9}$$

Equations (8) and (9) are amenable to solution by Complex Fourier Transformation and the deflection is given by (Bandyopadhyay 1978, 1980):

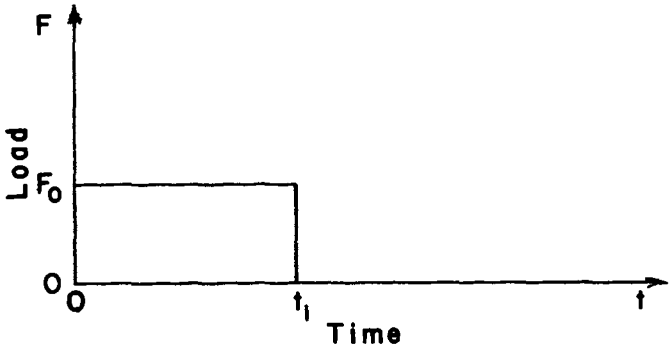

Fig. 2 Load-Unload Cycle

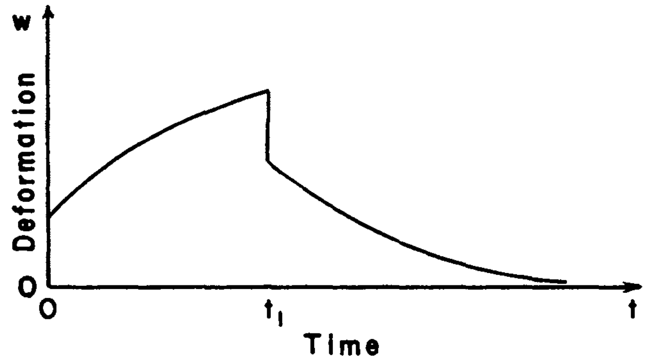

Fig. 5 Response of Three-Element Models

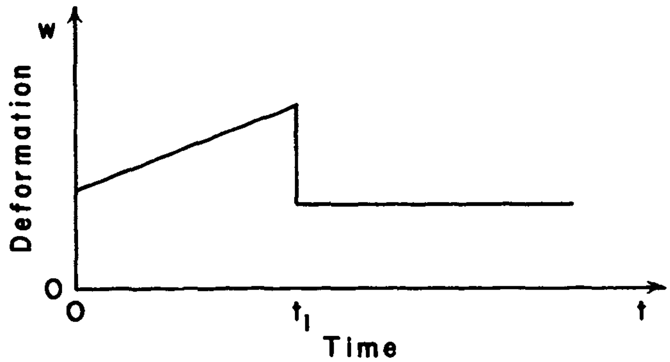

Fig. 3 Response of Maxwell Model

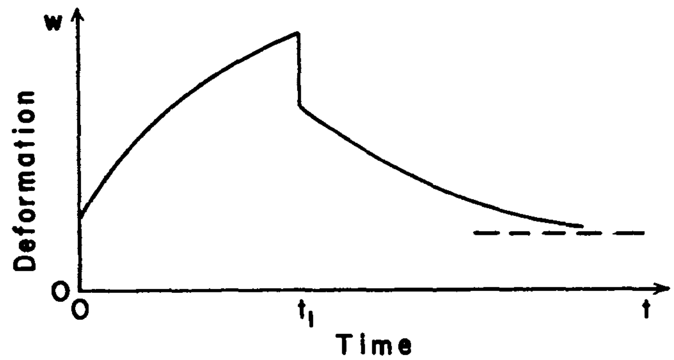

Fig. 6 Response of Four-Element Models

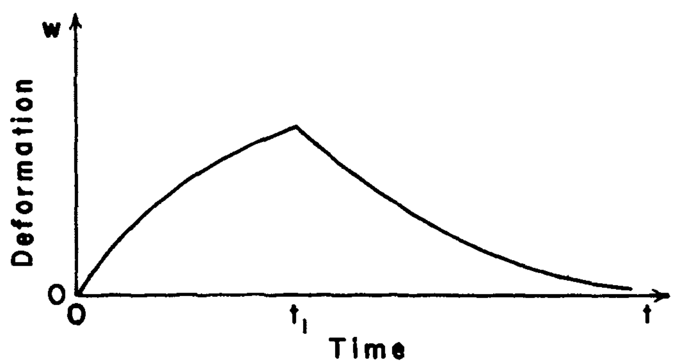

Fig. 4 Response of Kelvin Model

$$W = A_1 \,[sgn(a)]\, e^{-aR}\, H[sgn(a)R]$$
$$+ \,[sgn(c)]\, e^{-cR}\, [B_1 \cos dR + B_2 \sin dR]\, H[sgn(c)R]$$
$$+ \,[sgn(f)]\, e^{-fR}\, [B_3 \cos gR + B_4 \sin gR]\, H[sgn(f)R] \quad (10)$$

In Equation (10) sgn ( ) and H( ) are generalized functions defined by

$$sgn(v) = \begin{cases} 1 & \text{for } v > 0 \\ -1 & \text{for } v < 0 \end{cases}$$

$$H(R) = \begin{cases} 1 & \text{for } R > 0 \\ 0 & \text{for } R < 0 \end{cases}$$

The expressions for a, c, d, f, g, A_1, B_1, B_2, B_3 and B_4 are given elsewhere (Bandyopadhyay, 1978).

The dimensionless bending moment M is given by (Bandyopadhyay 1978, 1980)

$$M = -\frac{1}{2}\; W'' \quad (11)$$

Where, $M = \frac{M^*}{M_o}$, $M^* = -D\,\frac{d^2w}{dy^2}$ and $M_o = \frac{F_o}{4B}$. Mo is the bending moment just under a stationary load in a plate of unit width supported by an elastic foundation of spring constant k_2. Equation (10) and (11) with a different set of expressions for the constants holds good for Van Der Poel Model.

The deflection equation of the pavement supported on Burger's or the Four Element model is found to be

$$W = \frac{A_2}{2}[sgn(R)] - B[sgn(a)]e^{-aR}\, H[sgn(a)R]$$
$$- \,[sgn(c)]e^{-cR}\, [C_1 \cos dR + C_2 \sin dR]\, H[sgn(c)R]$$
$$- \,[sgn(f)]e^{-fR}\, [C_3 \cos gR + C_4 \sin gR]\, H[sgn(f)R] \quad (12)$$

Expressions for a, c, d, f, g, A_2, B, C_1, C_2, C_3, and C_4 for Burger's and Four Element Models are given elsewhere (Bandyopadhyay 1978). In addition to the dimensionless quantities already introduced for the Standard Solid Model, another dimensionless quantity, defined by:

$$\lambda = \frac{\eta_2}{\sqrt{k_2 \rho H}} \qquad (13)$$

was introduced in equation (12).

VISCOELASTIC MODEL EVALUATION

Base courses are used under concrete pavements for various reasons, including control of pumping, control of shrink and swell of the subgrade, drainage, etc. The base course (often called a subbase course) also lends some structural capacity to the pavement. In this section, the appropriate choice of models along with the values of the material constants will be discussed for different type of bases and subgrade.

The responses of different viscoelastic models to a load-unload cycle like that if Fig. 2 are shown in Fig. 3 to Fig. 6. The Maxwell Model represents a material which when subjected to stress, undergoes an instantaneous elastic deformation together with deformation increasing with time. The model can also be used to represent a material exhibiting relaxation of stress with time when the material is held at constant deformation. A permanent strain would result when the model is subjected to a load-unload cycle. A Kelvin Model, on the other hand, illustrates a material behavior characterized by elastic effects that are delayed by time. The use of Maxwell and Kelvin Models to simulate the deviatoric and volumetric behavior of soil media has been discussed by Schiffman (1959) and Soydemir and Schmid (1970). Emery (1966) discussed the use of these models in Rock Mechanics. For a Sand-Asphalt mixture, Wood and Goetz (1959) found that the Modulus of Recovery varied from 63,000 psi at 40^oF to 4180 psi at 140^oF. The corresponding viscoelastic parameter, Mixture Viscosity, varied from 1.702×10^7 lb-Sec/in^2 at 40^oF to 3.33×10^5 lbs-Sec/in^2 at 140^oF. For a Soil-Asphalt mixture, Abdel-Hady and Herrin (1965) found that the Modulus of Recovery ranged from 6.4×10^4 psi with 2% asphalt to 2.2×10^4 psi with 7% asphalt.

The Van Der Poel and the Standard Solid models are capable of instantaneous elastic deformation, retarded deformation and recovery. Secor and Monismith (1961) have given the following values of Standard Solid parameters for asphalt concrete: $K_1 = 2.50 \times 10^4$ psi, $K_2 = 1.30 \times 10^4$ psi, $\eta_1 = 9.0 \times 10^3$ psi-min per unit strain. A study of the stress-relaxation phenomenon in specimens of clay, loess, and a sand-clay mixture tested in a state of uniaxial compression has been made by Kondner and Stallknecht (1961). The data were analyzed with the aid of rheologic models, namely, Standard Solid and a number of Maxwell elements in parallel.

With certain materials, there appears to be a permanent set after creep recovery, which cannot be explained by the Three-Element models. Four-Element models can be appropriately used in those cases. Burger's model has the advantage that it displays under load instantaneous elastic deformation, retarded elastic deformation and viscous flow. The first two types of deformation are recoverable, whereas the viscous flow is, of course, irrecoverable. For Soil-Cement mixtures, George (1969) has evaluated the material constants of Burger's model - Sandy-Clay with 6% cement: $K_1 = 0.89 \times 10^6$ psi, $K_2 = 1.70 \times 10^6$ psi, $\eta_1 = 4.59 \times 10^9$ psi-min, $\eta_2 = 2.70 \times 10^9$ psi-min; Silty-clay with 10% cement: $K_1 = 0.30 \times 10^6$ psi, $K_2 = 0.33 \times 10^6$ psi, $\eta_1 = 1.0 \times 10^9$ psi-min; $\eta_2 = 1.0 \times 10^9$ psi-min. Lara-Tomas (1962) used a Maxwell Model with variable viscosity combined with a series of Kelvin elements to study the time-dependent deformation of clay soils. The instantaneous and delayed deformations computed for the model were compared with the experimental data. Good agreement with the experimental points were found after the first cycle of loadings. Tsai and Schmid (1969) indicated that a Burger's Model originally assumed for the soils under impact load can be simplified to a two-parameter Maxwell Model. The parameters of the model can be obtained quickly by a simple impact penetrometer test. Comparing the values of the spring constant and the dashpot constant, they concluded that the viscous part of the deformation predominates, because the soil was very much liquefied under impact loads.

The problem with the Burger's model, however, is that the elastic recovery is the same as that of instantaneous elastic deformation, which is generally not valid for most of the foundation materials. A Four-Element model (Fig. 1f), which can incorporate the variability of elastic recovery with time, would therefore be more appropriate. Secor and Monismith (1961) found the following values of material constants for asphalt concrete, treating it as a Four-Element model: $K_1 = 2.45 \times 10^4$ psi, $K_2 = 1.35 \times 10^4$ psi, $\eta_1 = 8.70 \times 10^3$ psi-min, $\eta_2 = 3.67 \times 10^7$ psi-min, Temperature = 77^oF.

It must be pointed out that the material constants discussed above are dependent, among others, on temperature and stress level. The variation of K_1, K_2, η_1, and η_2, of asphalt concrete with temperature and lateral pressure has been demonstrated by Secor and Monismith (1962). Using a Standard Solid model, Christensen and Wu (1964) evaluated the model parameters for glacial lake clays from Sault Ste. Marie and Detroit and Illite for different load increments. Though sufficient evidence is available to suggest that the behavior of foundation materials can be adequately represented by proper choice of viscoelastic models, more research is needed to identify and evaluate the model parameters for different foundation materials for a wide range of temperature and stress level.

RESULTS AND DISCUSSIONS

Both the Standard Solid and Van Der Poel models exhibit an initial elastic response and delayed elasticity. Two elastic responses are thus always associated with each model. The elastic reponses are limit cases of the viscoelastic responses because they correspond to $\zeta = \infty$ and $\zeta = 0$. The elastic response associated with Van Der Poel model has been discussed by Achenbach and Sun (1965) and that with Standard Solid Model has been discussed by Bandyopadhyay (1980).

A subgrade of Kelvin elements correspond to a limit case of a foundation of Standard Solid elements, the limit being obtained by letting the constant of elasticity K_1 increase beyond bounds. Accordingly, the Kelvin foundation yields a value of m equal to unity. It can be shown that the difference of a Kelvin foundation response with that of a Standard Solid or Van Der Poel foundation is the first term of Equation (10). The response of the plate on the Standard Solid or Van Der Poel foundation includes an exponentially decaying a nonperiodic response behind the load. This response is absent for the plate on the Kelvin foundation. It can be seen from equation

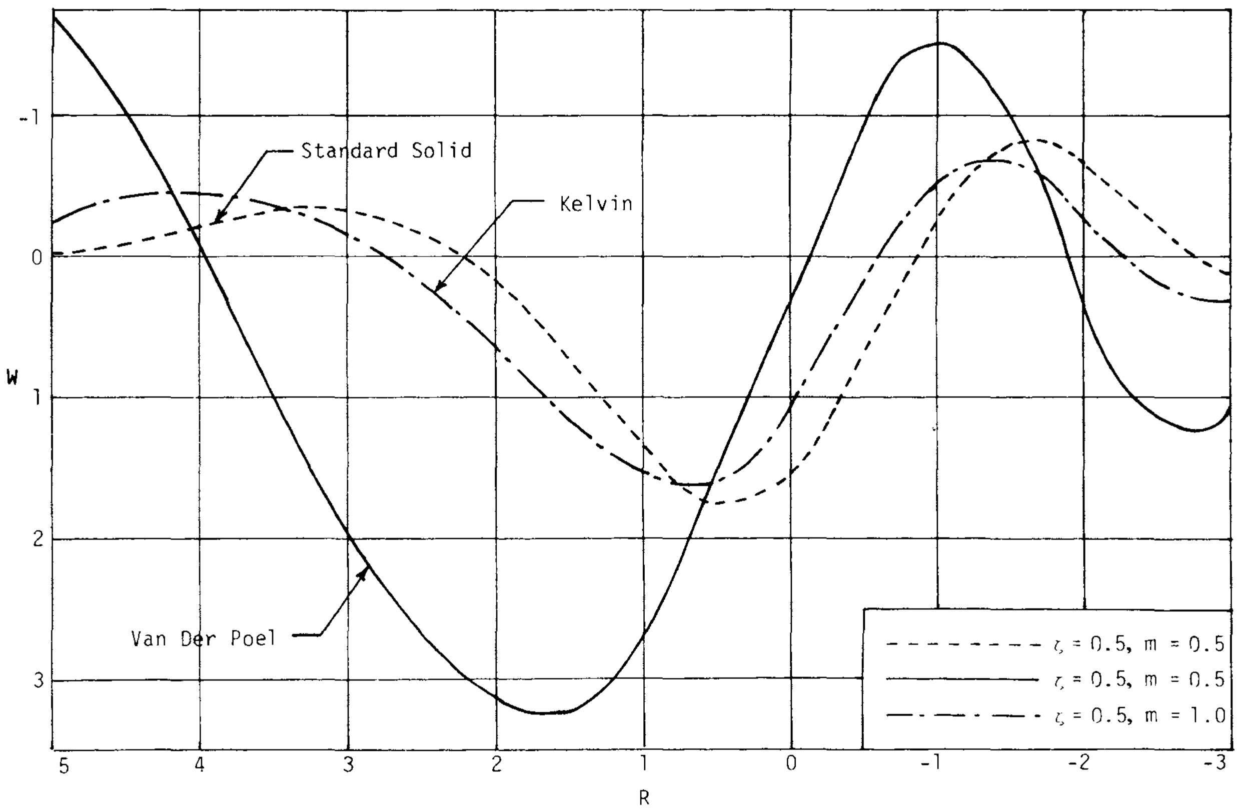

Fig. 7. Deflection W as a Function of R for θ = 1.0

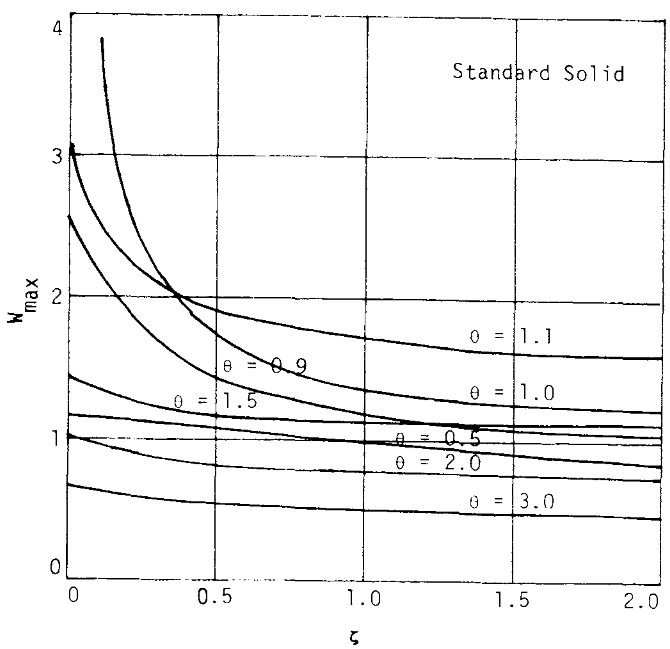

Fig. 8. Maximum Deflection as a Function of ζ for Various Values of θ, m = 0.5

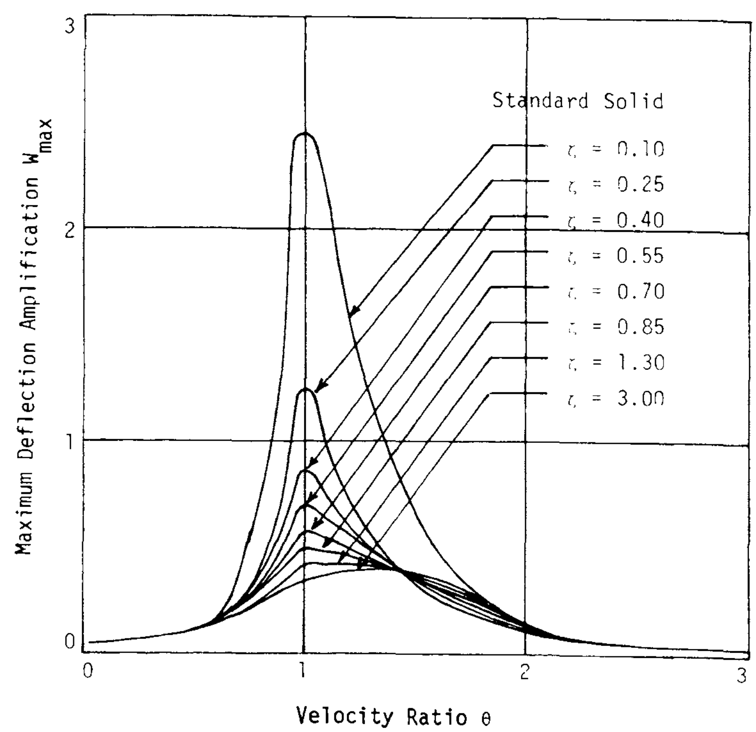

Fig. 9. Stability Diagram for Surface Displacement Ahead of Load

(12) that there is always some residual deformation associated with the Burger's or the Four Element model, as can be expected. For the range of parameter values for asphalt concrete given by Secor and Monismith (1961), the deflections obtained with the Four Element model are virtually the same when the model is replaced by its corresponding three element model. Fig. 7 shows the deflection profiles for θ = 1.0 for Kelvin, Van Der Poel and Standard Solid models.

An important feature of the road vibrations that occur because of a moving load is that the deflections are not symmetrical about the load. While the wavy profile of the pavement does propagate along the road with the same velocity as the load, the waves ahead of the load have a shorter wave length and smaller amplitude, in general, than the waves behind the load. At static conditions (θ = 0), the maximum deflection occurs under the load(at R = 0) with the deflection curve being symmetrical about the position R = 0. As the velocity increases, the point of maximum deflection falls behind the load. Also with increasing speed the wave length becomes shorter ahead of the load and becomes longer behind the load to the extent that at supercritical velocity, no oscillatory waveform will ever be obtained. The magnitude of the maximum deflection increases with speed increasing up to the vicinity of critical velocity and then decreases with further increase in the velocity. The damping of the foundation has a pronounced influence on the pavement deflection for load velocities in the neighborhood of the critical velocity. This effect is clearly shown in Fig. 8, where the maximum deflection has been plotted as a function of ζ for various values of θ for the Standard Solid model. At light damping and with speed increasing up to the vicinity of critical value, the maximum deflection, which is located behind the load, increases up to three times the static deflection depending upon the rheological model. For heavy damping, the maximum deflection for the Kelvin model is always less than the static value, never gets lower than the static value for the Van Der Poel model and is between these two values for the Standard Solid model. Another way of viewing the results obtained in this study is shown in Fig. 9 called the Stability Diagram. The maximum deflection behind the load occurs in the region of positive deflection whereas the maximum deflection ahead of the load occurs in the region of negative deflection.

CONCLUSION:

The equation of motion of a long, narrow, elastic pavement uniformly supported by viscoelastic subgrade has been formulated. Complex Fourier Transformation has been used to solve the resulting differential equations. The results are presented in non-dimensional form. The effect of different parameters, namely, the velocity ratio, and the elastic and viscous constants of the foundation on the response of the pavement has been discussed. A detailed study regarding the appropriate choice of models and the corresponding material constant values for different types of foundations has been made.

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