29 Apr 1981, 9:00 am - 12:30 pm

Dynamic Analysis of Foundations for Heavy Duty Diesel Engines

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Proceedings: First International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 26 - May 3, 1981, St. Louis, Missouri

Dynamic Analysis of Foundations for Heavy Duty Diesel Engines

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SYNOPSIS Coupled motion analysis of Machine-Foundation-Soil systems subjected to periodic dynamic forces and moments which are of general nature (both in amplitudes and periods) has been carried out using Fourier analysis and numerical techniques. Free vibration as well as forced vibration analysis in all the coupled and uncoupled modes of the system have been presented. The procedure is illustrated in the case of an eight cylinder heavy duty diesel engine generator assembly and results have been graphically shown in terms of frequency ratios and maximum steady state responses in the all the relevant modes of vibration.

1. INTRODUCTION

Foundations for machine assemblies can be directly supported by soil medium if the soil is (a) of medium to high strength (b) of low compressibility (c) is not made up or reclaimed and is a natural formation. Otherwise such foundations have to be supported by piles taken deeper into the soil. Analysis of rigid machine-foundation assemblies subjected to periodic dynamic loads and moments (each of a single period or a combination of several periods) of general nature (both amplitudes and periods can be very general), directly supported on soil medium is presented in this investigation. Equations of motion of the Machine-Foundation-Soil system (MFS) and the response analysis have been discussed. Coupled and uncoupled motions of such a system subjected to forces and moments of general nature are considered for free and forced vibration analysis. Dynamic responses of foundations for an eight cylinder heavy duty diesel engine power generating set manufactured by M/s. Kirloskar Oil Engines Ltd., Pune, India have been computed and are graphically shown as an illustration.

2. GENERAL EQUATIONS OF MOTION

A multi-cylinder diesel engine with sign convention is shown in Fig.1. A typical assembly of diesel generator set is shown in Fig. 2. A rigid foundation block supporting a machine and resting on a homogeneous, isotropic and linearly elastic, semi-infinite soil medium is shown in Fig. 3. Point A is the combined centre of gravity (C.G.) of the machine and foundation (assumed as a rigid body) and the reference coordinates x, y, z pass through A (Fig.3). Point B with coordinates x_B, y_B, z_B is any point of the system. The displacements of B can be expressed in terms of the displacements of A, the combined C.G., and rotations of the rigid body as

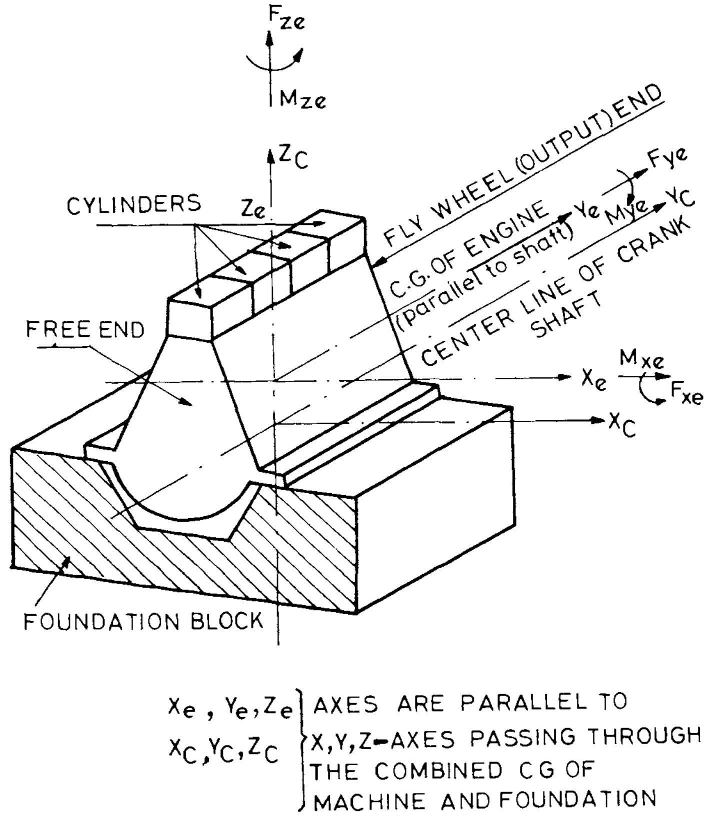

Fig.1. Multi-Cylinder Diesel Engine

$$u_B = u_A - y_B \psi_z + z_B \psi_y$$
$$v_B = v_A + x_B \psi_z - z_B \psi_x \qquad (1)$$
$$w_B = w_A - x_B \psi_y + y_B \psi_x$$

where u,v,w are displacements and ψ_x, ψ_y, ψ_z are rotations along and about x,y,z-axes

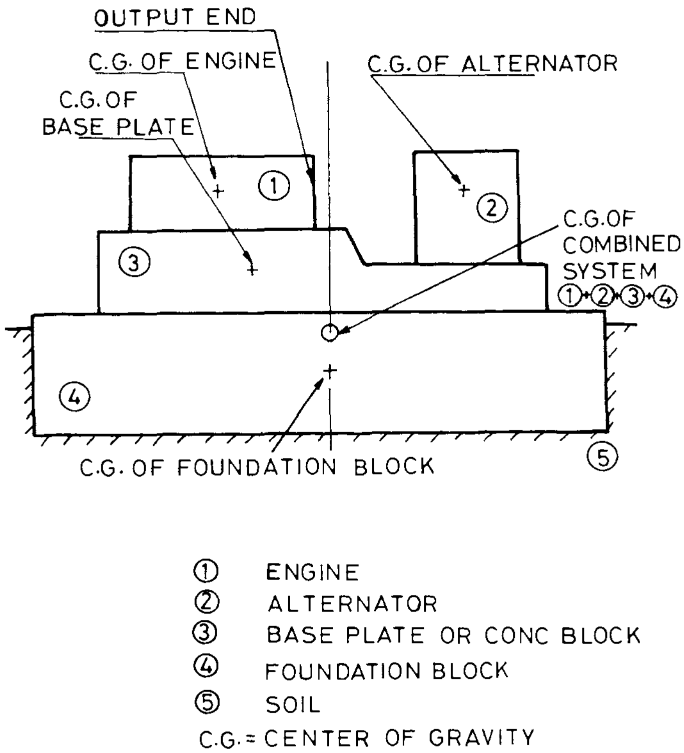

Fig. 2. Typical Assembly of Diesel Generator Set

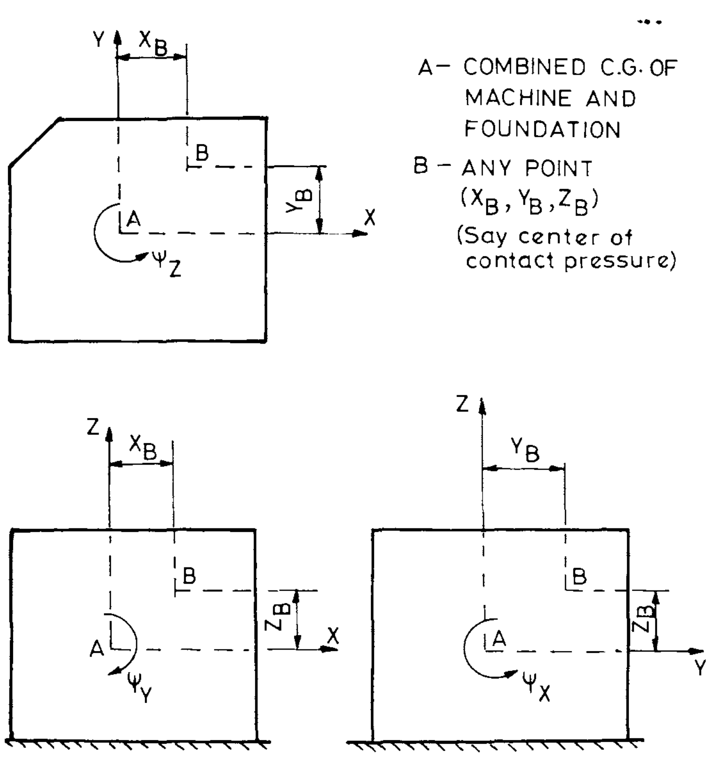

Fig. 3. Machine-Foundation-Soil System

respectively.

The soil reaction can be resolved in general into six components acting at the centre of contact area (i.e.) three forces P_x, P_y, P_z and three moments M_x, M_y, M_z. Taking B, as the centre of contact area, these can be expressed as (Kameswara Rao(1977)).

$$
\begin{aligned}
P_x &= -\,c_{xx}\frac{du_B}{dt} - k_{xx}u_B \\
P_y &= -\,c_{yy}\frac{dv_B}{dt} - k_{yy}v_B \\
P_z &= -\,c_{zz}\frac{dw_B}{dt} - k_{zz}w_B \\
M_x &= -\,c_{\psi x}\frac{d\psi_x}{dt} - k_{\psi x}\psi_x \\
M_y &= -\,c_{\psi y}\frac{d\psi_y}{dt} - k_{\psi y}\psi_y \\
M_z &= -\,c_{\psi z}\frac{d\psi_z}{dt} - k_{\psi z}\psi_z
\end{aligned}
\tag{2}
$$

where 't' is the time variable, and c_{xx}, c_{yy}, k_{xx}, k_{yy} etc. are the damping constants and spring constants of the equivalent single degree of freedom analogues and are given by Kameswara Rao (1977), Richart et al. (1970). The unbalanced forces and moments from the machine (or from the various units of the machine assembly) can be reduced into generalised forces and moments acting of the combined centre of gravity, A, of the machine-foundation system, consisting of three forces $\bar{P}_x$, $\bar{P}_y$, $\bar{P}_z$ and three moments $\bar{M}_x$, $\bar{M}_y$, $\bar{M}_z$ along and about the respective coordinate axes. From dynamic equilibrium of the rigid body, using equations (1) and (2), the general equations of motion of the machine-foundation-soil system can be written in all the six modes of vibration (Kameswara Rao (1977)). All these six equations can be seen to be linear and coupled and uniquely determine the responses u_A, v_A, w_A, ψ_x, ψ_y, ψ_z. While solutions of these six coupled equations can be attempted using the analysis presented in this investigation, these can be simplified considerably by a proper choice of the foundation dimensions. For example, if the foundation block dimensions are chosen such that the centre of contact area, B, lies on the vertical line passing through A, the combined C.G., thus making $x_B = 0$, $y_B = 0$, these equations of motion get simplified as.

$$m\frac{d^2w_A}{dt^2} + c_{zz}\frac{dw_A}{dt} + k_{zz}w_A = \bar{P}_z \qquad \text{(a)}$$

$$m\frac{d^2u_A}{dt^2} + c_{xx}\frac{du_A}{dt} + k_{xx}u_A + z_B\left(c_{xx}\frac{d\psi_y}{dt} + k_{xx}\psi_y\right) = \bar{P}_x \qquad \text{(b)}$$

$$I_{\psi y} \frac{d^2\psi_y}{dt^2} + (c_{\psi y} + z_B^2 c_{xx}) \frac{d\psi_y}{dt} + (k_{\psi y} + z_B^2 k_{xx})$$

$$\psi_y + z_B(c_{xx} \frac{du_A}{dt} + k_{xx} u_A) = \overline{M}_y \qquad (c)$$

$$m \frac{d^2 v_A}{dt^2} + c_{yy} \frac{dv_A}{dt} + k_{yy} v_A - z_B(c_{yy} \frac{d\psi_x}{dt}$$

$$+ k_{yy} \psi_x) = \overline{F}_y \qquad (d)$$

$$I_{\psi x} \frac{d^2\psi_x}{dt^2} + (c_{\psi x} + z_B^2 c_{yy}) \frac{d\psi_x}{dt}$$

$$+ (k_{\psi x} + z_B^2 k_{yy})\psi_x - z_B(c_{yy} \frac{dv_A}{dt} + k_{yy} v_A) = \overline{M}_x \quad (e)$$

$$I_{\psi z} \frac{d^2\psi_z}{dt^2} + c_{\psi z} \frac{d\psi_z}{dt} + k_{\psi z} \psi_z = \overline{M}_z \quad (f)$$

(3)

in which z_B is the z-coordinate of the centre of contact area with reference to coordinate axes passing through combined C.G. (Fig.3), m is the total mass, $I_{\psi x}$, $I_{\psi y}$, $I_{\psi z}$ are the mass moments of inertia about x,y,z-axes respectively, of the machine-foundation system.

It can be seen from equations (3), that equations of motion for translation along z-axis (vertical direction) and rotation about z-axis are completely uncoupled (equations (3a) and (3f)). Accordingly w_A and ψ_z can be independently solved from the governing equations (3a) and (3f) respectively. Further, equations (3b) and (3c) involving u_A and ψ_y are coupled. Similarly equations (3d) and (3e) involving v_A and ψ_x are also coupled. Hence the translation of point A (combined C.G.) along any horizontal axis (x or y-axis) is always associated with rotation about the horizontal axis perpendicular to the axis of translation (i.e. y or x-axis) and vice versa. These are called coupled sliding and rocking modes of motion. Thus by making $x_B = y_B = 0$ (i.e. making the system symmetric in the horizontal plane), equations (3) are in a considerably simplified form and are well suited for easier analysis. This symmetry can be easily prescribed by the designer (even if x_B and (or) y_B are of the order of 5 per cent (or less) of the corresponding dimensions of the contact area, symmetry can be assumed (Barkan (1962)).

3. SOME SOLUTIONS FOR RESPONSES OF SINGLE DEGREE OF FREEDOM SYSTEM

Some solutions of the classical single degree of freedom system (SDF)(Kameswara Rao (1977) Tse et al.(1963)) which are relevent to the analysis of equations (3) are presented below. The equation of motion of SDF can be written in the usual notation as,

$$\overline{m}\,\ddot{Z} + C\,\dot{Z} + KZ = F(t) \qquad (4)$$

where $\overline{m}$, C, K, Z, F(t) are mass of the rigid body, damping constant, spring constant displacement along the direction of motion and the dynamic load respectively. Dots denote derivatives with respect to time t. Solutions of equation (4) for various cases are readily available in any of the standard books (Tse et al. (1963)), but only some relevant solutions are reviewed below.

c_c = critical damping = $2\sqrt{K\,\overline{m}}$;

ς = damping factor = C/c_c

ω_n = undamped natural frequency = $\sqrt{K/\overline{m}}$;

ω_d = damped natural frequency = $\omega_n\sqrt{1-\varsigma^2}$

ω_m = resonent frequency in the case of constant amplitude dynamic load (such as $F(t) = F_o \sin\omega t$) = $\omega_n\sqrt{1-2\varsigma^2}$

ω_{mr} = resonent frequency in the case of frequency dependent dynamic load (such as $F(t) = m_e e\omega^2 \sin\omega t$) = $\omega_n/\sqrt{1-2\varsigma^2}$

Frequency ratios: $r_n = \frac{\omega}{\omega_n}$; $r_d = \frac{\omega}{\omega_d}$; $r_m = \frac{\omega}{\omega_m}$; $r_{mr} = \frac{\omega}{\omega_{mr}}$.

where ω = operating frequency of the dynamic load F(t).

z = dynamic displacement of the body
= $z_p \sin(\omega t - \phi)$, in the case of constant amplitude dynamic load
= $z_{pr} \sin(\omega t - \phi)$; in the case of frequency dependent dynamic load.

$$\text{where } z_p = \text{amplitude} = \frac{F_o}{K} M_p$$

$$= \frac{F_o}{K} \frac{1}{\left[\left\{1-\left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(2\frac{\varsigma\omega}{\omega_n}\right)^2\right]^{1/2}}$$

$$z_{pr} = \text{amplitude} = \frac{m_e e}{\overline{m}} M_{pr} = \frac{m_e e}{\overline{m}}\left(\frac{\omega}{\omega_n}\right)^2 M_p \qquad (5)$$

$$\phi = \text{phase difference} = \tan^{-1}$$

$$= \tan^{-1}\left[\frac{2\varsigma(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$$

in which M_p and M_{pr} are called magnification factors.

Periodic dynamic load of general nature

Any dynamic load F(t) which is periodic with **period** τ can be expressed in general as a sum of several harmonic components (called harmonics) using Fourier series (Tse et al.(1963)), as

$$F(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (6)$$

where $a_o = \frac{2}{\tau} \int_0^{\tau} F(t)\, dt$; $a_n = \frac{2}{\tau} \int_0^{\tau} F(t) \cos n\omega t\, dt$;

$$b_n = \frac{2}{\tau} \int_0^{\tau} F(t) \sin n\omega t\, dt \text{ and } \omega = 2\pi/\tau \quad (7)$$

a_o, a_n, b_n are called Fourier coefficients. n is referred to as the number of harmonic. If F(t) is in the form of a function, integrations in equations (7) can be carried out either exactly or numerically as long as F(t) satisfies the standard mathematical requirements. If the load data is available at discrete time intervals, as is the case of many engine assemblies such as diesel engines, numerical integration techniques have to be necessarily resorted to. If once the load is thus converted into harmonic components, as given by equation (6), the solutions presented above for harmonic loads can be directly adopted and the steady state response of SDF can be obtained using the principle of superposition as

$$z = \frac{1}{K}\left[\frac{a_o}{2} + \sum_{n=1}^{\infty}\left\{\frac{a_n \cos(n\omega t - \phi_n) + b_n \sin(n\omega t - \phi_n)}{\sqrt{\left(1-\left(\frac{n\omega}{\omega_n}\right)^2\right)^2 + \left(\frac{2\xi n\omega}{\omega_n}\right)^2}}\right\}\right] \quad (8)$$

$$\text{where } \phi_n = \tan^{-1}\left[\frac{2\xi n \frac{\omega}{\omega_n}}{1-\left(\frac{n\omega}{\omega_n}\right)^2}\right]$$

If summing up the series (equation (8)), care must be taken not to ignore the important harmonics (not necessarily the first few) which may contribute significantly to the response. This can be ensured by summing up the series beyond that value of n (say $n = n_s$) for which the denominator in equation (8) may be very small or zero from which n_s can be obtained as

$$n_s = \sqrt{\frac{(2r_n^2 - 4\xi^2 r_n^2) + \sqrt{(2r_n^2 - 4\xi^2 r_n^2)^2 - 4r_n^4}}{2 r_n^4}} \quad (9)$$

where $r_n = \omega/\omega_n$.

If any of the quantities under square root in equation (9) are negative, n_s can be taken as 1. However, the summation of the series (equation (8)) should be carried out beyond $n = n_s$, untill the series converges as may be stipulated by an appropriate convergence criterion.

If the dynamic load F(t) is of general nature with two or more periods, the solution for each period can be obtained independently as explained above and the total solution can be obtained by superposing these solutions obtained independently for each period.

4. RESPONSE ANALYSIS OF MACHINE-FOUNDATION-SOIL SYSTEM

Machine-Foundation block, being taken as rigid, will have six degrees of freedom, three components of translation and three components of rotation. The equations of motion in all these six degrees of freedom are given by equations (3), solutions of which will yield the responses at the combined C.G. of the machine-foundation block. Then the responses at any other point of the rigid body can be readily obtained from equations (1). Solutions of equations (3) are presented below.

(i) Vertical Translation (motion along z-axis)

The equation of motion is given by (3a), which can be seen to be in an uncoupled form similar to that of SDF (equation (4)). Hence all the solutions presented in section 3 can be directly used by replacing $\overline{m}$, C, K, Z, F(t) by m, c_{zz}, k_{zz}, w_A, $\overline{P}_z$ (given by manufacturer or can be obtained from the engine data (Richart, et al. (1970)). In particular, the frequency ratios and the dynamic response w_A, can be computed from equations (5) and (8) respectively. ($\overline{P}_z$ is usually periodic but of a general nature as in the case of diesel engines (Appendix-I))

(ii) Rotation about the Vertical Axis (torsional motion about z-axis)

The equation of motion is given by equation (3f) which also can be seen to be in an uncoupled form and is similar to that of SDF (equation (4)). Accordingly all the solutions presented in section 3 can be directly used by replacing $\overline{m}$, C, K, Z, F(t) by m, $c_{\psi z}$, $k_{\psi z}$, ψ_z, $\overline{M}_z$ respectively. In particular, the frequency ratios and dynamic response ψ_z can be computed from equations (5) and (8) respectively. ($\overline{M}_z$ is usually periodic but of a general nature, as in the case of diesel engines (Appendix -I)).

(iii) Translation along x-axis coupled with Rotation about y-axis (coupled sliding (along x-axis) and rocking (about y-axis) motion)

As mentioned in section 2, equations (3b) and (3c) describe this coupled motion. This type of motion can occur either due to $\overline{P}_x$ or $\overline{M}_y$ or due to free vibrations. $\overline{P}_x$ and $\overline{M}_y$ are usually periodic but of general nature, as in the case of diesel engines (Appendix -I).

Free vibration analysis

(a) Undamped, Free Vibrations

Omitting damping terms and forcing functions in equations (3b) and (3c), and following the undamped, free vibration analysis of coupled systems (Kameswara Rao (1980), Tse et al. (1963)) the undamped natural frequencies can be obtained as

$$(\omega_n)_{1,2} = \left\{ \frac{mk_{\psi y} + k_{xx}(mz_B^2 + I_{\psi y})}{2m\, I_{\psi y}} \mp \frac{\sqrt{\left(mk_{\psi y}+k_{xx}(mz_B^2+I_{\psi y})\right)^2 - 4mI_{\psi y}k_{xx}k_{\psi y}}}{2m\, I_{\psi y}} \right\}^{1/2} \quad (10)$$

The corresponding frequency ratios can be expressed as

$$r_{n1} = \omega/\omega_{n1};\; r_{n2} = \omega/\omega_{n2} \quad (11)$$

where ω is the operating frequency.

(b) Damped, Free Vibrations

Omitting the forcing functions $\bar{P}_x$ and $\bar{M}_y$ in equations (3b) and (3c) and following the damped, free vibration analysis of coupled systems (Kameswara Rao (1980)), the characteristic determinant in terms of the parameter's'(from which damped natural frequencies can be computed) can be obtained as

$$\begin{vmatrix} ms^2+c_{xx}s+k_{xx} & z_B(c_{xx}s + k_{xx}) \\ z_B(c_{xx}s + k_{xx}) & I_{\psi y}s^2+(c_{\psi y}+z_B^2c_{xx})s+k_{\psi y}+z_B^2\, k_{xx} \end{vmatrix} = 0 \quad (12)$$

Solving the resulting quartic equation in s, damped natural frequencies exist only if the four roots of s are complex conjugate with negative real part, the complex parts giving the damped natural frequencies ω_{d1} and ω_{d2}. If they exist the corresponding frequency ratios can be expressed as

$$r_{d1} = \omega/\omega_{d1};\; r_{d2} = \omega/\omega_{d2} \quad (13)$$

where ω is the operating frequency.

Forced Vibration Analysis

Referring to equations (3b) and (3c), solutions are presented below taking the forcing functions $\bar{P}_x$ and $\bar{M}_y$ to be of general nature but of the same period (this includes the case when either $\bar{P}_x$ or $\bar{M}_y$ may be zero). Solutions for forcing functions of general nature but of different periods, can be obtained by repeating the analysis presented below (for the case when $\bar{P}_x$ and $\bar{M}_y$ are of the same period or frequency) for each period and superposing all such solutions to get the total solution .

Since the forcing functions are taken to be of general nature but of the same period, τ (and hence the same frequency $\omega = 2\pi/\tau$), $\bar{P}_x$ and $\bar{M}_y$ can be expressed using Fourier series (Tse et al.(1963)) as

$$\bar{P}_x = \frac{a_{10}}{2} + \sum_{n=1}^{\infty} (a_{1n}\cos n\omega t + a_{2n}\sin n\omega t)$$

$$\bar{M}_y = \frac{b_{10}}{2} + \sum_{n=1}^{\infty} (b_{1n}\cos n\omega t + b_{2n}\sin n\omega t) \quad (14)$$

in which a_{10}, a_{1n}, a_{2n}, b_{10}, b_{1n}, b_{2n} are the Fourier coefficients which can be obtained using the Fourier analysis knowing $\bar{P}_x$ and $\bar{M}_y$ as defined in equations (7). Solutions of equations (3b) and (3c) can be sought in the form

$$u_A = \frac{A_{10}}{2} + \sum_{n=1}^{\infty} (A_{1n}\cos n\omega t + A_{2n}\sin n\omega t)$$

$$\psi_y = \frac{B_{10}}{2} + \sum_{n=1}^{\infty} (B_{1n}\cos n\omega t + B_{2n}\sin n\omega t) \quad (15)$$

Substituting equations (15) in equations (3b) and (3c) and using the orthogonality of harmonics, the coefficients A_{10}, A_{1n}, A_{2n}, B_{10}, B_{1n}, B_{2n} can be obtained for all values of n equal to 1 to ∞ from the following matrix equations (Kameswara Rao (1980)).

$$\begin{bmatrix} k_{xx} & z_B\, k_{xx} \\ z_B k_{xx} & k_{\psi y}+z_B^2 k_{xx} \end{bmatrix} \begin{Bmatrix} A_{10} \\ B_{10} \end{Bmatrix} = \begin{Bmatrix} a_{10} \\ b_{10} \end{Bmatrix} \quad (16)$$

$$\begin{bmatrix} -mn^2\omega^2+k_{xx} & n\omega c_{xx} & z_B k_{xx} & n\omega z_B c_{xx} \\ -n\omega c_{xx} & -mn^2\omega^2 + k_{xx} & -n\omega z_B c_{xx} & z_B k_{xx} \\ z_B k_{xx} & n\omega z_B c_{xx} & -I_{\psi y}n^2\omega^2 + k_{\psi y} + z_B^2 k_{xx} & n\omega(c_{\psi y} + z_B^2\, c_{xx}) \\ -n\omega z_B c_{xx} & z_B k_{xx} & -n\omega(c_{\psi y} + z_B^2\, c_{xx}) & -I_{\psi y}\, n^2\omega^2 + k_{\psi y} + z_B^2 k_{xx} \end{bmatrix} \begin{Bmatrix} A_{1n} \\ A_{2n} \\ B_{1n} \\ B_{2n} \end{Bmatrix} = \begin{Bmatrix} a_{1n} \\ a_{2n} \\ b_{1n} \\ b_{2n} \end{Bmatrix} \quad (17)$$

Knowing A_{10}, A_{1n}, A_{2n}, B_{10}, B_{1n}, B_{2n}, the steady state responses u_A and ψ_y can be computed from equations (15).

However, in summing up the series given by equations (15), care must be taken not to ignore the important harmonics (not necessarily the first few) which may contribute significantly to the components of motion. This can be ensured by summing up the series (equations (15)) beyond that value of n (say $n = n_s$) for which the coefficient matrix in equations (17) becomes singular or value of the determinant becomes very small. Accordingly n_s can be obtained as

$$n_s = \sqrt{\frac{p_1 + \sqrt{p_1^2 - 4p_2}}{2}} \qquad (18)$$

$$\text{where } p_1 = \frac{1}{\omega^2}\left[\frac{k_{\psi y} + z_B^2 k_{xx}}{I_{\psi y}} + \frac{k_{xx}}{m}\right];$$

$$p_2 = \frac{k_{xx}\, k_{\psi y}}{m I_{\psi y}\, \omega^4} \qquad (19)$$

If $p_1^2 - 4p_2$ in equation (18) is negative, n_s can be taken as 1. The summation of the series (equations (15)) should be carried out beyond $n = n_s$, untill the series converges as may be stipulated by an appropriate convergence criterion, thus giving the coupled responses u_A and ψ_y.

(iv) Translation along y-axis coupled with Rotation about x-axis (coupled sliding (along y-axis) and rocking (about x-axis) motion)

The relevant equations of motion are given by equations (3d) and (3e) and the analysis is just similar to the coupled motion analysis discussed in the above subsection (iii). Accordingly, following the same steps indicated in the above subsection, the final solutions can be obtained by replacing x, y, u, z_B by $y, x, v, -z_B$ respectively whereever they occur in all the equations of subsection (iii).

5. RESULTS

A computer program has been developed incorporating all the aspects of the analysis discussed in the present investigation and some results have been presented to illustrate the same. Computations have been carried out for the eight cylinder diesel engine (turbocharged) power generating set manufactured by M/s KIRLOSKAR OIL ENGINES LIMITED, PUNE, INDIA. The assembly is shown in Fig. 2. Some relevant details of the engine, generator set assembly, dynamic forces and moments coming from the engine are given in Appendix-I. Foundation block of the shape of a rectangular parallelopiped with dimensions a,b,c (along x, y, z respectively) is considered. While a and b are taken as 2.00 and 5.985 meters, two values of thickness c are considered for the analysis, (i) c=1.5 meters (ii) c=0.3 meters. The unit weight, ρ and Poisson's ratio, γ of the soil are taken as 1700 kg/m^3 and 0.3 respectively. Results have been obtained for a range of values of shear of modulus of the soil, G between 0.1×10^7 kg/m^2 to 1.0×10^7 kg/m^2 (medium to dense soils). Frequency ratios r_n and r_d and maximum responses in the various modes of vibration have been presented in Figs. 4 and 5.

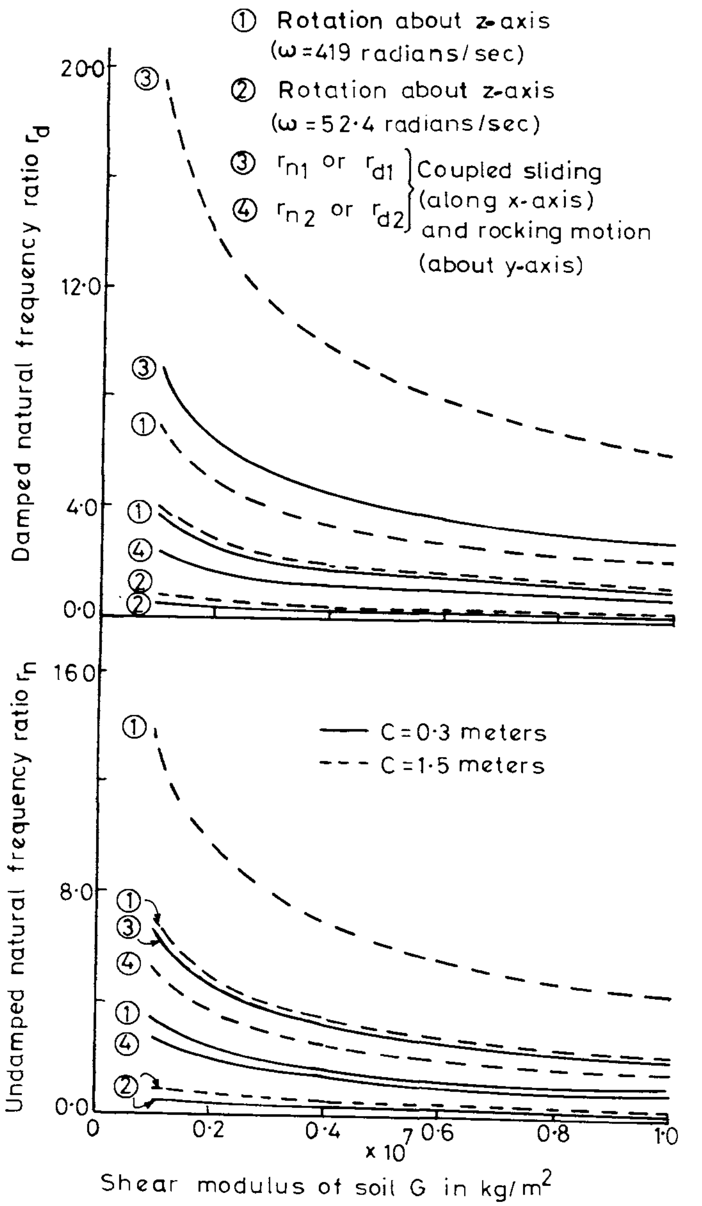

Fig. 4. Variation of Frequency Ratios with Shear Modulus of Soil

As is evident from the dynamic load data of the engine, (Appendix-I), only the three modes of vibration of the machine-foundation-soil system are of interest in this example namely, coupled sliding (along x-axis) and rocking (about y-axis) motion and the rotation about z-axis. Here also, the dynamic moment about z-axis coming from the engine has two components with different periods (i.e.) the one due to M_{ze} with a frequency of 500 cycles per minute (ω = 52.4 radians/sec)and the second due to F_{xe} with a frequency of 4000 cycles per minute (ω= 419 radians/sec). As can be seen from the Appendix-I all the dynamic forces and moments are of a general nature and hence the program carries out the analysis using Fourier

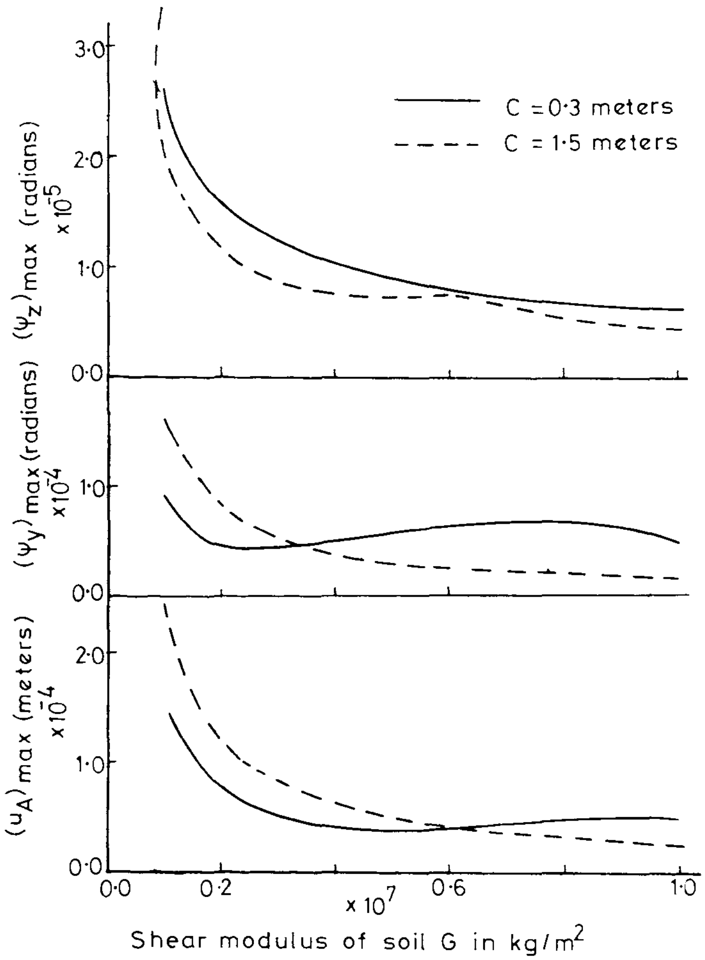

Fig. 5. Variation of Maximum Responses with Shear Modulus of Soil

series in all the modes of vibration as described in section 4.

6. CONCLUSIONS

The versatality of the analysis is brought out by illustrating the same in the case of a typical diesel engine installation for power generation application. From the results presented in Figs. 4 and 5, it can be seen that the thickness of the foundation block has higher influence both on the frequency ratios as well as on the maximum responses at the combined C.G. in the lower range of values of shear modulus, G than in the higher range of G values. However, it can also be seen from these figures that increased thickness of the foundation block does not necessarily improve the dynamic performance of the system. The analysis presented is quite general and can be used successfully for evaluating the responses of machine-foundation-soil systems of general nature.

7. ACKNOWLEDGEMENTS

The author acknowledges with thanks the support extended by M/s. Kirloskar Oil Engines Limited, Pune, India in carrying out this investigation. The encouragement given by the authorities of Indian Institute of Technology, Kanpur in pursuing this work is appreciated.

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APPENDIX-I : DETAILS OF THE MACHINE ASSEMBLY AND DYNAMIC LOAD DATA

The following data is provided by M/s. Kirloskar Oil Engines Limited, Pune, India.

Engine: Eight cylinder (Turbo-charged) diesel engine manufactured by M/s. Kirloskar Oil Engines Limited, Pune, India.

Revolutions per minute (r.p.m) = 1000; Brake horse power = 372.

Static weights and coordinates of the C.G. of the various units of the assembly with reference to x_C, y_C, z_C axes (Fig.1 and Fig. 2).

Sl. No.	Unit	Weight kg.	x meters	y meters	z meters
1	Diesel engine	3603	0.0	0.0	0.3
2	Alternator	2445	0.0	2.545	0.0
3	Base plate	1500	0.0	0.125	-0.421
4	Foundation block (size a x b x c)	2400abc	0.0	0.425	*

* i) -1.546 for c=1.5m
ii) -0.946 for c=0.3m

The dynamic loads and moments coming at the C.G. of the engine F_{xe}, M_{ye}, M_{ze} (as per the convention shown in Fig. 1) are given below for various angles of the crank shaft for one cycle, after which they repeat. Knowing these values and the coordinates of the C.G. of the engine with reference to x, y, z-axes passing through the combined C.G. of the machine and foundation, the forcing functions at A, the combined C.G. (Fig.2), $\bar{P}_x$, $\bar{P}_y$, $\bar{P}_z$, $\bar{M}_x$, $\bar{M}_y$, $\bar{M}_z$ can be easily computed which are used in the analysis (Section 4).

F_{xe} (dynamic load along the x-axis in kg. acting at the C.G. of the engine as shown in Fig. 1) is given below for various crank angles in intervals of 5 degrees starting from 0 degrees to 90 degrees after which the cycle repeats itself. Hence frequency of F_{xe}, ω = 419 radians/sec (4000 cycles/minute) and period $\tau = \frac{2\pi}{\omega}$ = 0.015 sec.

-375, -685, -1031, 1398, -1658, -1745, -1663, -1483, -1216, -928, -648, -429, -242, -73, 41, 101,49, -123, -375.

M_{ye}(dynamic moment about the y-axis in kg. meters acting at the C.G. of the engine as shown in Fig. 1) is given below for various crank angles in intervals of 5 degrees starting from 0 degrees to 90 degrees after which the cycle repeats itself. Hence frequency of M_{ye}, ω = 419 radians/sec (4000 cycles/minute) and period $\tau = 2\pi/\omega$ = 0.015 sec.

-28.3, -81.2, -141.8, -207.2, -253, -267.9, -253.1, -221.9, -177.9, -132.8, -89.8, -56.7, -27.1, 2.8, 26.6, 42.8, 39.4, 13.2, -28.3.

M_{ze} (dynamic moment about the z-axis in kg. meters acting at the C.G. of the engine as shown in Fig. 1) is given below for various crank angles in intervals of 5 degrees starting from 0 degrees to 720 degrees after which the cycle repeats itself. Hence frequency of M_{ze}, ω = 52.4 radians/sec (500 cycles/minute) and period $\tau = 2\pi/\omega$=0.12 sec.

-51.5, 201.4, 480.7, 777.7, 1006.3, 1122.3, 1122.7, 1052.6, 924.3, 790.8, 660.1, 558.4, 478.1, 414.9, 372.5,341.6, 337.4, 361.0, 403.9, 4 460.5, 527.6, 602.1, 659.8, 684.7, 684.5, 673.6, 662.6, 674.5, 689.9, 691.9, 701.9, 726.6, 730.6, 697.2, 583.2, 379.4, 118.7, 182.1, -509.4, -854, -1117.3, -1252.4, -1252.6, -1165.3, -1006.9, -836.1 -667.6, -537.3, -430.6, -337.4, -267.7, -213.7, -195.8, -215.4, -258.8, -317.5, -384.6, -455.1, 489.2, -465.4, -388.7, -288.6, -155, 1.5, 155.1, 269.6, 373.6, 478.9, 542.2, 551.6, 448.4, 222.5, -83.8, -448, -852, -1281.1,-1503.5, -1750.8, -1721.3, -1586.8,-1362.8, -1125.2, -893.4, -713, -565.5, -441.3, -357.5, -304.9, -319.5, 405.4, -539.3, -707.1, -898.9, -1105.5, -1257, -1313.2, -1283.2, -1207.8, -1100.6, -1008.9, -923.2,-846.4, -789.3, -753, -715.6, -660.5, -565.3, -423.8, -254, -64.5, 1[illegible]8.1, 350.6, 520.2, 623.9, 653.9, 631.1, 568.9, 501.7, 434.3, 382.8, 343.1, 311, 282.8, 250.4, 213.7, 171, 123.4, 70.9, 13.3, -48.3, -107.9, -163.1, -210, -245.7, -283, -335.9, -388.4. -424.1, -461.1, -505.3, -527.3, -514.[illegible], -430.5, -26[illegible], -51.5.