# Control of Nonholonomic Mobile Robot Formations Using Neural Networks

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*Abstract*—***In this paper the control of formations of multiple nonholonomic mobile robots is attempted by integrating a kinematic controller with a neural network (NN) computed-torque controller. A combined kinematic/torque control law is developed for leader-follower based formation control using backstepping in order to accommodate the dynamics of the robots and the formation in contrast with kinematic-based formation controllers. The NN is introduced to approximate the dynamics of the follower as well as its leader using online weight tuning. It is shown using Lyapunov theory that the errors for the entire formation are uniformly ultimately bounded, and numerical results are provided.***

***Index Terms* —Neural network, formation control, Lyapunov methods, kinematic/dynamic controller.**

## I. INTRODUCTION

Over the past decade, the attention has shifted from the control of a single nonholonomic mobile robot [1-2] to the control of multiple mobile robots because of the advantages a team of robots offer such as increased efficiency and more systematic approaches to tasks like search and rescue operations, mapping unknown or hazardous environments, and security and bomb sniffing.

There are several methodologies [3-9] to robotic formation control which include behavior-based [3], generalized coordinates [4], virtual structures [5], and leader-follower [6-10] to name a few. Perhaps the most popular and intuitive approach is the leader-follower method. In this method, a follower robot stays at a specified separation and bearing from a designated leader robot.

In [6] and [9], local sensory information and a vision based approach to leader-following is undertaken respectively. In both the approaches, the sensory information was used to calculate velocity control inputs. A modified leader follower control is introduced in [7] where Cartesian coordinates are used rather than polar. In [8], it is acknowledged that the separation-bearing methodologies of leader-follower formation control closely resemble a tracking controller problem and a reactive tracking control strategy that converts a relative pose control problem into a tracking problem between a virtual robot and the leader is developed. A drawback of this controller is the need to define a virtual robot and the fact that dynamics are not considered. A characteristic that is common in many formation control papers [6-9] is the design of a kinematic controller thus requiring a perfect velocity tracking assumption and formation dynamics are ignored. In [10], the dynamics of the follower robot are considered and a neural network (NN) is introduced to estimate its dynamics; however, the dynamical effects of the leader and the formation are ignored.

In this paper, the frame work developed for controlling single nonholonomic mobile robots is expanded to leader follower formation control, and the dynamics of all robots have been considered thus incorporating the formation dynamics in the controller design. The dynamical extension introduced in this paper provides a rigorous method of taking into account the specific vehicle dynamics to convert a steering system command into control inputs via backstepping. Both feedback velocity control inputs and velocity following control laws are presented, and a neural network (NN) is introduced to learn the dynamics of the follower robots well as their leaders' online. The formation errors are shown to be uniformly ultimately bounded using Lyapunov methods, and simulation results are provided.

## II. LEADER-FOLLOWER FORMATION CONTROL

The two popular techniques in leader-follower formation control include separation-separation and separation-bearing [9]. The goal of separation-bearing formation control is to find a velocity control input such that

$$\lim_{t\to\infty}(L_{ijd}-L_{ij})=0 \quad \text{and} \quad \lim_{t\to\infty}(\Psi_{ijd}-\Psi_{ij})=0 \qquad (1)$$

where $L_{ij}$ and $\psi_{ij}$ are the measured separation and bearing of the follower robot with $L_{ijd}$ and $\psi_{ijd}$ represent desired distance and angles respectively [6][9]. Only separation-bearing techniques are considered, but our approach can be extended to separation-separation control. To avoid collisions, separation distances are measured from the back of the leader to the front of the follower, and the kinematic equations for the front of the $j^{th}$ follower robot can be written as

$$\dot{q}_j=\begin{bmatrix}\dot{x}_j\\ \dot{y}_j\\ \dot{\theta}_j\end{bmatrix}=S_j(q_j)v_j=\begin{bmatrix}\cos\theta_j & -d_j\sin\theta_j\\ \sin\theta_j & d_j\cos\theta_j\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_j\\ \omega_j\end{bmatrix} \qquad (2)$$

where $d_j$ is the distance from the rear axle to the to front of the robot, $x_j$ , $y_j$ , and $\theta_j$ are actual Cartesian position and orientation of the physical robot, and $v_j$ , and $\omega_j$ are linear and angular velocities respectively. Many robotic systems can be characterized as a robotic system having an $n$-dimensional configuration space $\mathcal{C}$ with generalized coordinates $(q_1,...q_n)$ subject to $m$ constraints [1] where after

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applying the transformation described in [1], the dynamics are given by

$$\bar{M}_j(q_j)\dot{v}_j + \bar{V}_{m_j}(q_j,\dot{q}_j)v_j + \bar{F}_j(v_j) + \bar{\tau}_{d_j} = \bar{B}_j(q_j)\tau_j. \quad (3)$$

where $\bar{M}_j \in \Re^{rxr}$ is a symmetric positive definite inertia matrix, $\bar{V}_{mj} \in \Re^{rxr}$ is the centripetal and coriolis matrix, $\bar{F}_j \in \Re^{rx1}$ is the friction vector, $\bar{\tau}_{dj}$ represents unknown bounded disturbances, and $\bar{\tau}_j = \bar{B}_j\tau \in \Re^{rx1}$ is the input vector. It is important to highlight the *skew symmetric property* common to robotic systems [1] as $\dot{\bar{M}}_j - 2\bar{V}_{m_j}(q_j,\dot{q}_j) = 0$.

*A. Controller Design*

Standard approaches [6-9] to leader follower formation control deal only with (11) and assume that perfect velocity tracking holds. In other words, the dynamics of mobile robot leader *i* on follower *j* are ignored, and this paper overcomes this assumption by defining the nonlinear feedback control input

$$\tau_j = \bar{B}_j^{-1}(\bar{M}_j u_j + \bar{V}_{mj}v_j + \bar{F}_j(v_j) + \bar{\tau}_d) \quad (4)$$

where $u_j$ is an auxiliary input. Applying this control law to (3) allows one to convert the dynamic control problem into the kinematic control [1] such that

$$\begin{aligned} \dot{q}_j &= S_j(q_j)v_j \\ \dot{v}_j &= u_j. \end{aligned} \quad (5)$$

*Backstepping Design:* To incorporate the dynamics of the mobile platform, it is desirable to convert a control velocity $v_c(t)$ into a control torque, $\tau_c(t)$ for the physical robot. Contributions in single robot frameworks are now considered and expanded upon in the development a kinematic controller for the separation-bearing formation control technique. Our aim to design a NN based torque controller such that (2) and (3) exhibit the desired behavior for a given control $v_c(t)$ thus removing perfect velocity tracking.

In a single robot control, steering control input $v_c(t)$ is designed to solve three basic problems: path following, point stabilization, and trajectory following such that $lim_{t\to\infty}(q_r-q_j)=0$ and $lim_{t\to\infty}(v_c-v_j)=0$ [1]. If the mobile robot controller can successfully track a class of smooth control velocity inputs, then the problems can be solved with the same controller [1].

Consider the tracking controller error system presented in [1] used to control a single robot as

$$\begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} \cos\theta_j & \sin\theta_j & 0 \\ -\sin\theta_j & \cos\theta_j & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_j \\ y_r - y_j \\ \theta_r - \theta_j \end{bmatrix} \quad (6)$$

$$\dot{x}_r = v_r\sin\theta_r,\ \dot{y}_r = v_r\sin\theta_r,\ \dot{\theta}_r = \omega_r,\ \dot{q}_r = \begin{bmatrix}\dot{x}_r & \dot{y}_r & \dot{\theta}_r\end{bmatrix}^T \quad (7)$$

where $x_j$, $y_j$, and $\theta_j$ are actual position and orientation of the physical robot, and $x_r$, $y_r$, and $\theta_r$ are the positions and orientation of a virtual reference cart *j* seeks to follow [1][2].

The three basic tracking control problems can be extended to formation control as follows. The virtual reference cart is replaced with a physical mobile robot acting as the leader *i*, and $x_r$ and $y_r$ are defined as points at a distance $L_{ijd}$ and a desired angle $\psi_{ijd}$ from the lead robot. Now the three basic navigation problems can be introduced for leader-follower formation control as follows.

*Tracking:* Let there be a leader *i* for follower *j* such that

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -d_i\sin\theta_i \\ \sin\theta_i & d_i\cos\theta_i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \quad (8)$$

$$\begin{aligned} x_{jr} &= x_i - d_i\cos\theta_i + L_{ijd}\cos(\Psi_{ijd}+\theta_i) \\ y_{jr} &= y_i - d_i\sin\theta_i + L_{ijd}\sin(\Psi_{ijd}+\theta_i) \\ \theta_{jr} &= \theta_i \end{aligned} \quad (9)$$

$$v_{jr} = [|v_i| \quad |\omega_i|]^T \quad (10)$$

where $v_{jr}$ is the time varying linear and angular speeds of the leader such that $v_{jr} > 0$ for all time. Then define the actual position and orientation of follower *j* as

$$\begin{aligned} x_j &= x_i - d_i\cos\theta_i + L_{ij}\cos(\Psi_{ij}+\theta_i) \\ y_j &= y_i - d_i\sin\theta_i + L_{ij}\sin(\Psi_{ij}+\theta_i) \\ \theta_j &= \theta_j \end{aligned} \quad (11)$$

where $L_{ij}$ and $\Psi_{ij}$ is the actual separation and bearing of follower *j*. In order to solve the formation tracking problem with one follower, find a smooth velocity input $v_{jc}=f(e_p, v_{jr}, K)$ such that $lim_{t\to\infty}(q_{jr}-q_j)=0$, where $e_p$, $v_{jr}$, and $K$ are the tracking position errors, reference velocity for follower *j* robot, and gain vector respectively. Then compute the torque $\tau_j(t)$ for the dynamic system of (3) so that $lim_{t\to\infty}(v_{jc}-v_j)=0$. Achieving this for every leader *i* and follower *j=1,2,..N* ensures that the entire formation tracks the formation trajectory.

The contribution in this paper lies in deriving an alternative control velocity, $v_{jc}(t)$, for separation-bearing leader follower formation control, and calculating the specific torque $\tau_j(t)$ to control (3) which accounts for the $i^{th}$ leader's dynamics as well as the $j^{th}$ follower's. It is common in the literature to assume perfect velocity tracking which does not hold in real applications. To remove this assumption, integrator backstepping is applied.

Using (9), (11) and simple trigonometric identities the error system (6) can be rewritten as

$$e_j = \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} L_{ijd}\cos(\Psi_{ijd}+e_{j3}) - L_{ij}\cos(\Psi_{ij}+e_{j3}) \\ L_{ijd}\sin(\Psi_{ijd}+e_{j3}) - L_{ij}\sin(\Psi_{ij}+e_{j3}) \\ \theta_i - \theta_j \end{bmatrix} \quad (12)$$

The transformed error system now acts as a formation tracking controller which not only seeks to remain at a fixed desired distance $L_{ijd}$ with a desired angle $\psi_{ijd}$ relative to the lead robot *i*, but also achieves the same orientation as the lead robot which is desirable when $\omega_i = 0$.

In order to calculate the error dynamics given in (12), it is necessary to calculate the derivatives of $L_{ij}$ and $\psi_{ij}$, and it is assumed that $L_{ijd}$ and $\psi_{ijd}$ are constant. It is shown in [12] that

$$\dot{L}_{ij} = v_j \cos\gamma_j - v_i \cos\Psi_{ij} + d_j w_j \sin\gamma_j$$
$$\dot{\Psi}_{ij} = \frac{1}{L_{ij}}(v_i \sin\Psi_{ij} - v_j \sin\gamma_j + d_j w_j \cos\gamma_j - L_{ij} w_i) \tag{13}$$

where $\gamma_j = \Psi_{ij} + e_{j3}$.

Now, using the derivative of (12), equation (13) and applying simple trigonometric identities, the error dynamics can be expressed as

$$\begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = \begin{bmatrix} -v_j + v_i \cos e_{j3} + \omega_j e_{j2} - \omega_i L_{ijd} \sin(\Psi_{ijd} + e_{j3}) \\ -\omega_j e_{j1} + v_i \sin e_{j3} - d_j \omega_j + \omega_i L_{ijd} \cos(\Psi_{ijd} + e_{j3}) \\ \omega_i - \omega_j \end{bmatrix}. \tag{14}$$

Examining (14) and the error dynamics of a tracking controller for a single robot in [1], one can see that dynamics of a single follower with a leader is similar to [1], except additional terms are introduced as a result of (2) and (13).

To stabilize the kinematic system, we propose the following velocity control inputs for follower robot $j$ to achieve the desired position and orientation with respect to leader $i$ as

$$v_{jc} = \begin{bmatrix} v_{jc} \\ \omega_{jc} \end{bmatrix} = \begin{bmatrix} v_i \cos e_{j3} + k_1 e_{j1} \\ \omega_i + (v_i + k_v) k_2 e_{j2} + (v_i + k_v) k_3 \sin e_{j3} \end{bmatrix} + \begin{bmatrix} \gamma_{vjc} \\ \gamma_{wjc} \end{bmatrix} \tag{15}$$

where $$\gamma_{vjc} = -\omega_i L_{ijd} \sin(\Psi_{12d} + e_{j3}) \tag{16}$$

$$\gamma_{\omega jc} = -\frac{\left|e_{j2}\right| \left(\omega_i (d_j + L_{ijd}) + (v_i + k_v) k_3 d_j + 1\right)}{1/k_2 + \left|e_{j2}\right| d_j} \tag{17}$$

Before we proceed, the following assumptions are needed.

*Assumption 1.* Follower $j$ is equipped with sensors capable of measuring the separation distance $L_{ij}$ and bearing $\Psi_{ij}$ and that both leader and follower are equipped with instruments to measure their linear and angular velocities as well as there orientations $\theta_i$ and $\theta_j$.

*Assumption 2.* Wireless communication is available between the $j^{th}$ follower and $i^{th}$ leader with communication delays being zero.

*Assumption 3.* The $i^{th}$ leader communicates its linear and angular velocities $v_i$, $w_i$ as well as its orientation $\theta_i$ and control torque $\tau_i(t)$ to its $j^{th}$ follower.

*Assumption 4*. For the nonholonomic system of (2) and (3) with $n$ generalized coordinates $q$, $m$ independent constraints, and $r$ actuators, the number of actuators is equal to the number of degrees of freedom ( $r = n - m$ ).

*Assumption 5.* The reference linear and angular velocities measured from the leader $i$ are bounded and $v_{jr}(t) \geq 0$ for all $t$.

*Assumption 6.* $K = [k_1 \quad k_2 \quad k_3]^T$ is a vector of positive constants.

*Assumption 7.* Let perfect velocity tracking hold such that $\dot{v}_j = \dot{v}_{jc}$ (this assumption is relaxed later).

*Theorem 1[12]:* Given the nonholonomic system of (2) and (3) with $n$ generalized coordinates $q$, $m$ independent constraints, and $r$ actuators, along with the leader follower criterion of (1), let *Assumption 1-7* hold. Let a smooth velocity control input $v_{jc}(t)$ for the $j^{th}$ follower be given by (15), (16), and (17). Then the origin $e_j$=0 consisting of the position and orientation error for the follower is asymptotically stable.

Now assume that the perfect velocity tracking assumption does not hold making *Assumption 7* invalid. A two-layer NN is considered here consisting of one layer of randomly assigned constant weights $V \in \Re^{axL}$ in the first layer and one layer of tunable weights $W \in \Re^{Lxb}$ in the second with $a$ inputs, $b$ outputs, and L hidden neurons. The *universal approximation property* for NN's [11] states that for any smooth function $f(x)$, there exists a NN such that $f(x) = W^T \sigma(V^T x) + \varepsilon$ where $\varepsilon$ is the NN functional approximation error and $\sigma(\cdot): \Re^a \to \Re^L$ is the activation function in the hidden layers. The sigmoid activation function is considered here. For complete details of the NN and its properties, see [11].

*Remark:* $\|\cdot\|$ and $\|\cdot\|_F$ will be used interchangeably as the Frobenius vector and matrix norms [11].

Define the velocity tracking error as

$$e_{jc} = v_{jc} - v_j \tag{18}$$

Differentiating (18) and adding and subtracting $\overline{M}_j(q_j)\dot{v}_{jc}$ and $\overline{V}_{mj}(q_j)v_{jc}$ to (3) allows the mobile robot dynamics to be written in terms of the velocity tracking error and its derivative as

$$\overline{M}_j(q_j)\dot{e}_{jc} = -\overline{V}_{mj}(q_j, \dot{q}_j)e_{jc} + f_j(x) + \overline{\tau}_{dj} \tag{19}$$

where $$f_j(x_j) = \overline{M}_j(q_j)\dot{v}_{jc} + \overline{V}_{mj}(q_j, \dot{q}_j)v_{jc} + \overline{F}_j(v_j) \tag{20}$$

Define $x_j = [\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, q_j, v_j, w_j, e_j, \dot{e}_j]$. The function $f_j(x_j)$ in (20) will be used to bring in the dynamics of leader $i$ through $\dot{v}_{jc}$ by observing that

$$\dot{v}_{jc} = f_{vcj}(\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, e_j, \dot{e}_j). \tag{21}$$

The leader $i$'s dynamics can be written in the form of (3) as

$$\dot{v}_i = \overline{M}_i^{-1}(q_i)(\overline{B}_i(q_i)\tau_i - \overline{\lambda}_i - \overline{V}_{m_i}(q_i, \dot{q}_i)v_i - \overline{F}_i(v_i) - \overline{\tau}_{d_i}) \tag{22}$$

Substituting (22) into (21) results in the dynamics of the $i^{th}$ leader robot to become apart of $\dot{v}_{jc}$ as

$$\dot{v}_{jc} = f_{vcj}(v_i, \omega_i, \theta_i, \tau_i, e_j, \dot{e}_j) \tag{23}$$

A conventional computed torque controller with velocity tracking could be defined as [12]

$$\tau_j = \overline{B}_j^{-1}(\overline{M}_j K_4 e_{jc} + f_j(x_j)) \tag{24}$$

where $f_j(x_j)$ is defined by (20) and $K_4$ is a positive gain matrix. However, the $j^{th}$ follower is not able to construct $\dot{v}_{jc}$ since knowledge of the dynamics of leader $i$ is required, making (24) unavailable.

*Remark:* In [1] and [2], the reference velocity is taken as a constant by ignoring the dynamics of the reference cart.

That assumption is not valid here since the reference cart has been replaced by a physical robot $i$ which appears to be the leader. Thus, the dynamics of leader robot $i$ must be considered in follower $j$'s torque command.

Therefore, the NN is introduced to approximate the dynamics of the mobile robots—both leader and followers. Define a control torque for follower $j$ to be as

$$\bar{\tau}_j = \hat{W}_j^T \sigma(\bar{x}_j) + K_4 e_{jc}, = \hat{f}_j + K_4 e_{jc} \tag{25}$$

where

$$\bar{x}_j = V^T [v_{jr}^T \quad \theta_i \quad \tau_i^T \quad e_j^T \quad \dot{e}_j^T \quad v_j^T \quad v_{jc}^T \quad \dot{v}_{jc}^T\big|_{\dot{v}_{jr}=0}] \tag{26}$$

and $K_4$ is a positive definite matrix defined by $K_4 = k_4 I$ and $\hat{f}_j$ is the NN estimate of (20). The last element of the NN input vector (26) is a preprocessed derivative of control velocity (15), (16) and (17) assuming the leader's acceleration is zero (i.e. $\dot{v}_{jr} = 0$). Since the leader's acceleration is not always zero, the first four terms of (26) are introduced to accommodate the dynamics of the leader and the omitted terms of $\dot{v}_{jc}^T$. Substituting the torque control (25) into the mobile robot error system (19), the closed loop equations become

$$\overline{M}_j \dot{e}_{jc} = -(K_4 + \overline{V}_{mj}) e_{jc} + \tilde{f}_j + \bar{\tau}_d + \varepsilon_j \tag{27}$$

where the velocity tracking error $e_{jc}$, is driven by the NN functional estimation error

$$\tilde{f}_j = f_j - \hat{f}_j \tag{28}$$

According to [11] and [2], applying control (25) does not guarantee that the $\bar{\tau}_j$ will make the velocity tracking error (18) small. In order to guarantee that (18) is small, it is required to specify a method of selecting $K_4$ and $\hat{f}_j$ such that the velocity tracking error is bounded. Before proceeding, the following definitions and mild assumptions are required.

The weight estimation errors for follower $j$ can be defined similarly to (28), such that

$$\tilde{W}_j = W_j - \hat{W}_j \tag{29}$$

*Definition 1:* An equilibrium point $x_e$ is said to be *uniformly ultimately bounded (UUB)* if there exists a compact set $S \subset \Re^n$ so that for all $x_0 \in S$ there exists a bound $B$ and a time $T(B,x_o)$ such that $\|x(t) - x_e\| \le B$ for all $t \ge t_0 + T$ [11].

*Assumption 8.* On any compact subset of $\Re^n$, the ideal NN weights are bounded by known positive values for all followers $j=1,2,\ldots N$ such that $\|W_j\|_F \le W_M$ [11].

*Assumption* 9. The NN reconstruction error for all followers $j$ is bounded such that $\|\varepsilon_j\| < \varepsilon_N$, and the disturbances are bounded such that $\|\bar{\tau}_{dj}\| \le d_M$ [2].

*Assumption 10.* Let the NN approximation property (8) hold for the function $f_j(x_j)$ (20) with accuracy $\varepsilon_N$ for all followers $j$ for all $x_j$ $j=1,2,\ldots N$ in the compact set $S$ [11].

*Theorem 2:* Let *Assumptions 1-6* and *8-10* hold and let $k_4$ be a sufficiently large positive constant. Let a smooth velocity control input $V_{jc}(t)$ for the $j^{th}$ follower be defined by (15), (16) and (17). Let the torque control for the $j^{th}$ follower robot (25) be applied to the mobile robot system (3) and let the weight tuning law be given as

$$\dot{\hat{W}}_j = F\sigma_j e_{jc}^T - \kappa F \|e_{jc}\| \hat{W}_j \tag{30}$$

where $F = F^T > 0$ and $\kappa > 0$ a small design parameter. Then $e_j$ , $e_{jc}$ and $\tilde{W}$ which are the position, orientation , and velocity tracking errors as well as the NN weight estimates respectively for follower $j$ are UUB. Furthermore, the velocity tracking errors can be made as small as desired by increasing the gain matrix $K_4$.

*Proof* : Consider the following Lyapunov candidate:

$$V_j' = V_j + V_{jNN} \tag{31}$$

where $V_j$ is the Lyapunov candidate from *Theorem 1* and defined in [12] as $V_j = \frac{1}{2}(e_{j1}^2 + e_{j2}^2) + \frac{1-\cos e_{j3}}{k_2}$. $V_{jNN}$ is defined as

$$V_{jNN} = \frac{1}{2} e_{jc}^T \overline{M}_j e_{jc} + \frac{1}{2} tr\{\tilde{W}_j^T F^{-1} \tilde{W}_j\}. \tag{32}$$

Differentiating (31) yields $\dot{V}_j' = \dot{V}_j + \dot{V}_{jNN}$, and in *Theorem 1,* it was stated and proved in [12] that $\dot{V}_j < 0$, therefore, we will focus on $\dot{V}_{jNN}$ which is

$$\dot{V}_{jNN} = e_{jc}^T \overline{M}_j \dot{e}_{jc} + \frac{1}{2} e_{jc}^T \dot{\overline{M}}_j e_{jc} + tr\{\tilde{W}_j^T F^{-1} \dot{\tilde{W}}_j\} \tag{33}$$

Substitution of the closed loop error dynamics of follower $j$ (27) and the weight tuning law (30) into (33) and application of the *skew symmetric property* produces

$$\dot{V}_{jNN} = -e_{jc}^T K_4 e_{jc} + \kappa \|e_{jc}\| tr\{\tilde{W}_j^T (W_j - \tilde{W}_j)\} + e_{jc}^T (\varepsilon_j + \bar{\tau}_{dj}) \tag{34}$$

after simplifications. Applying *Assumptions 8* and *9* and noting that [11]

$$tr\{\tilde{W}_j^T (W_j - \tilde{W}_j)\} = \langle \tilde{W}_j, W_j \rangle_F - \|\tilde{W}_j\|_F^2 \le \|\tilde{W}_j\|_F \|W_j\|_F - \|\tilde{W}_j\|_F^2$$

allows (34) to be written as

$$\dot{V}_{jNN} \le -\|e_{jc}\| [K_4 \|e_{jc}\| + \kappa \|\tilde{W}\|_F (\|\tilde{W}\|_F - W_M) - (\varepsilon_N + d_M)] \tag{35}$$

Completing the square with respect to $\|\tilde{W}\|_F$ produces

$$\dot{V}_{jNN} \le -\|e_{jc}\| [K_{4\min} \|e_{jc}\| + \kappa \left( \|\tilde{W}_j\|_F - \frac{W_M}{2} \right)^2 - \kappa \frac{W_M^2}{4} - (\varepsilon_N + d_M)] \tag{36}$$

where $K_{4_{\min}}$ is the minimum singular value of $K_4$. Equation (36) is less than zero if the terms in the braces are greater than zero. The term in the braces is guaranteed to be positive if

$$\|e_{jc}\| > \frac{\kappa \frac{W_M^2}{4} + \varepsilon_N + d_M}{K_{4_{\min}}} \equiv b_{ecj} \tag{37}$$

or

$$\|\tilde{W}_j\|_F > \frac{W_M}{2} + \sqrt{\kappa \frac{W_M^2}{4} + \frac{\varepsilon_N + d_M}{K_{4_{\min}}}} \equiv b_{Wj} \tag{38}$$

Examining (37), it is evident that $\left\| e_{jc} \right\|$ can be made arbitrarily small by increasing the gain matrix $K_4$. Therefore, it can be concluded that $\dot{V}_{jNN}$ is negative outside of a compact set. Selecting the gain matrix $K_4$ such that (37) and (38) are satisfied ensures that the compact set defined by $\left\| e_{jc} \right\| \le b_{ecj}$ is contained in *S* so that the approximation property holds throughout [11]. Thus, the position, orientation, velocity tracking errors and NN weight estimates for follower *j* are UUB.

*Leader Control Structure*: In every formation, we assume there is leader *i* such that the following assumptions hold:

*Assumption 11.* The formation leader follows no physical robots, but follows the virtual leader described in [1].
*Assumption 12.* The formation leader is capable of measuring its absolute position via instrumentation like GPS so that tracking the virtual robot is possible.

The kinematics and dynamics of the formation leader *i* are defined similarly to (2) and (3) respectively. From [1], the leader tracks a virtual reference robot with the kinematic constraints of (7), and the control velocity $v_{ic}(t)$ can be defined as

$$v_{ic} = \begin{bmatrix} v_{ir} \cos e_{i3} + k_{i1} e_{i1} \\ \omega_{ir} + k_{i2} v_{ir} e_{i2} + k_{i3} v_{ir} \sin e_{i3} \end{bmatrix} \tag{39}$$

Defining the error system for leader *i* using similar steps used to form (19) and (20) for follower *j,* the control torque for leader *i* can be defined similarly to follower *j*'s as

$$\bar{\tau}_i = \hat{W}_i^T \phi(\bar{x}_i) + K_{i4} e_{ic}, = \hat{f}_i + K_{i4} e_{ic} \tag{40}$$

where $\bar{x}_j = V^T [v_i^T \quad v_{ic}^T \quad \dot{v}_{ic}^T], K_{i4} = k_{i4} I$, and $e_{ic}$ is defined similarly to (18). Let the NN weight updates for the leader *i* be given by

$$\dot{\hat{W}}_i = F \phi_i e_{ic}^T - \kappa F \left\| e_{ic} \right\| \hat{W}_i \tag{41}$$

*Remark:* Since the formation leader tracks a virtual robot, it is able to calculate $\dot{v}_{ic}^T$ since the virtual robot does not have dynamics. Therefore, for the formation leader only, any stable dynamical tracking controller developed for single robot application could be used. Here we choose to define a NN torque controller with the same properties as the followers so that proving the entire formation is stable is simplified.
*Assumption 13.* The reference linear velocity $v_{ir}$ is greater than zero and bounded and the reference angular velocity $\omega_{ir}$ is bounded for all *t*.
*Assumption 14.* $K=[k_{i1} \ k_{i2} \ k_{i3}]^T$ is a vector of positive constants.

*Theorem 3*: Given the kinematic system of (8) and dynamic system in the form of (3) for leader *i* with *n* generalized coordinates $q_i$, *m* independent constraints, and *r* actuators, let *Assumption 4* and *Assumptions 8-14* hold for leader *i.* Let $k_{i4}$ be a sufficiently large positive constant. Let there be a smooth velocity control input $v_{ic}(t)$ for the leader *i* given by (39), and let the torque control for the lead robot *i* (40) be applied to the mobile robot system in the form of (3). Then leader's position, orientation, and velocity tracking errors as well as the NN weight estimates error are UUB.

*Proof* : Due to page limitations, the proof of *Theorem 3* is not included. However, the theorem can be proved by selecting the Lyapunov candidate $V_i' = V_i + V_{iNN}$

where
$$V_i = \frac{1}{2}(e_{i1}^2 + e_{i2}^2) + \frac{1 - \cos e_{i3}}{k_{i2}} \tag{42}$$

and
$$V_{iNN} = \frac{1}{2} e_{ic}^T \overline{M}_i e_{ic} + \frac{1}{2} tr\{\widetilde{W}_i^T F^{-1} \widetilde{W}_i\} \tag{43}$$

and noting the similarities between *Theorems 2* and *3.*

*Remark: T*he stability of a formation consisting of 1 leader and N followers can be proved as well as the stability of the formation for the case when follower *j* becomes a leader to follower *j+1*. Proofs of these claims are not presented here due to length constraints, but they follow as a result of *Theorems 2* and *3*.

## III. SIMULATION RESULTS

A wedge formation of five identical nonholonomic mobile robots is considered where the leader's trajectory is the desired formation trajectory and simulations are carried out in MATLAB under two scenarios. First, perfect velocity tracking in the presence of dynamics examined. In this case, the mass, coriolis, and input transformation matrices are assumed to be known by both the leader and its followers so that the control torque $\tau = \overline{B}^{-1}(\overline{M}(q)\dot{v}_c + \overline{V}_m(q,\dot{q})v_c)$ can be calculated. In the second case, only the input transformation matrix is assumed to be known, perfect velocity tracking is not assumed, and the control torques (25) and (40) are applied. Under both scenarios, unmodeled dynamics are introduced in the form of friction as

$$F = \begin{bmatrix} \mu_1 sign(v) + \mu_2 v \\ \mu_3 sign(\omega) + \mu_4 \omega \end{bmatrix}$$

where $\mu_i$ varied between 0 and 1 for each robot. The leader's reference linear velocity is 5 m/s while the reference linear velocity is allowed to vary.

A simple wedge formation is considered such that follower *j* should track its leader at separation of $L_{ijd}$=2 meters and a bearing of $\Psi_{ijd} = \pm 120^o$ depending on the follower's location, and the formation leader is located at the apex of the wedge. The following gains are used for the controllers:

| | | | | | |
|---|---|---|---|---|---|
| Leader | $K_{i4}$=*daig{40}* | $K_{i1}$=*10* | $K_{i2}$=*5* | $K_{i3}$=*4* | |
| Follower *j* | $K_4$=*diag{40}* | $k_1$=*7* | $k_2$=*20* | $k_3$=*.01* | $k_v$=*1* |

For the NN controllers, F=diag{40}, $\kappa$=0.1 are used for both leader and follower controllers. The following robotic parameters are considered for the leader and its followers: m=5 kg, I = 3 kg$^2$, R=.175 m, r = 0.08 m, and d=0.45 m.

Figure 1 shows the resulting trajectories for both scenarios. In both cases, the robots start in the bottom left corner of Figure 1 and travel towards the top right corner of the figure. A steering command in the form of angular acceleration is given to the formation at *x=2* symbolizing an obstacle avoidance maneuver. Examining Figure 1, it is apparent that perfect velocity tracking does not hold in presence of dynamics as the formation not only forms incorrectly, but also does not follow its trajectory. Even if a velocity tracking loop is introduced, knowledge of the full dynamics is necessary for conventional torque controllers, and full

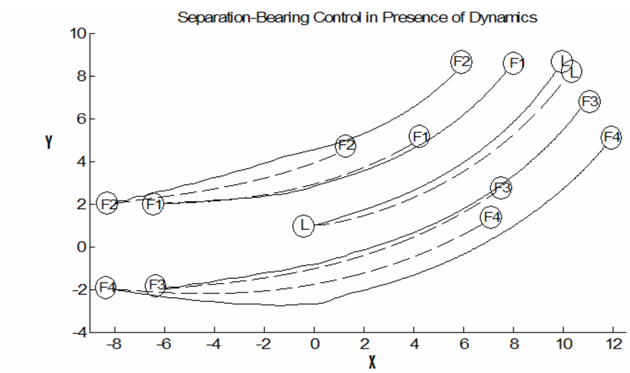


Figure 1: Scenario 1: Perfect velocity tracking-Dashed, Scenario 2: NN controller-Solid.

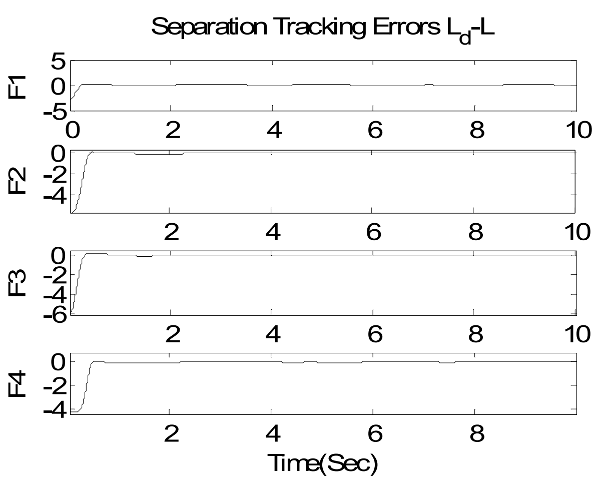


Figure 2: Separation tracking errors

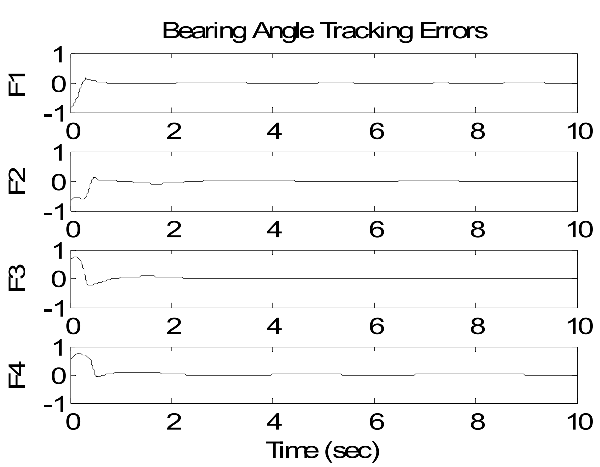


Figure 3: Bearing tracking errors

information is very unlikely and impractical. In scenario 2, only the torque input transformation matrix is known. All other dynamics, including terms like friction, are learned online. With the NN dynamical controllers, the wedge formation was achieved and maintained, and small, bounded errors are observed in Figures 2 and 3.

## IV. Conclusions

A stable tracking controller for leader-follower based formation control was presented that considers the dynamics of the leader and the follower using backstepping. The feedback control scheme is valid even when the dynamics of the followers and their leader are unknown since the NN learns them all online. Numerical results were presented and the stability of the system was verified. Simulation results verify the theoretical conjecture and expose the flaws in ignoring the dynamics of the mobile robots as well as the effects unmodeled dynamics have on conventional computed torque controllers with perfect velocity tracking assumption.

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