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## A Parametric Study of an Effective Stress Liquefaction Model

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# A Parametric Study of an Effective Stress Liquefaction Model

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**SYNOPSIS** A method for evaluating the soil parameters required for effective stress analyses of the earthquake liquefaction potential of saturated sands is described. By means of parametric studies, it is demonstrated that the drained volume change and rebound constants required, may be backfitted to match a given field liquefaction strength curve. By means of this technique, fully coupled effective stress response analyses and site liquefaction evaluations can become a more routine engineering tool.

## INTRODUCTION

The analytical approach for evaluating the liquefaction potential of saturated sand deposits to earthquake ground motions, has developed historically through total stress methods. This approach is based on a comparison between field liquefaction strengths established from undrained cyclic laboratory tests on soil samples, and earthquake induced shearing stresses, estimated from one or two dimensional seismic response calculations (Seed, 1979). As laboratory tests are performed by applying uniform cyclic stress amplitudes to soil samples, time histories of earthquake shearing stresses computed from dynamic response programs such as SHAKE, must be converted to an equivalent number of uniform stress cycles. The development of a field liquefaction strength curve from laboratory test results, also requires data adjustment to account for factors such as correct cyclic stress simulation, possible sample disturbance, aging effects, field cyclic stress history, and the magnitude of insitu lateral stresses. These adjustments require in many cases, a considerable degree of engineering judgement.

The development of a better understanding of the fundamental mechanisms leading to the generation of pore pressures during undrained cyclic loading of sands (Martin et al., 1975), resulted in the development of an alternative analytical approach based on an effective stress model (Finn et al., 1977). In this approach, pore pressure increases are coupled to dynamic response solutions, enabling the complete time history of pore pressure increases to be computed during an earthquake. This method also allows the effects of soil stiffness degradation resulting from pore pressure increases to be reflected in the dynamic response solutions. Furthermore, the effects of pore pressure redistribution and dissipation can also be taken into account.

Effective stress models for computing pore pressure increases and liquefaction potential of saturated sands during earthquake loading,

continue to be the subject of research interest (Liou et al., 1977; Zienkiewicz et al., 1978; Ghaboussi and Dikmen, 1978; and Egan and Sangrey, 1978). However, in this presentation, attention is confined to the model developed by Martin et al., and its practical application to earthquake liquefaction problems through the use of the one dimensional non-linear dynamic response program DESRA II (Lee and Finn, 1978).

Details and applications of the model have been documented elsewhere (Lee and Finn, 1978; Finn et al., 1978; Finn and Martin, 1978; Martin et al., 1978; and Seed et al., 1975), and are not fully described here. In this paper, attention is focussed on the method of computing progressive pore pressure increases during undrained cyclic loading. This is achieved through the use of soil parameters defining the magnitude of permanent volume changes arising from cyclic shearing stresses and the magnitude of elastic rebound of sands under drained conditions.

It has been shown (Martin et al., 1975) that the increment of permanent pore pressure increase resulting from one cycle of undrained simple shear loading is approximately given by

$$\Delta u = \bar{E}_r \Delta \epsilon_{vd} \quad (1)$$

where  $\Delta \epsilon_{vd}$  is the increment of volumetric compaction strain arising from a cycle of the same strain amplitude during a drained test, and  $\bar{E}_r$  is the one dimensional rebound modulus corresponding to the initial effective stress level at the start of the cycle.

It has been shown experimentally by Martin et al. (1975) that under simple shear conditions, the volumetric strain increment  $\Delta \epsilon_{vd}$ , is a function of the total accumulated volumetric strain,  $\epsilon_{vd}$ , and the amplitude of the shear strain cycle,  $\gamma$ . The relationship has the form

$$\Delta \epsilon_{vd} = C_1 (\gamma - C_2 \epsilon_{vd}) + \frac{C_3 \epsilon_{vd}^2}{\gamma + C_4 \epsilon_{vd}} \quad (2)$$

in which  $C_1$ ,  $C_2$ ,  $C_3$  and  $C_4$  are constants that depend on the sand type and the relative density. An analytical expression for the rebound modulus  $\bar{E}_r$  at any effective stress level,  $\sigma_v'$ , is given by the equation

$$\bar{E}_r = \frac{(\sigma_v')^{1-m}}{mk_2} (\sigma_{vo}')^{m-n} \quad (3)$$

in which  $\sigma_{vo}'$  is the initial value of the vertical effective stress prior to unloading, and  $k_2$ ,  $m$  and  $n$  are experimental constants for a given sand. The use of the above equations coupled with a dynamic response analysis of a given site to compute the time history of cyclic strain amplitudes  $\gamma$ , enables pore pressure generation time histories to be computed, and hence the evaluation of liquefaction potential.

It is evident from the above summary, that the use of the effective stress model necessitates a series of laboratory tests to determine the several soil constants defining volume change and unloading behavior under drained conditions. In practice, these tests are relatively difficult to perform on undisturbed soil samples. In addition, the problem remains as to how these soil constants should be adjusted to reflect insitu field conditions, in a manner consistent with adjustments used for the total stress approach. The total stress method has an advantage in this respect; as the effects of disturbance and other factors affecting liquefaction strengths measured in the laboratory, have been the subject of considerable research (Seed, 1979). Furthermore, the use of empirical correlations between standard penetration tests and liquefaction strengths may be used as a guide in assessing field liquefaction strength curves for use in analyses.

Whereas the conventional total stress analytical method has been adapted to develop a simplified procedure for effective stress analyses of ground response (Martin and Seed, 1979) where drained soil parameters are not required, this approach retains the "equivalent number of uniform cycles" concept, and does not fully take into account the potential effects of soil degradation and the time history effects which may be associated with different earthquake records. In this respect, it has been found desirable for more complex problems to retain the advantageous features implicit in the effective stress approach, and to explore ways and means to overcome the practical difficulties of assigning field values to the required drained soil constants. In this paper, by means of a series of parametric studies, it is demonstrated that the drained constants required for effective stress analyses may be backfitted to match a given field liquefaction strength curve and pore pressure buildup function. By means of this technique, fully coupled effective stress response analyses and site liquefaction evaluations become a more routine engineering tool.

#### SITE CHARACTERISTICS USED FOR RESPONSE STUDIES

Figure 1 shows the maximum shear modulus  $G_{max}$

profile utilized to idealize a representative saturated uniform sand deposit used as an example in the paper. A transmitting boundary (Joyner & Chen, 1976; and Lee & Finn, 1978) at a depth of 200 ft. is used as an earthquake input motion interface, and a constant shear wave velocity of 1800 ft/sec is assumed for depths greater than 200 ft.

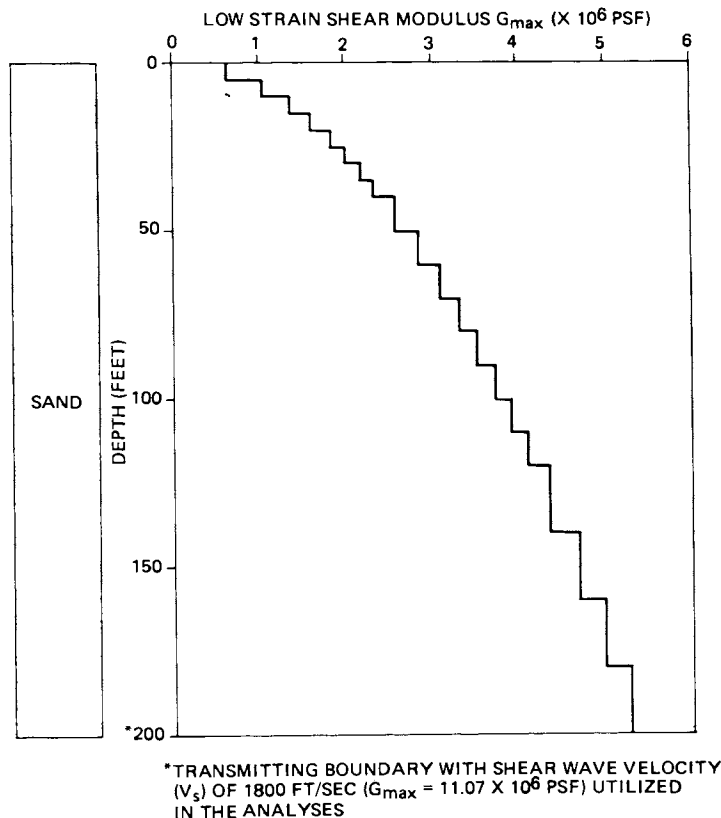


FIG. 1 IDEALIZED SOIL PROFILE NO. 1

Figure 2 shows the modulus ratio ( $G/G_{max}$ ) versus shearing strain amplitude ( $\gamma$ ) curve used to characterize the initial loading hyperbolic shear stress-shear strain curve used for all studies, as given by the equation

$$\tau = \frac{G_{max} \gamma}{1 + \frac{G_{max}}{\tau_{max}} \gamma} \quad (4)$$

where  $\tau$  is the shear stress at strain amplitude  $\gamma$ ,  $G_{max}$  is the initial maximum shear modulus,  $\tau_{max}$  is the maximum shear stress that can be applied without failure, and  $G$  is the shear modulus for a strain amplitude  $\gamma$ . This initial loading or backbone curve is used to define the hysteretic unloading and reloading behavior in shear by use of a Masing model (Masing, 1926).

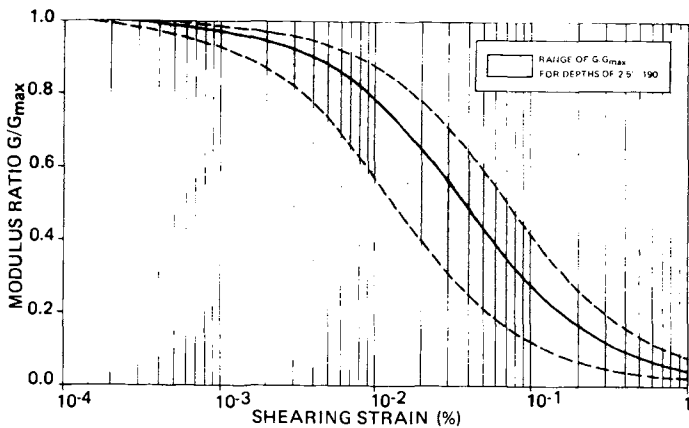
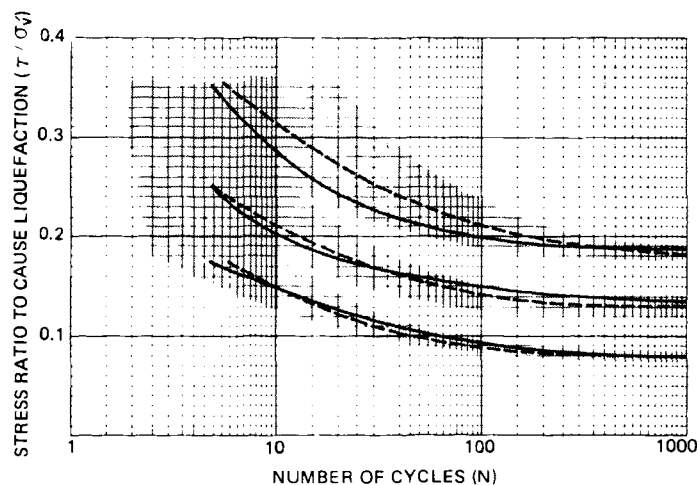


FIG.2 RANGE OF STRAIN DEPENDENT MODULUS RATIO CURVES ASSUMED IN THE ANALYSES

Figure 3 shows the three basic assumed field liquefaction strength curves used for the parametric studies. These curves are expressed in the conventional non-dimensional manner and are representative of relatively loose through dense sands. Such liquefaction strength curves may be predicted from a knowledge of the basic effective stress soil constants described above. However, the problem is to now work in reverse to determine a set of constants which are consistent with a pre-determined liquefaction strength curve and pore pressure build up function.



--- STRENGTH ASSUMED AT THE DEPTH OF 45  
 — STRENGTH UTILIZED IN THE ANALYSES AT THE DEPTH OF 45  
 ... RANGE OF STRENGTH FOR DIFFERENT DEPTHS (2.5 TO 190) RESULTING FROM THE VOLUME CHANGE CONSTANTS ASSUMED

| STRENGTH | VOLUME CHANGE CONSTANTS |                |                |                |
|----------|-------------------------|----------------|----------------|----------------|
|          | C <sub>1</sub>          | C <sub>2</sub> | C <sub>3</sub> | C <sub>4</sub> |
| HIGH     | 0.346                   | 3.536          | 3.707          | 3.025          |
| MEDIUM   | 0.520                   | 2.518          | 4.569          | 3.706          |
| LOW      | 0.666                   | 1.968          | 4.761          | 3.865          |

FIG.3 LIQUEFACTION STRENGTH CURVES

The back fitting of the volume change constants  $C_1$  through  $C_4$  defined by equation (2) for a given liquefaction strength curve, may be achieved using a trial and error basis for the corresponding and given pore pressure build up function as defined in the manner shown in Figure 4, and a given rebound curve as expressed in Figure 5. As the backfitting cannot be achieved in an explicit manner, to minimize the number of iterations in the trial and error procedure, a reasonable level of tolerance must be accepted in the matching of the pore pressure build up and liquefaction strength curves. Using the rebound curve no. 1 in Figure 5 for example, the constants  $C_1$  through  $C_4$  computed to approximately match the three liquefaction strength curves used for the study (for a depth of 45 ft), are shown tabulated in Figure 3. The pore pressure build up functions consistent with these three liquefaction curves, all lie within the shaded area shown in Figure 4. It is noted that most measured pore pressure buildup curves during simple shear tests, would lie in the shaded area.

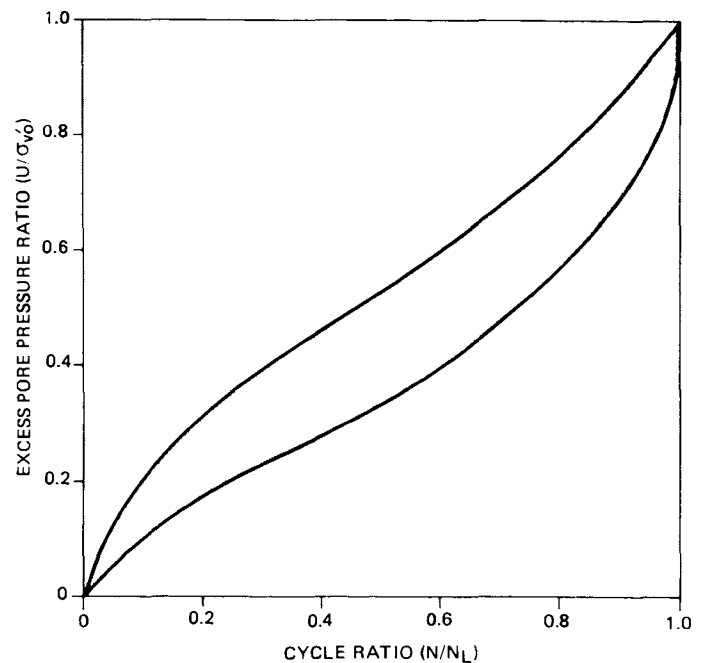
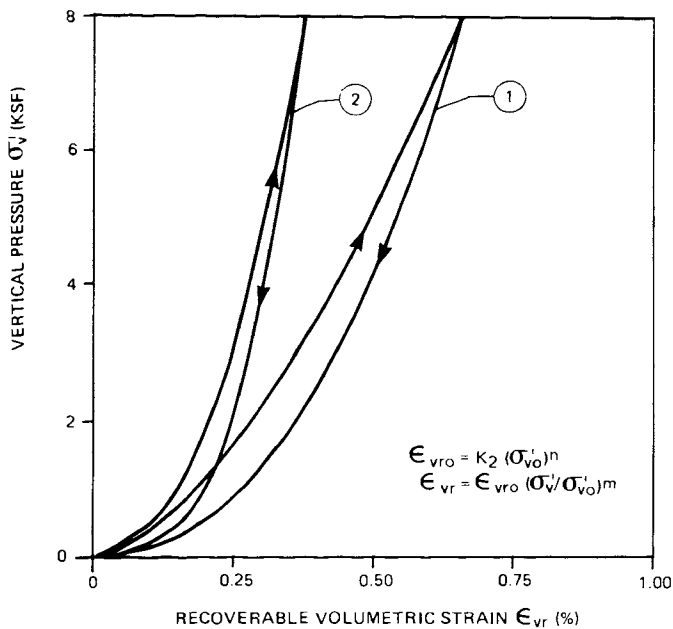


FIG.4 RANGE OF PORE PRESSURE PATHS ASSUMED IN THE ANALYSES

It is also noted that the liquefaction strength curves predicted by the above volume change constants, vary with initial confining stress over the depth of the uniform soil deposit chosen for the study, in the manner shown by the shaded zones in Figure 3. Such reductions in liquefaction strength (as expressed by the stress ratio  $\tau'/\sigma_{v0}'$  causing liquefaction in a given number of cycles) with initial confining stress  $\sigma_{v0}'$ , are commonly observed in laboratory liquefaction strength tests (Seed et al., 1978).



| CURVE | REBOUND MODULUS CONSTANTS |       |       |
|-------|---------------------------|-------|-------|
|       | k                         | m     | n     |
| 1     | 0.0025                    | 0.430 | 0.620 |
| 2     | 0.0079                    | 0.283 | 0.428 |

FIG. 5 REBOUND MODULUS PARAMETERS USED IN THE ANALYSES

## EFFECTIVE STRESS RESPONSE STUDIES

The earthquake accelerogram used for response studies is shown in Figure 6. The accelerogram represents the San Martin record from the 1979 Coyote Lake earthquake scaled to 0.4g. For all analyses, the permeability of the sand has been assumed as  $10^{-3}$  cm/sec ( $3.3 \times 10^{-5}$  ft/sec). This value is representative of a fine uniform sand where only small pore pressure dissipation and re-distribution effects would occur during an earthquake. The effects of permeability on pore pressure response have been discussed elsewhere (Finn et al., 1977; Martin & Seed, 1979).

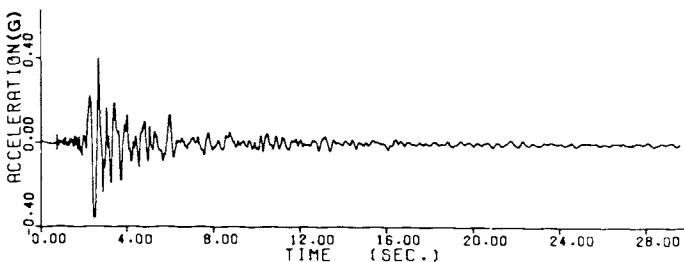


FIG. 6 INPUT MOTION (MODIFIED SAN MARTIN RECORD OF THE 1979 COYOTE LAKE EARTHQUAKE)

Using the idealized soil profile shown in Figure 1, together with the three sets of liquefaction strength curve constants shown in Figure 3, the pore pressure build up responses to the modified San Martin record were computed using the program DESRA II. The distribution of pore pressure response at two instants of time (4 seconds and 12 seconds after the earthquake started) are shown plotted in Figure 7, where it may be seen that for the two denser sands liquefaction does not occur, while for the loose sand, liquefaction occurs to a depth of about 30 ft.

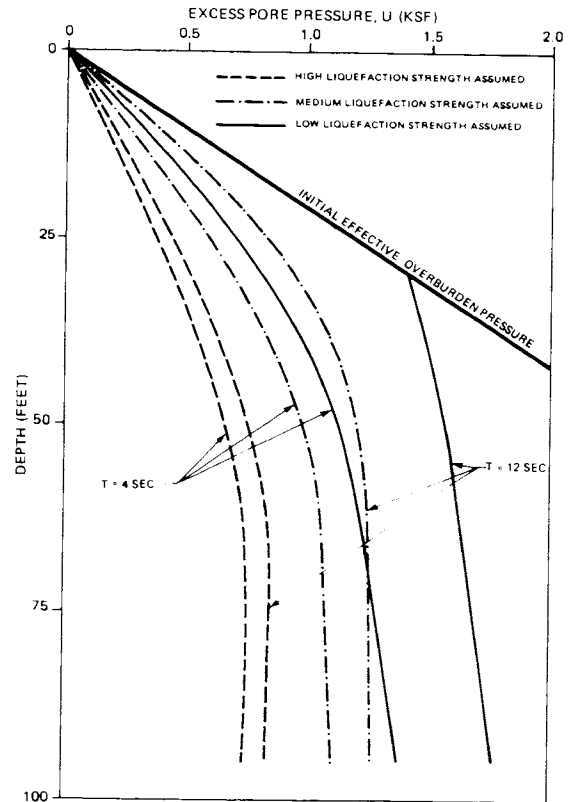


FIG. 7 SUMMARY OF PORE PRESSURE PROFILES OBTAINED FROM DESRA II ANALYSES

The above results reflect a case where given liquefaction strength curves were matched by calculating volume change parameters for a given rebound curve, shown as curve 1 in Figure 5. However, as the shape and magnitude of an appropriate rebound function would be unknown unless laboratory tests were undertaken, the question arises as to the sensitivity of pore pressure response to rebound characteristics. To investigate this question, a further set of volume change constants  $C_1$  through  $C_4$  were computed using rebound curve 2 (Figure 5) and the medium liquefaction strength curve (Figure 3). The corresponding pore pressure buildup functions (at the selected 45 ft. depth) for the two rebound curves resulting from the iterative backfitting process, were almost identical as may be seen in Figure 8. Similarly, the liquefaction strength curves were matched almost identically, as shown in Figure 9.

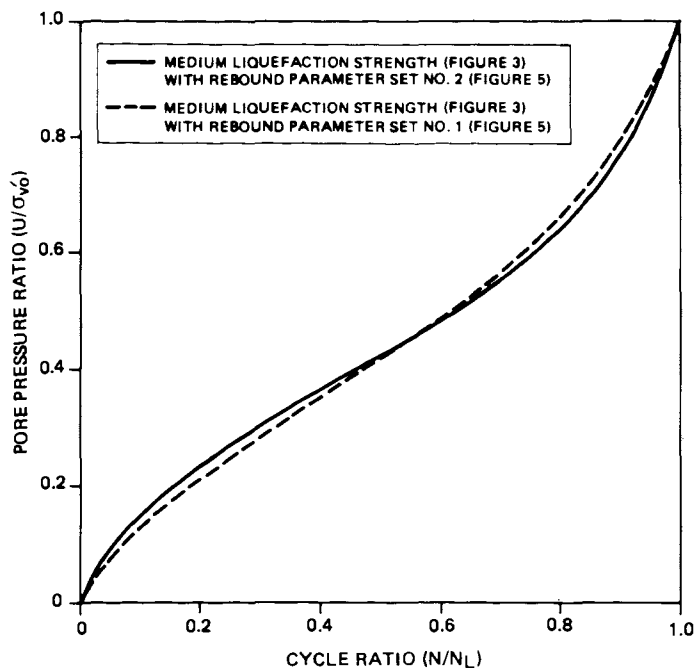


FIG. 8 EFFECTS OF REBOUND PARAMETERS ON PORE PRESSURE PATHS (DEPTH = 45')

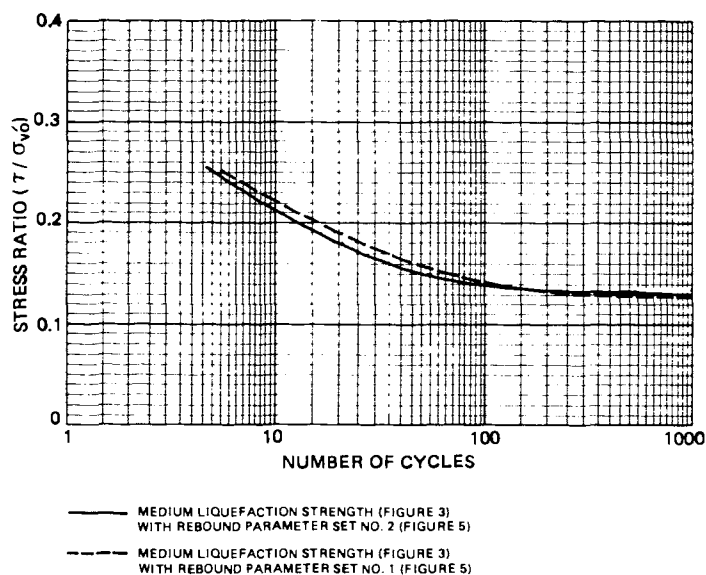


FIG. 9 EFFECTS OF REBOUND PARAMETERS ON LIQUEFACTION STRENGTH VARIATION (DEPTH = 45')

In principle, it would seem that provided a consistent set of effective stress parameters (volume change and rebound characteristics) were chosen to match a given field liquefaction strength curve and corresponding pore pressure build up function (representative of uniform cyclic loading under simple shear conditions), then the values of the effective stress parameters should not significantly effect the results of an earthquake response analysis. To illustrate this point, the site profile shown in Figure 10 was chosen for study. By isolating a sand layer at a depth of 45 ft, the influence of confining stress on results is avoided.

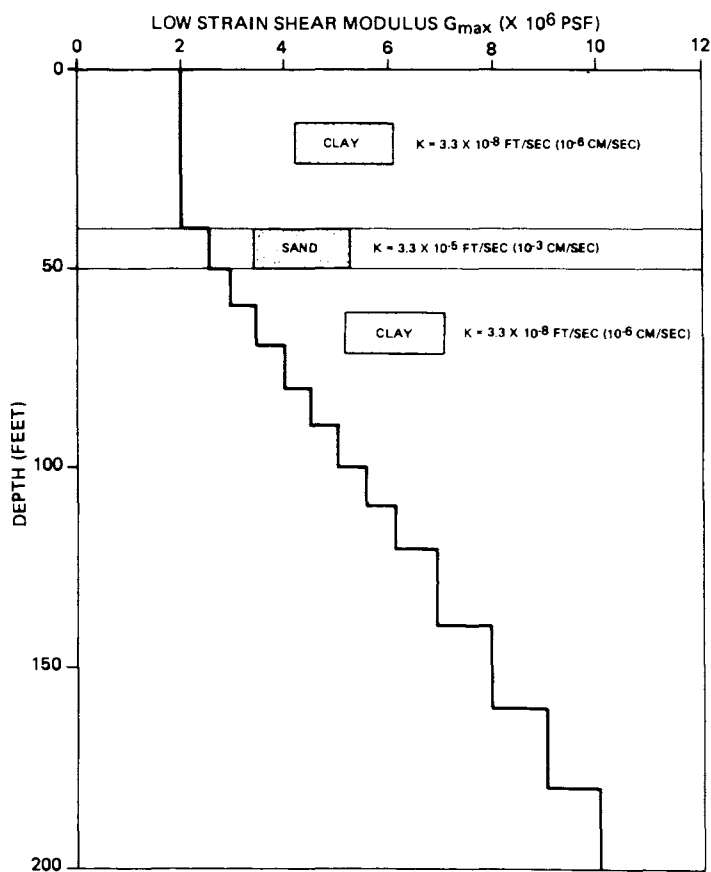


FIG. 10 IDEALIZED PROFILE NO. 2 (CLAY-SAND) FOR REBOUND VARIATION STUDY

The earthquake response of the above profile was again computed using the modified San Martin accelerogram as input, and the pore pressure response of the sand layer at 45 ft. plotted for the two sets of effective stress parameters corresponding to the two rebound curves. Results are shown plotted in Figure 11, where it may be seen that the pore pressure build up is similar for both cases.

## CONCLUSIONS

Whereas it is clearly recognized that the total stress approach has a well established and practical role in evaluating the earthquake liquefaction potential for saturated cohesionless soil sites, the effective stress approach has several advantageous features when assessing more complex problems. In particular, the approach utilized in the dynamic response pro-

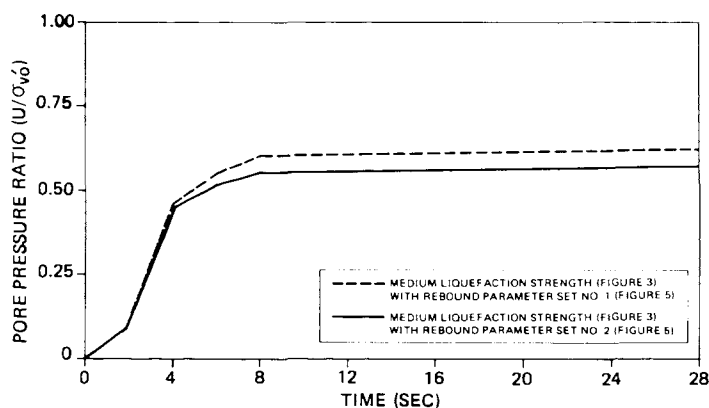


FIG.11 EFFECTS OF REBOUND PARAMETERS ON PORE PRESSURE TIME HISTORY AT THE DEPTH OF 45'(SAND LAYER)

gram DESRA, can take into account the effects of soil stiffness degradation, the effects of pore pressure distribution on dissipation, and also computes the complete time history of pore pressure increase at any depth.

A recognized inconvenience usually associated with the effective stress approach, is the uncertainty in assessing the appropriate soil parameters representative of in situ soil conditions. However, it has been demonstrated that this may be overcome in a practical manner by iterative backfitting of effective stress volume change parameters to match given conventional liquefaction strength curves and pore pressure build up functions. By means of this technique, fully coupled effective stress response analyses and site liquefaction evaluations may be used as a more routine engineering tool.

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