28 Apr 1981, 9:00 am - 12:30 pm

# Dilation Angle and Liquefaction Potential

Y. P. Vaid
*University of British Columbia, Canada*

P. M. Byrne
*University of British Columbia, Canada*

J. M. O. Hughes
*Situ Technology Inc., Canada*

### Recommended Citation

# Dilation Angle and Liquefaction Potential

## Y. P. Vaid and P. M. Byrne

Associate Professors of Civil Engineering, University of British Columbia, Vancouver, B.C.,
Canada

## J. M. O. Hughes

Situ Technology Inc., Vancouver, B.C., Canada

SYNOPSIS  Most of our understanding of the liquefaction phenomenon has come from laboratory tests.
It would be desirable to express liquefaction resistance in terms of a parameter which can be
measured both in the laboratory and in the field.  It is proposed that the dilation angle or expan-
sion rate of the sand is such a parameter.  It is readily measured in the laboratory from drained
simple shear or triaxial tests and in the field from self boring pressuremeter tests.  Based on
laboratory tests on Ottawa sand a chart is presented for estimating the liquefaction resistance of
saturated sands in terms of dilation angle in addition to the usual parameters relative density and
blow count.  When the chart was used in conjunction with pressuremeter tests, a conservative esti-
mate of liquefaction resistance of a hydraulic fill dam was obtained.

## INTRODUCTION

Cyclic loading of saturated sands under condi-
tions of no drainage results in a marked reduc-
tion of its strength or resistance.  This
strength loss is associated with a rise in
porewater pressure, and if this rise should
cause the effective stress to drop to zero, the
strength will be zero, and the sand is said to
have liquefied.  Such strength loss and lique-
faction resulted in major damage during the
Alaska, Niigata and San Fernando earthquakes.

Laboratory cyclic load tests indicate that the
relative density of the sand is an important
parameter controlling its resistance to lique-
faction.  In the field it is difficult to
measure relative density and it is common prac-
tice to infer the relative density and lique-
faction resistance of sand from penetration
tests.  The most commonly used relationship be-
tween penetration resistance and relative den-
sity is the one proposed by Gibbs and Holtz
(1957).  More recently Seed et al. (1975) have
proposed that the penetration resistance

corrected or normalized to 1 $T/ft^2$ be used
directly as a measure of liquefaction resis-
tance.  Penetration resistance tests cannot be
performed on small laboratory samples and thus
cannot be correlated directly with laboratory
cyclic load test behaviour.  It would be desir-
able to have a single parameter describing the
initial state of the sand which could be
obtained in both the laboratory and the field
and which would give a measure of liquefaction
resistance.  It is proposed herein that the
dilation angle or expansion rate of the sand
during shear is such a measurement.  It can
readily be obtained in the laboratory from
drained triaxial or simple shear tests and in
the field it can be obtained from self boring
pressuremeter tests as described by Hughes et
al. (1977).

A liquefaction resistance chart similar to that
proposed by Seed (1976) but based on dilation

angle rather than corrected blow count is
presented.  From this chart an estimate of the
field liquefaction resistance of saturated
sands can be obtained from dilation angles
measured from self boring pressuremeter tests.

## DILATION ANGLE AND RELATIVE DENSITY

An idealized shear behavior of sand at constant
confining stress $\sigma_3'$ in drained triaxial com-
pression is shown in Fig. 1.  Over a consider-
able range of strain, both initially loose and
dense samples undergo volume expansion (dila-
tion), and at very large shear strains tend to
approach an ultimate strength and void ratio.
However, the rate of volume expansion with shear
strain is larger for the dense than for the
loose sand.  The dilation rate of sand is
characterized by the dilation angle $\nu$, which is
the inverse sine of the slope of volume expan-
sion curves in Fig. 1 (Hansen, 1958), i.e.

$$\sin \nu = \frac{dv}{d\gamma} \qquad\qquad (1)$$

If triaxial compression tests were carried out
using a higher value of confining pressure,
results similar to Fig. 1 will be obtained; but,
at a given initial void ratio (or relative
density $D_r$), increase in $\sigma_3'$ will result in a
decrease in $\nu$ and also the ultimate void ratio.
The dilation angle $\nu$ is thus a function both of
relative density and confining pressure.

The shear behavior of sand under drained simple
shear conditions at constant vertical confining
stress $\sigma_{vo}'$ is also similar to that under the
triaxial conditions.  The results of such tests
on Ottawa sand C-109 are shown in Fig. 2 for a
range of relative densities.  It may be seen
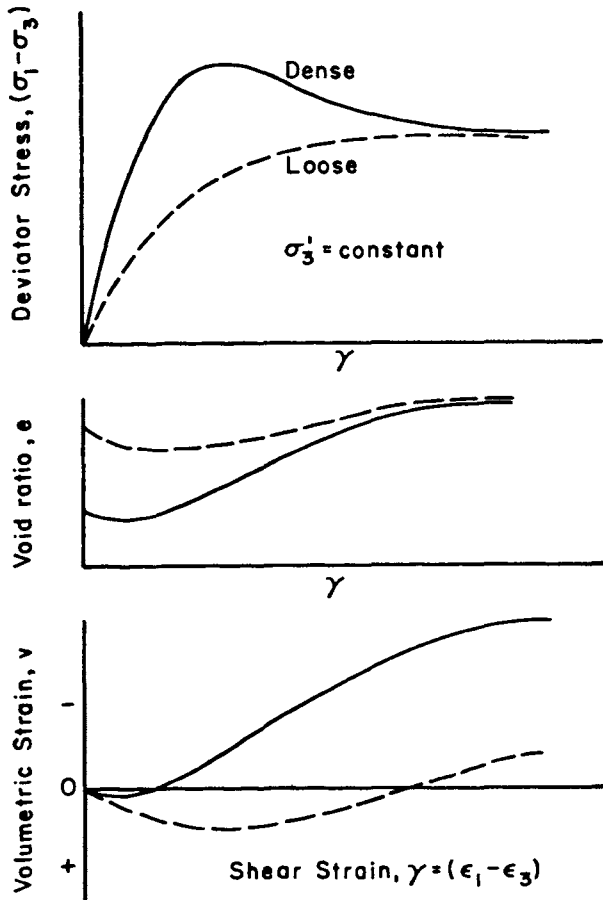that for each relative density the slope of vol-

Fig. 1.  Idealised behaviour of sand in drained triaxial compression



Fig. 2.  Stress-strain behaviour of Ottawa Sand in drained simple shear



Fig. 3.  Relationship between dilation angle and relative density

umetric strain versus shear strain curve, and hence $\nu$, is essentially constant over a wide range of shear strain.  A notable feature of the behavior in simple shear, however, is the relative flatness of the stress-strain curve for dense sand in contrast to a peak followed by a reduction in strength under the triaxial conditions (Fig. 1).  This implies that under simple shear conditions, the sand deforms at a constant stress ratio with the result that the dilation angle $\nu$ also remains constant over a considerable strain range.

It is clear from the results shown in Fig. 2, that the dilation angle $\nu$ of sand is closely related to its relative density.  The value of $\nu$ was computed at a shear strain $\gamma = 10\%$, and is shown plotted against the corresponding relative density in Fig. 3.  A linear relationship was obtained for the sand tested.  This relationship, however, applies for a vertical confining stress of 200 kPa.  As pointed out earlier, the dilation angle also depends on the level of confining stress.  The variation of dilation angle with relative density and vertical confining pressure for Leighton Buzzard sand is shown in Fig. 4, and is based on a comprehensive series of simple shear tests by Cole (1967).  It is clear that for this sand also a linear relationship exists between $\nu$ and $D_r$ at each confining pressure (Fig. 4a).
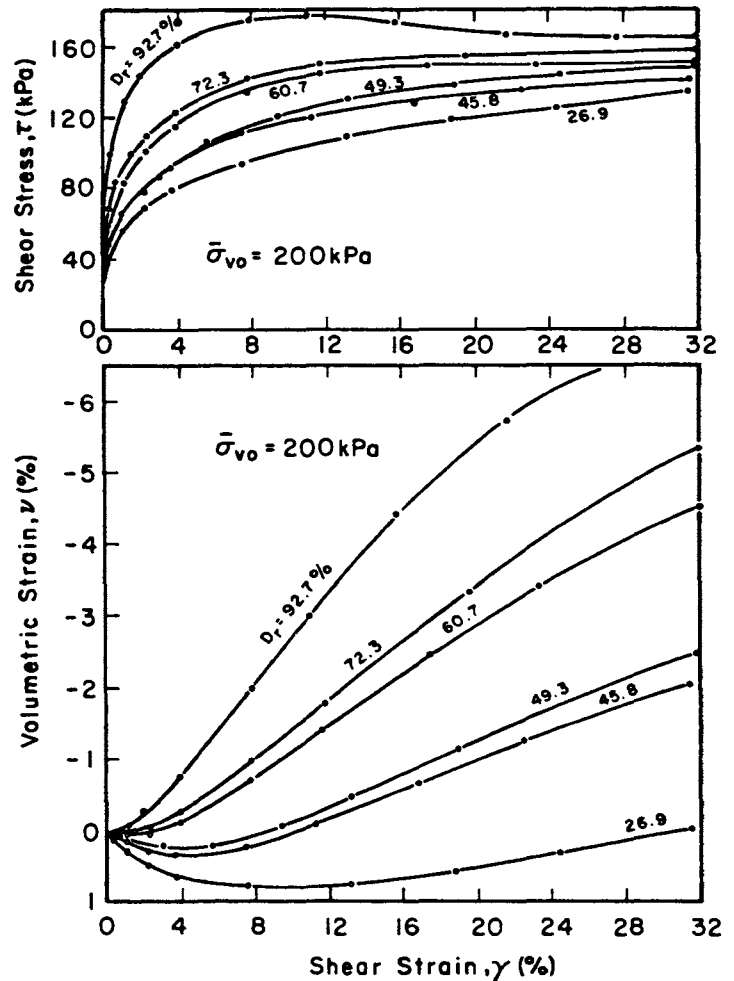
Furthermore, at each relative density, $\nu$

decreases linearly with confining pressure and the slope of the straight line is essentially independent of relative density (Fig. 4b).  The dilation angle $\nu_1$ at a confining pressure of
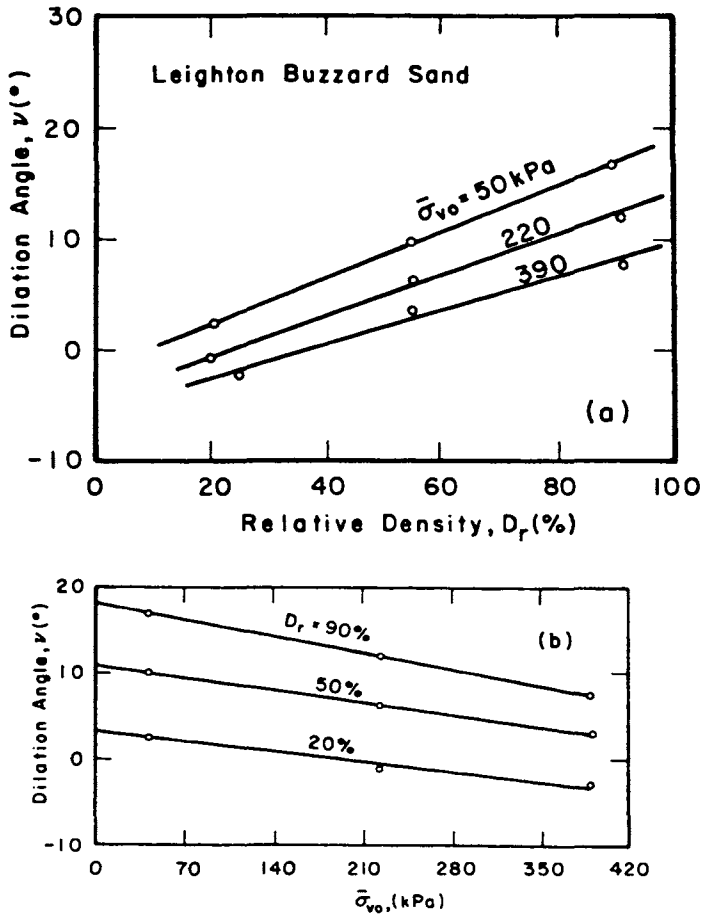
Fig. 4. Dependence of dilation angle on relative density and confining pressure (Data from Cole, 1967)

$1 \text{ T/ft}^2$ (100 kPa) is about 2 1/2 degrees higher than at $2 \text{ T/ft}^2$ (200 kPa). If it is assumed that Ottawa sand behaves in a manner similar to Leighton Buzzard sand with respect to the dependence of $\nu$ on confining pressure, a linear relationship between dilation angle corrected to a confining pressure of $1 \text{ T/ft}^2$ (100 kPa) and relative density will be obtained. This is shown dotted in Fig. 3.

## ANALYSIS PROCEDURE

The liquefaction resistance of saturated sand as obtained from laboratory tests has been expressed as a function of relative density by many researchers including Seed (1976) and Finn and Vaid (1978). By performing drained static simple shear tests on the same sand over a range of relative densities and observing the volume change characteristics, its dilation angle as a function of relative density can be obtained. This allows the liquefaction resistance to be expressed in terms of dilation angle and thus by obtaining in situ measurements of dilation angles from self boring pressuremeter tests, an estimate of the in situ liquefaction resistance of saturated sand is obtained.

## LIQUEFACTION RESISTANCE, RELATIVE DENSITY AND DILATION ANGLE

The liquefaction resistance of Ottawa sand as obtained from constant volume cyclic simple shear tests is shown in Fig. 5.
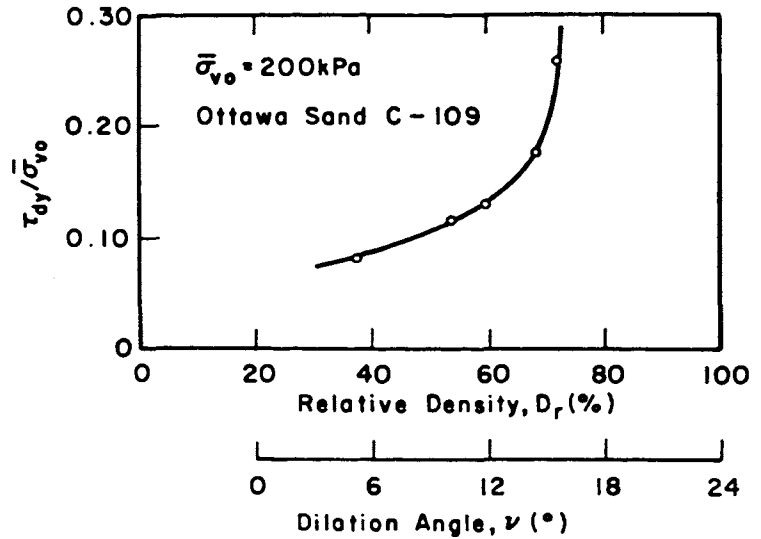


Fig. 5. Resistance to liquefaction as a function of relative density and dilation angle.

Here the cyclic stress ratio, $\tau_{dy}/\sigma'_{vo}$, to cause liquefaction or 10 percent double amplitude shear strain in 10 cycles is shown as a function of relative density. It may be seen that the liquefaction resistance increases with increasing relative density and very markedly so for relative densities in excess of about 70 percent. These results are in close agreement with the analyses of field records of liquefaction resistance shown in Fig. 6, which is based on data presented by Christian and Swiger (1975). This field experience was analysed by examining sites that had been subjected to earthquakes and which were underlain by granular deposits. Liquefaction had occurred at some of these sites and not at others. The equivalent stress ratio, $\tau_{dy}/\sigma'_{vo}$ was computed from the earthquake acceleration level, and the relative density was inferred from the blow count based on Gibbs and Holtz (1957) correlation. The field liquefaction resistance curve represents the lower limit at which liquefaction was observed to occur. The equivalent uniform stress ratio was taken as 0.65 times the maximum value as suggested by Seed (1976).

Since from Fig. 3 the dilation angle corrected to a normal pressure of $1 \text{ T/ft}^2$ (100 kPa) is uniquely related to the relative density, the liquefaction resistance can also be related to the dilation angle. This is shown in Fig. 5 by an added scale for dilation angle $\nu$, besides the relative density scale. This figure is thought to give a low or conservative estimate of the liquefaction resistance of saturated medium sands. Thus by obtaining the dilation angle in the field as described by Hughes et al. (1977) a conservative estimate of liquefaction
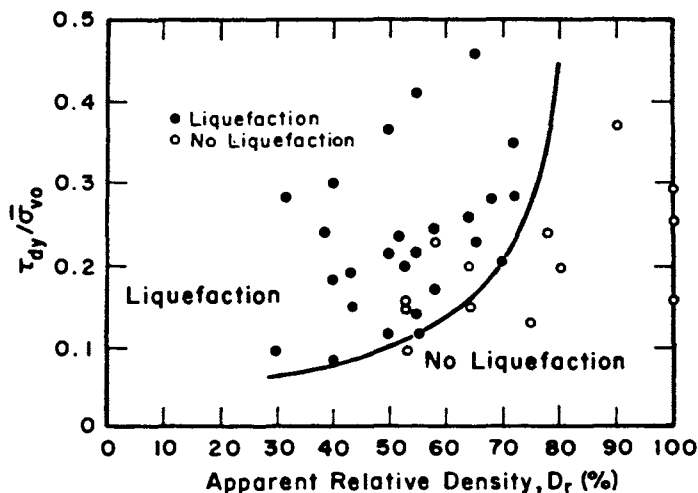
Based on Data by Christian and Swiger, 1975

Fig. 6. Analysis of field records of sites where liquefaction did and did not occur.



Fig. 7. Relationship between blow count corrected to $1T/ft^2$ and relative density (After Gibbs and Holtz, 1957)

resistance is obtained.

The chart shown in Fig. 5 was used to estimate the liquefaction resistance of a dam in British Columbia. This dam was constructed by the hydraulic fill method and the core is comprised of non plastic silt. Dilation angles measured from self boring pressuremeter tests and corrected to an effective confining stress of $1$ $T/ft^2$ were in the range of 0 to $5°$. From Fig. 5 this material could be expected to have a dynamic resistance ratio of about 0.08. Laboratory cyclic load tests on undisturbed samples obtained from locations close to the site for pressuremeter tests, indicate that the dynamic resistance ratio is 0.10, and thus the chart gives a reasonable but conservative estimate of liquefaction resistance for this case.

RELATIVE DENSITY, BLOW COUNT AND LIQUEFACTION RESISTANCE

The most commonly used relationship between relative density, blow count and confining pressure is that proposed by Gibbs and Holtz (1957). For a confining pressure of 1 $T/ft^2$, the relationship is as shown in Fig. 7. Using this relationship combined with the liquefaction resistance versus relative density curve of Fig. 5, the liquefaction resistance in terms of blow count normalized to 1 $T/ft^2$ is obtained. This is shown by the solid line in Fig. 8. Field experience presented by Seed (1976) together with his liquefaction resistance curve (dashed line) is also shown on this figure and are seen to be in very close agreement with the solid line for $N_1$ values less than about 20.

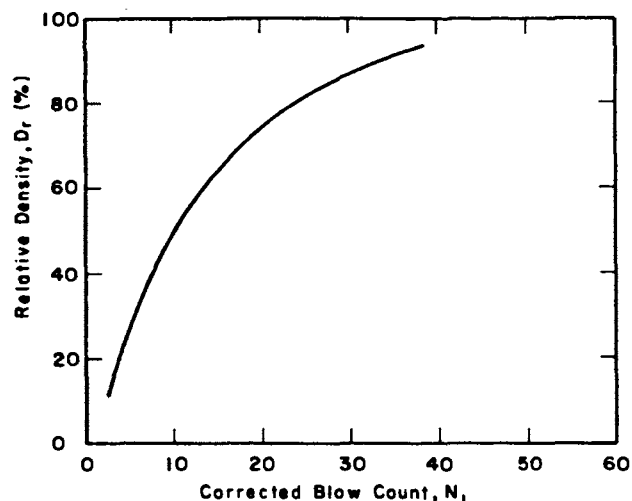Liquefaction resistance can be correlated with relative density, corrected dilation angle or

corrected blow count and a chart showing liquefaction resistance in terms of these 3 parameters is shown in Fig. 9. It indicates that liquefaction is not likely to occur regardless of the stress level provided the relative density is in excess of 75 percent, the corrected dilation angle is greater than $16°$, or the corrected blow count exceeds 20. Such a conclusion seems apparent from analyses of field records of liquefaction by Seed (Fig. 8), Christian and Swiger (Fig. 6) and Castro (1975).

CONCLUSIONS

Dilation angle or expansion rate of sand during shear has been shown to be uniquely related to its relative density and confining pressure. Its resistance to liquefaction can therefore be expressed in terms of dilation angle in addition to the usual blow count. Laboratory test results on Ottawa sand have been used to illustrate such a correlation. In the field, the dilation angle can be measured by the self boring pressuremeter. Such measurements were performed on a hydraulic fill dam and its liquefaction resistance predicted by the established correlation. A reasonable agreement was obtained between the predictions and the results of liquefaction tests on undisturbed samples.

REFERENCES

Castro, G. (1975), "Liquefaction and Cyclic Mobility of Saturated Sands," Journal of the Geotechnical Engineering Division, ASCE, Vol. 101, GT6, pp. 551-569.

Christian, J.T. and Swiger, W.F. (1975), "Statistics of Liquefaction and SPT Results."

Journal of the Geotechnical Engineering
Division, ASCE, Vol. 101, GT11, pp. 1135-
1150.

Cole, E.R.L. (1967), "The Behavior of Soils in
the Simple Shear Apparatus," Ph.D. Thesis,
Cambridge University, England.

Finn, W.D.L. and Vaid, Y.P. (1978), "Constant
Volume Cyclic Simple Shear Testing", Pro-
ceedings 2nd International Conference on
Microzonation for Safer Construction, San
Francisco, U.S.A., Vol. 2, pp. 839-851.

Gibbs, H.J. and Holtz, W.G. (1957), "Research
on Determining the Density of Sands by Spoon
Penetration Testing," Proceedings, 4th
International Conference on Soil Mechanics
and Foundation Engineering," Vol. 1, London,
England, pp. 35-39.

Hansen, B. (1958), "Line Ruptures Regarded as
Narrow Rupture Zones; Basic Equation Based
on Kinematic Considerations," Proceedings,
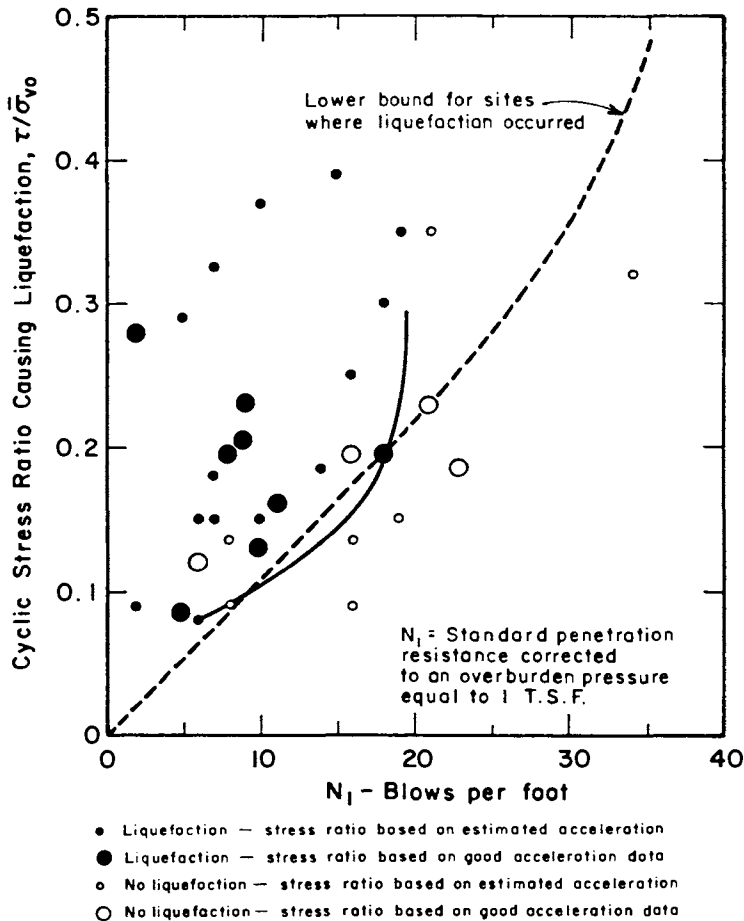Brussels Conference on Earth Pressure Prob-
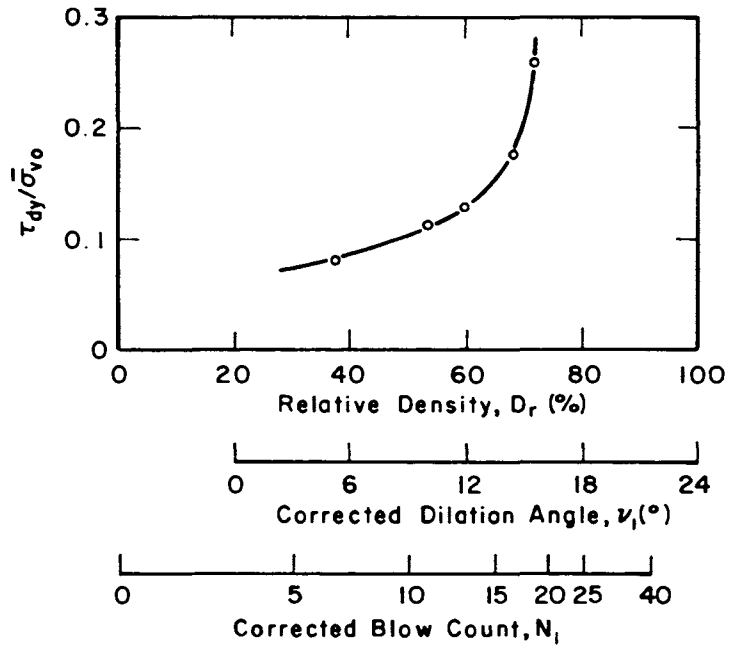lems, Vol. 1, pp. 39-48.

Fig. 9.    Resistance to liquefaction of sand as
a function of relative density, dila-
tion angle or penetration resistance.

Hughes, J.M.O., Wroth, C.P. and Windle, D.
(1977), "Pressuremeter Tests in Sand," Geo-
techniques, Vol. 27, No. 4, pp. 455-477.

Seed, H.B. (1976), "Evaluation of Soil Lique-
faction Effects on Level Ground During
Earthquakes," State-of-the-Art Paper, ASCE,
National Convention, Philadelphia.

Seed, H.B., Mori, K. and Chan, C.K. (1975),
"Influence of Seismic History on the Lique-
faction Characteristics of Sands," Earthquake
Engineering Research Centre, Report No.
EERC 75-25, University of California,
Berkeley.

● Liquefaction — stress ratio based on estimated acceleration
● Liquefaction — stress ratio based on good acceleration data
o No liquefaction — stress ratio based on estimated acceleration
O No liquefaction — stress ratio based on good acceleration data

Fig. 8.    Correlation between stress ratio
causing liquefaction in the field and
penetration resistance of sand (After
Seed et al. 1975)