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Displacement Analysis of Rigid Retaining Walls in Rocking

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SYNOPSIS The paper presents a simple approach for computing rotational displacements of rigid retaining walls during an earthquake, an aspect that had not been considered so far (1981). The values of rotational displacements using the proposed method for various combinations of wall geometry, backfill material and ground motion parameters have been worked out. The values of rotational displacements have been compared with the values obtained by using available approaches for displacement analysis (sliding or overall) for rigid retaining walls and it is shown that the contribution of rotation to the overall displacement of the retaining wall may be quite significant in some cases and should therefore, be accounted for. The necessity to develop a rational displacement analysis considering combined rocking and sliding is stressed.

INTRODUCTION

Till recently, the design of rigid retaining walls in seismic zones was based upon the pseudo-static approach in which the additional increment (or decrement) in earthpressure due to an earthquake is replaced by an equivalent static force of constant magnitude. The stability analysis of the retaining wall is then made as for a static case and the wall is considered safe if the factor of safety in sliding and overturning are equal to or greater than the specified values. Modified Coloumb's approach due to Mononobe (1929) and Okabe (1926) is generally used for computation of earth pressures. The most important consideration favoring the use of pseudostatic approach is its simplicity. However, a consideration of factor of safety alone under earthquake loading conditions gives only an incomplete picture and information on the likely displacements is an important consideration. This aspect attracted attention of the geotechnical profession as far back as 1965 and attempts have been made to develop analysis for estimating displacements of rigid retaining walls under earthquakes (Newmark, 1965, Nandakumaran, 1973, Richard and Elms, 1974, Prakash et al., 1981). The displacement analyses proposed by Nandakumaran (1973) and Prakash et al (1981) take into account the ground motion and wall parameters in computing the displacement of the retaining wall. They have also presented charts for computing displacements per cycle of motion from which total displacement may be computed based upon the number of total equivalent uniform effective cycles of ground motion. The analysis however is limited only to sliding displacements and effects of rotation have been omitted. Richard and Elms (1979) have used the approach due to Newmark (1965) for computing overall displacement of rigid retaining walls irrespective of the retaining wall parameters and its behavior in failure by sliding or tilting. The analysis thus considers only the ground motion parameters and the effects of wall-soil interaction are neglected. The resulting motion of the retaining wall due to earthquake loading is rather complex and may be more reasonably idealized as consisting of combined effects of sliding or translational motion and rotational or rocking motion. Any rational approach for estimating wall displacements during an earthquake must account for these two modes of vibration.

No effort has been made so far (1981) to assess the effect of rotational vibrations on the overall displacement of rigid retaining walls during an earthquake. The authors have attempted to estimate the contribution of rocking motion of the retaining wall towards its displacements during an earthquake based on certain simplifying assumptions. The displacements due to rotational vibrations have been compared with the sliding displacements estimated using the approaches due to Prakash et al (1981) and Richard and Elms (1979) for a few typical cases. It is felt that displacements due to rocking may be quite significant compared to displacements in sliding and need to be considered. Suitable displacement analysis accounting for combined effects of rocking and sliding should therefore develop. All these details are discussed subsequently.

PROPOSED METHOD

Assumptions: The proposed method is based on the following assumptions:

1. Rocking vibrations are independent of sliding vibrations and the stiffness to rocking is not affected by sliding.

2. The earthquake motion may be considered as an equivalent sinusoidal motion with uniform peak accelerations and the total displacement = residual displacement per cycle x number of cycles.

3. Wall rotates about the heel.

4. Soil stiffness for rotational displacement of wall away from the backfill may be computed corresponding to average displacement for fully active conditions.

5. Soil stiffness for rotational displacements towards the backfill may be computed corresponding to average displacements for development of fully passive conditions.

6. The stiffness values computed in (4) and (5) remain unchanged when the wall rotates towards or away from the backfill as the case may be.

7. Soil participating in vibrations may be neglected.

These assumptions are not valid in the strict sense. Both sliding and rocking are excited simultaneously and the soil stiffness does not remain constant. The soil stiffness depends upon the magnitude of shear strains induced in the soil and will therefore vary during different phases of displacement. Soil mass participating in the vibrations effects the dynamic response of the system. However these assumptions may be considered reasonable as a first approximation.

MATHEMATICAL MODEL

To arrive at an appropriate mathematical model for the soil-wall system subjected to ground motion, the mechanism of rotation of the wall needs consideration. Figure 1 shows a typical rigid retaining wall of height 'H', base width 'b' and top width 'b_t'. One cycle of idealized ground motion is represented in Figure 2. Figure 3 shows in a schematic manner the positions of the retaining wall during different stages of rotational oscillation due to one cycle of ground motion. AB is the position of the retaining wall just before the motion starts. Assuming that the wall starts its oscillations about 'A' by virtue of rotation away from the backfill, it may occupy a position A-1 at time $T_p/4$ where T_p=period of ground motion (Fig. 2). During this phase of rotational vibration, resistance is mobilized at the base and on the side. Active conditions govern the behavior of the soil in the backfill. During the time $T_p/4$ to $T_p/2$ (Figure 2), the rotation of the wall is towards the backfill and this leads to development of passive conditions in the backfill. At time $T_p/2$ the wall may be at some position A-2. This trend of wall rotation towards the backfill continues through the time $T_p/2$ to $3T_p/4$ and the wall rotates to position A-3. During the time $3T_p/4$ to T_p, the direction of wall rotation is away from the backfill and active conditions again govern the backfill behaviour. During the next quarter cycle of motion the backfill remains in active condition and during successive phases of ground motion the conditions in the backfill change between passive and active every half cycle of motion. Therefore for a number of cycles of ground motion, it may be considered that during one complete cycle, the rotation is away from the backfill during the first half cycle and it is towards the backfill during the other half of the cycle.

During rotation away from the backfill, active conditions are generated in the backfill while for rotation toward the backfill, the conditions developing there correspond to passive conditions. However for development of fully active or passive condition, certain displacement criteria needs to be satisfied. This is illustrated in Figure 4 wherein fully active conditions are assumed to develop at an average displacement of 0.25% of the height of the retaining wall and fully passive conditions may be considered to develop at an average displacement of 2.5 percent of the height of the wall. Accordingly the effective soil springs of the backfill may be calculated as follows

$$K_A = \frac{\frac{K_o \gamma H^2}{2} - \frac{(k_a \cos\delta)\gamma H^2}{2}}{(\frac{0.25\ H}{100})} \qquad (1)$$

in which

K_A = soil spring for displacement away from the backfill.

K_o = coefficient of earth pressure at rest

k_a = coefficient of active earth pressure

γ = unit weight of soil

H = height of retaining wall, and

δ = angle of wall friction.

Similarly

$$K_p = \frac{\frac{(k_p \cos)\delta \gamma H^2}{2} - \frac{k_o \gamma H^2}{2}}{\frac{2.5\ H}{100}} \qquad (2)$$

in which

K_p = soil spring of the backfill for displacement towards the backfill, and

k_p = coefficient of passive earth pressure.

EQUATIONS OF MOTION

1. Rotation Away From the Backfill:

The equation of motion can be obtained by considering moments of the various resisting and actuating forces about the heel.

a. Moment due to soil reaction at the base: M_b

$$M_b = C_\phi \cdot I \cdot \phi_A \qquad (3)$$

in which

C_ϕ = coefficient of elastic nonuniform shear

I = moment of inertia of base contact area about the axis through the heel and perpendicular to the plane of vibrations, and

ϕ_A = angle of rotation.

b. Moment due to soil resistance on the side of the wall: M_s

$$M_s = K_A \cdot \frac{H\phi}{2} \left(\frac{2H}{3}\right) = K_A \cdot \frac{H^2 \cdot \phi}{3} \qquad (4)$$

c. Moment due to inertia of the wall: M_i

$$M_i = M_o \ddot{\phi} \qquad (5)$$

in which

M_o = mass moment of inertia of the retaining wall about the axis of rotation through the heel, and

$\ddot{\phi}$ = angular acceleration.

d. Actuating moment M(t). M(t) is the moment of inertia force about the assumed point of rotation. Equation of motion may then be written as

$$M_o \ddot{\phi}_A + \left(C_\phi I - \frac{K_A H^2}{3}\right) \phi_A = M(t) \qquad (6)$$

Damping may be included in Eq. 6 as follows:

$$M_o \ddot{\phi}_A + C_A \dot{\phi}_A + \left(C_\phi I - \frac{K_A H^2}{3}\right) \phi_A = M(t) \qquad (7)$$

in which

C_A = damping coefficient for rotation away from the backfill.

e. Rotation towards the backfill. Equation of motion for rotational vibrations of the wall-soil system for the case when wall is rotating towards the backfill may similarly be written as

$$M_o \ddot{\phi}_p + \left(C_\phi I + K_p \cdot \frac{H^2}{3}\right) \phi_p = M(t) \qquad (8)$$

in which

ϕ_p = angle of rotation

Equation 8 may be modified to include damping as

$$M_o \ddot{\phi}_p + C_p \dot{\phi}_p + \left(C_\phi \cdot I + K_p \frac{H^2}{3}\right) \phi_p = M(t) \qquad (9)$$

in which

C_p = damping coefficients for rotations towards the backfill.

Displacement: Equations 7 and 9 can be solved to obtain the values of residual displacement Φ

$$\Phi = (\phi_A - \phi_p) \qquad (10)$$

in which

ϕ_A = residual displacement during one half cycle (rotation away from backfill), and

ϕ_p = residual displacement during the second half of the cycle (rotation towards the backfill).

Horizontal displacement at the top of the wall during one cycle = Δy

$$\Delta y = H(\phi_A - \phi_p) \qquad (11)$$

Cumulative displacement 'y' is given for N cycles by Eq. 12

$$y = N \cdot \Delta y = NH(\phi_A - \phi_p) \qquad (12)$$

PARAMETRIC STUDY

A parametric study was made to investigate the effect of ground motion period, geometry of the retaining wall and type of material at the base and in the backfill. The following values chosen for the study were:

a. Wall Geometry:

Height in 'm'	3.0	5.0	7.5	10.0
Base width	1.0	1.67	2.5	3.3
Top width	0.30	0.40	0.40	0.50

b. Backfill Material:

Angle of internal friction ϕ = 30°, 33°, 36°
Angle of wall friction $\delta = 2/3\ \phi$

c. Material below the base:

C_ϕ = 3.0, 4.0, 5.0, 6.0 and 8.0 kg/cm^3

d. Ground Motion Characteristics:

Period of ground motion T_p sec = 0.15, 0.2, 0.25, 0.30, 0.35, 0.40, 0.50, 0.6, 0.8, 1.0
Peak ground acceleration = 0.10 g

e. Dumping Values:

ξ_A%	0	5	10	15
ξ_p%	0	0	5	10

Values of the displacement at the top of the retaining wall for the following properties of backfill, base material and damping values for different periods of ground motion are listed in Table 1.

Values of the displacements at the top of the retaining wall in Table 1 illustrate that the rotational displacements are not insignificant depending upon the geometry of the retaining wall, soil properties and ground motion characteristics. Similar data is obtained for other cases of study and is not reported here for want of space. The magnitude of rotational displacement for a 3 m high wall has been compared with sliding displacements (Nandakumaran, 1974, Prakash et al., 1981) and overall displacements (Richards and Elms, 1979) in Table 2 for the following case:

Angle of internal friction ϕ of backfill = 36°

C_ϕ = 5.0 kg/cm^3

Base width = 1.0m

ξ_A = 10%
ξ_p = 5%

Peak horizontal ground acc^n = 0.25g

Period of ground motion = 0.3 sec

Table 1. Typical Values of Displacement at Top of the Retaining Wall Due to Rotation

Peak horizontal acceleration = 0.1 g
Angle of internal friction of backfill ϕ = 33°
Coefficient elastic non-uniform compression c_ϕ = damping in percent of critical damping:

ξ_A = 10%

ξ_P = 5%

Period of ground motion T_p = 0.2, 0.3, 0.4, 0.5, 1.0 sec

Height of wall m	3.0	5.0	7.5	10.0
Base width m	1.0	1.67	2.50	3.33
Top width m	0.30	0.40	0.40	0.50
K_A kg/cm	1110	1850	2770	3700
K_P kg/cm	7600	12680	19020	25030
Period $T^1_{\phi A}$ sec	0.26	0.32	0.36	0.52
Period $T^2_{\phi P}$ sec	0.11	0.14	0.16	0.19
Displacement$^+$ cm for T^3_p = 0.2 sec	7.26	6.09	5.08	2.90
Displacement$^+$ cm for T_p = 0.3 sec	14.78	27.75	19.53	18.95
Displacement$^+$ cm for T_p = 0.4 sec	8.26	18.05	35.77	34.36
Displacement$^+$ cm for T_p = 0.5 sec	7.02	13.13	20.67	88.11
Displacement$^+$ cm for T_p = 1.0 sec	6.45	10.08	13.89	33.99

$^+$Displacement at top of retaining wall in 15 cycles due to rotation

$^1T_{\phi A}$ = Period of wall rotation away from the backfill

$^2T_{\phi p}$ = Period of wall rotation towards backfill

^3T_p = Period of ground motion

Table 2. Comparison of Displacements of 3m High Using Different Approaches

Rotational Displacement			Sliding*		Displacement Analysis**
$T_{\phi A}$/sec	$T_{\phi p}$/sec	Displacement at top after 15 cycles/sec	T	Displacement in 15 cycles cm	Displacement cm
0.18	0.094	9.82	0.979	21.30	8.2

DISCUSSION: An examination of the data in Table 1 and similar data (not included here) shows that rotational displacements are not necessarily negligible. The contribution of rotation towards total displacement of the retaining wall may be quite significant under certain soil conditions. Similarly for the typical example, comparison of displacements in rotation and displacements computed by other methods again points out towards the fact that neglecting effects of rotational displacements may be absolutely unconservative in certain cases.

It may be mentioned here that the actual problem of the displacements of a rigid retaining wall during an earthquake is rather complex. It must be treated as a problem of displacements due to combined rocking and sliding. Nevertheless the study signifies that the omission of rocking vibrations and its effects on displacement may prove rather unconservative.

CONCLUSION

1. Rocking or rotational vibration as a parameter in working out displacements of rigid walls should be recognized as effectively demonstrated from the present study.

*Prakash et al., (1981), **Richards & Elms (1979)

2. An analytical model for displacement due to combined rocking and sliding should be developed. Authors have already initiated research in this direction.

3. The proposed analysis which treats rocking independent of sliding may be used along with the displacement analysis for sliding due to Prakash et al (1981) to make reasonable estimate of the overall displacements of the retaining wall.

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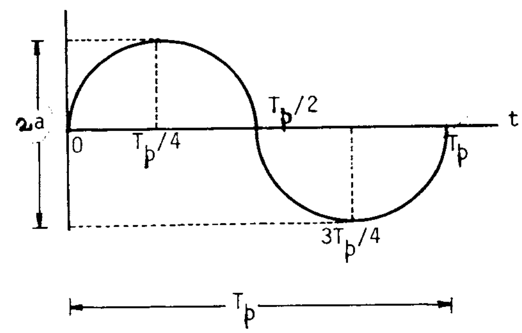

Fig. 2 1-cycle of idealized ground motion.

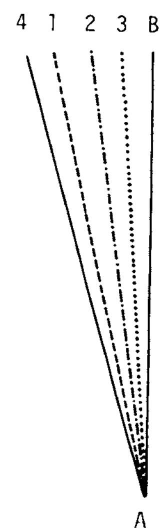

Fig. 3. Schematics of Wall Rotation

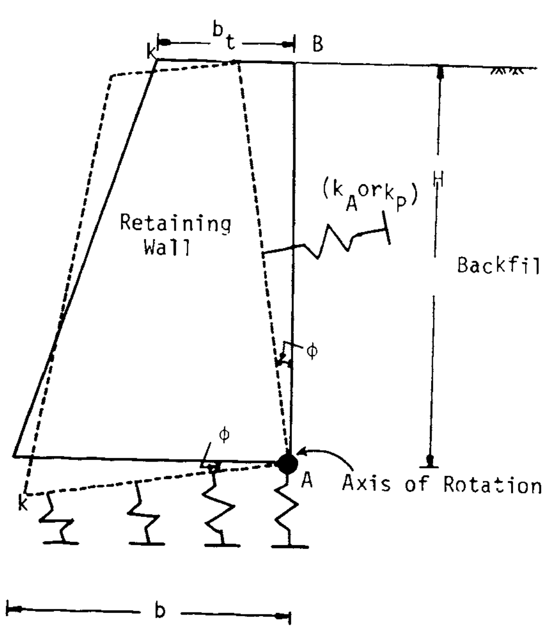

Fig. 1. Retaining Wall

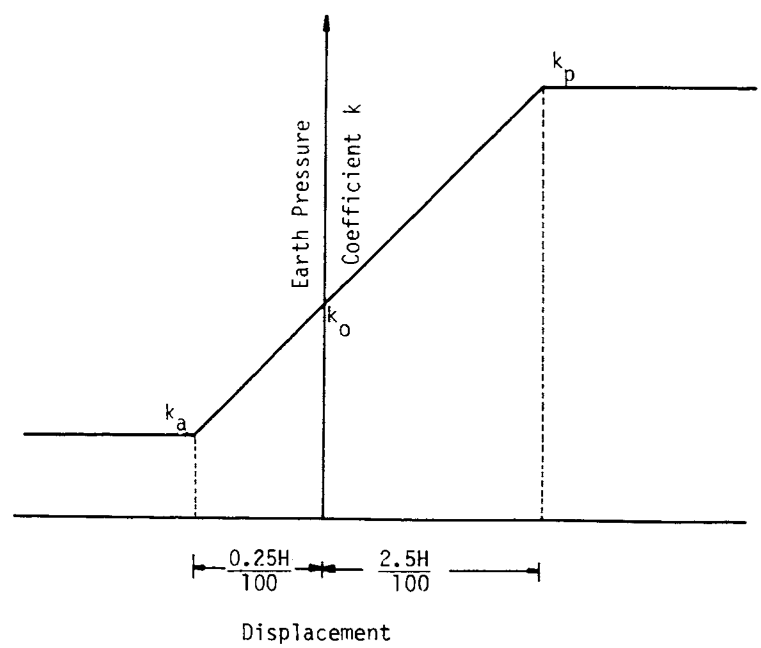

Figure 4. Soil Stiffness of Backfill