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SYNOPSIS. A simplified elastic model for analyzing static and dynamic interaction between earth-retaining structures and backfill within the range of small displacements is presented. The postulated model covers some of the available models as special cases. The model lends itself readily to the treatment of non-homogeneous backfills with elastic properties varying with depth. Internal (linear) damping in the backfill can be included without impairing the simplicity of the model. Radiation losses due to waves propagating horizontally in fills of semi-infinite extent are inherent to the postulated model. The solutions for some statical and dynamical problems of practical importance show satisfactory agreement with results based on the classical theory of elasticity.

INTRODUCTION

Interaction between earth-retaining structures and backfill in the range of small displacements has been treated by Wood (1973) within the frame of classical elasticity. Matsuo and Ohara (1960) have proposed a simplified model assuming vanishing vertical displacements. More radical simplifying assumptions have led Scott (1973) to represent the backfill as a cantilever shear beam, coupled with the retaining wall by a system of Winkler springs. Tajimi (1973) gives solutions for the problem of a quarter of an elastic space, excited by prescribed rigid body displacements on a part of one of its boundaries. The corresponding static problem has been treated by Finn (1963). Ambraseys (1960) used a model consisting of horizontal slices that deform only in shear to study the seismic behaviour of earth dams.

An examination of the available solutions based on classical elasticity theory shows that this type of formulation is confronted with considerable analytical difficulties due to the fact that the equations of motion in terms of components of displacement (Navier's equations) are coupled by terms containing the mixed derivatives. It is this difficulty that led Matsuo and Ohara to their proposal. However, as stated by Wood (1973), the significance of the approximations involved in Matsuo and Ohara's model and the importance of the deviations from the results of classical elastic theory are known for only a limited number of problems and need further evaluation. For Poisson's ratio $\nu = 1/2$, the model of Matsuo and Ohara gives infinite thrust on the wall, in contradiction with the results of conventional elastic models. Within the frame of classical elasticity, problemes in which there is perfect adhesion between the fill and the retaining wall are particularly difficult. This difficulty has been circumvented employing finite elements (Wood, 1973). The finite element method is confronted with difficulties of another kind in the case of backfills of semi-infinite extent; these can be handled through the introduction of adequate boundaries at the remote end of the model.

On the other hand, Scott's model offers some difficulties concerning the evaluation of the length of the base of the shear beam and the stiffness of the Winkler medium. It can be shown that, to obtain results consistent with the theory of elasticity, the stiffness of the Winkler springs should be a function of wall height, H, varying approximately as H^{-1}, while the length of the base should be proportional to HV_p/V_s, where V_p and V_s are, respectively, the velocities of propagation of compressional and shear waves in the backfill. Furthermore, Scott's model does not inherently include radiation losses due to horizontally propagating waves in backfills of semi-infinite length. To include this type of effect, one must either account for it artificially through the addition of *ad-hoc* dashpots, or generalize the model by representing the backfill as an infinite sequence of elastically coupled shear beams. In the second case, the main advantage of Scott's model, i.e., its simplicity, is lost.

Thus, it appears that there is some ground to propose a model that does not lead to analytical difficulties as hard as those confronted with in the classical theory of elasticity, and that, on the other hand, does not exhibit some of the shortcomings of more radically simplified models. The guiding principle in the formulation of the model will be the assumption that the main earthquake effects on retaining structures are due to horizontal actions, and that the main effects of these actions on the backfill can be described ignoring the vertical displacements. An extended work on the model is being developed (Arias, *et al*, 1981).

FORMULATION OF THE MODEL

It will be assumed that the geometrical and mechanical characteristics of the backfill and of the retaining structure, as well as the forcing function (body forces, inertia forces, prescribed displacements at the boundaries, specified surface tractions) and the constraints are such that the system behaves in plane strain. This assumption is by no means essential to the model to be now postulated; it is adopted for the sake of convenience and simplicity, so that problems may be formulated and analyzed on a plane O_{xy}. For definiteness, O_y is chosen to point vertically upwards, while O_x is horizontal and is directed away from the retaining wall and towards the backfill.

The specific hypotheses that define the model herein proposed are the following:

H.1. The backfill behaves as a continuous deformable solid without couple stresses.

H.2. Vertical stresses in the backfill are equal to zero.

H.3. Stresses and strains in the backfill are related by the equations

$$\sigma_x = K_{xx} \frac{\partial u}{\partial x}, \quad \tau_{yx} = K_{yx} \frac{\partial u}{\partial y}. \qquad (1)$$

Here σ_x and τ_{yx} have the usual meaning; u is the component of displacement parallel to the x-axis, and K_{xx}, K_{yx} are elastic coefficients, which will be assumed to be given functions of x and y.

With the usual assumption of small displacements, Newton's second law of motion leads to the partial differential equation

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[K_{xx} \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yx} \frac{\partial x}{\partial y}\right] + \rho X, \qquad (2)$$

where ρ is the mass density of the backfill material, X is the horizontal component of body forces per unit of mass, and t is time.

<u>Homogeneous backfill</u>

If the backfill is homogeneous, K_{xx}, K_{yx}, and ρ are constants. It is convenient in that case to introduce two positive constants α, β such that

$$K_{xx} = \rho\alpha^2, \quad K_{yx} = \rho\beta^2 \qquad (3)$$

With this notation, Eq. 2 becomes

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} + \beta^2 \frac{\partial^2 u}{\partial y^2} + X, \qquad (4)$$

which, for zero body forces, can be reduced to the two-dimensional wave equation by a suitable change of the space variables.

If $X = 0$ and u does not depend on the coordinate x, Eq. 4 take the form

$$\frac{\partial^2 u}{\partial t^2} = \beta^2 \frac{\partial^2 u}{\partial y^2} \qquad (5)$$

In a similar way, when $X = 0$ and u does not depend on y, the equation is reduced to

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \qquad (6)$$

It follows that the postulated medium is capable of propagating body shear waves in the vertical direction with velocity β, and compressional body waves in the horizontal direction with velocity α.

It is easily verified that $F(mx + ny \pm ct)$, where $F(\cdot)$ is an arbitrary twice differentiable function, is a solution of Eq. 4 for the case of zero body forces, provided that $c^2 = m^2\alpha^2 + n^2\beta^2$ and $m^2 + n^2 = 1$. Therefore, plane waves can propagate in any direction defined by the direction cosines (m, n), with velocity c. It can be shown that $\beta \leq c \leq \alpha$, with equality holding if and only if the direction of propagation is either vertical (shear waves, $c = \beta$) or horizontal (compressional waves, $c = \alpha$). For intermediate directions of propagation, plane waves are not purely shear nor purely dilatational waves, particle displacements being in all cases horizontal.

Let us remark that the postulated model is not isotropic and does not strictly exhibit effects of the Poisson type. Therefore, even in the homogeneous case, it differs significantly from a Hookean isotropic solid. In order to establish a relation with the classical theory of elasticity, consider two simple cases of homogeneous strain: simple horizontal shear, and uniform horizontal compression (or dilatation). It is easily found that for these two cases the results of the proposed model coincide with those of classical elasticity if the constants K_{xx}, K_{yx} are chosen to be

$$K_{yx} = G, \quad K_{xx} = \frac{E}{1 - \nu^2} \qquad (7)$$

where E, G and ν have the usual meaning. It follows that the constants α and β are related by the equation

$$\alpha = \beta\sqrt{\frac{2}{1 - \nu}}. \qquad (8)$$

This interpretation of the elastic coefficients differs from that implicit in the model of Matsuo and Ohara. Both models coincide if and only if $\nu = 0$. As will be seen in the examples, the present interpretation leads to satisfactory agreement with the results of classical elasticity.

DISCRETIZATIONS OF THE MODEL

The postulated model can be discretized in several ways. One of them is shown in Fig. 1. This might

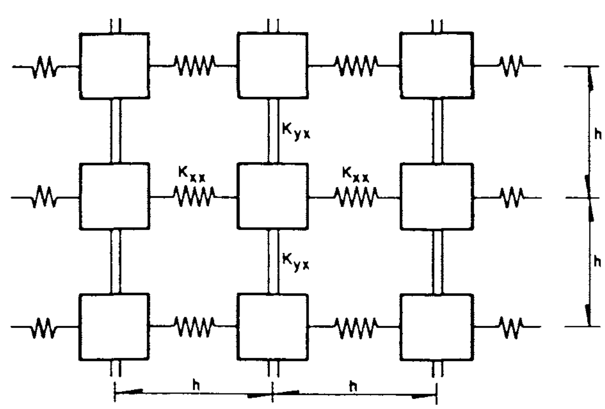

Fig. 1. Discretized elastic model

have been used as the original model. The continuum equations would then have been obtained by a limiting process letting $h \to 0$.

If Fig. 1 is partitioned by vertical planes, and the horizontal (compressional) springs inside each partition are replaced by rigid links, each partition will behave as a vertical shear beam. If now, the axes of contiguous beams are coupled by horizontal compressional springs of flexibilities equal to the sum of the flexibilities of the original compressional springs lying between the axes, the generalized shear beam model of Scott (1973) is obtained.

If Fig. 1 is partitioned by horizontal planes and the vertical (shear) springs are replaced by rigid links, each partition will behave as an elastic bar in compression and tension. Now, if contiguous bars are coupled by shear springs of flexibilities equal to the sum of the flexibilities of the original shear springs lying between the axes of the bars, the discrete model thus obtained is equivalent to that proposed by Ambraseys (1960) for the analysis of earth dams.

Thus, both of the above mentioned models are particular cases of the one postulated here. From the point of view of practical applications, discretization by horizontal bars has the advantage that horizontal stratified fills can be represented by a model in which the elastic properties of each bar are constants. This circumstance introduces significant analytical simplifications.

GENERALIZATIONS

The postulated model can be generalized in several ways without loosing simplicity or mathematical tractability. For example, to take account of the third dimension a term of the form

$$\frac{\partial}{\partial z} \left(K_{zx} \frac{\partial u}{\partial z}\right) ,$$

can be added to the right-hand side of Eq. 2. This generalization should prove to be useful in the analysis of seismic pressures on the front wall of bridge abutments, for example, when it is desired to account for the restraining effect of side walls.

Dissipative effects in the backfill can be simulated through the introduction of viscous dashpots acting in parallel with the springs of Fig. 1. Calling C_{xx}, C_{yx} the respective damping coefficients, the following equations hold instead of Eqs. 1

$$\sigma_x = K_{xx} \frac{\partial u}{\partial x} + C_{xx} \frac{\partial \dot{u}}{\partial x}, \quad \tau_{yx} = K_{yx} \frac{\partial u}{\partial y} + C_{yx} \frac{\partial \dot{u}}{\partial y} . \qquad (1')$$

The partial differential equation of motion becomes

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x} \left[K_{xx} \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y} \left[K_{yx} \frac{\partial u}{\partial y}\right] + \frac{\partial}{\partial t} \frac{\partial}{\partial x} \left[C_{xx} \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial t} \frac{\partial}{\partial y} \left[C_{yx} \frac{\partial u}{\partial y}\right] + \rho X \qquad (2')$$

If it is assumed that there exists a coefficient κ, not depending on the space variables x, y, such that

$$C_{xx} = \kappa K_{xx} , \qquad C_{yx} = \kappa K_{yx} \qquad (9)$$

and furthermore, that there are no external forces ($X = 0$), Eq. 2' takes the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \left[1 + \kappa \frac{\partial}{\partial t}\right] \left\{\frac{\partial}{\partial x}\left[K_{xx} \frac{\partial u}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yx} \frac{\partial u}{\partial y}\right]\right\} \qquad (10)$$

This equation is separable. In fact, writing

$$u(x, y, t) = \Phi(x, y) f(t) \qquad (11)$$

Eq. 10 is separated into a partial differential equation for Φ

$$\frac{\partial}{\partial x}\left[K_{xx} \frac{\partial \Phi}{\partial x}\right] + \frac{\partial}{\partial y}\left[K_{yx} \frac{\partial \Phi}{\partial y}\right] + \lambda^2 \rho \Phi = 0 \qquad (12)$$

and the ordinary differential equation for f:

$$\ddot{f} + \lambda^2 \kappa \dot{f} + \lambda^2 f = 0 , \qquad (13)$$

where λ^2 is the separation parameter. Putting

$$\kappa \lambda = 2\zeta \qquad (14)$$

Eq. 13 reduces to the well known differential equation for the free motion of a simple linear oscillator with viscous damping ζ (as a fraction of critical damping).

FIXED RIGID WALL. HORIZONTAL BODY FORCE

Consider a fixed rigid wall of infinite length backfilled with a material that satisfies hypotheses H.1-3. Let the forcing function be $X = -a$, where a is a constant. Three cases will be considered as shown in Fig. 2. The governing differential equation is

$$\alpha^2 u_{xx} + \beta^2 u_{yy} = a \qquad (15)$$

where subscripts denote partial differentiation. The following boundary conditions are valid for the three cases

$$u(0, y) = 0, \quad u(x, 0) = 0, \quad u_y(x, H) = 0 \qquad (16)$$

the condition at the far end of the backfill being different for each of the cases considered.

Semi-infinite backfill

The boundary condition at infinity is

$$\lim_{x \to \infty} u(x, y) = u_o(y) \equiv \frac{a}{2\beta^2} (y^2 - 2Hy) \qquad (17)$$

$u_o(y)$ can be interpreted as the displacement due to a body force a in a layer extended indefinitely in both senses of the x-axis.

The solution of Eqs. 15-17 is

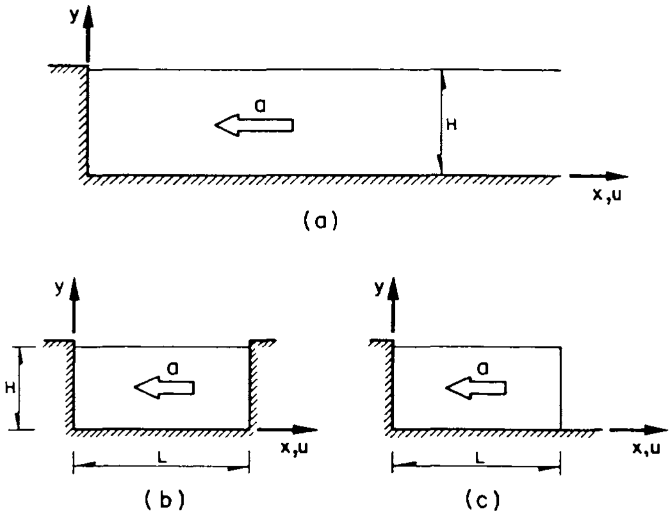

Fig. 2. Rigid wall and backfill under horizontal body force. a) Semi-infinite backfill; b) backfill fixed in x = L, and c) backfill free in x = L

$$u(x, y) = u_o(y) + \sum_{n=1}^{\infty} A_n \, e^{-\lambda_n x} \cdot \Phi_n(y) \qquad (18)$$

where

$$\Phi_n(y) = \sin \mu_n y, \quad \lambda_n = \beta\mu_n/\alpha, \quad \mu_n = (2n-1)\pi/2H \qquad (19)$$

and the coefficients A_n are obtained expanding $u_o(y)$ in a Fourier series of sines of the form

$$u_o(y) = \sum_{n=1}^{\infty} A_n \sin \mu_n y \; . \qquad (20)$$

Expressions for pressure distribution, thrust, and overturning moment about the toe of the wall may be readily obtained:

$$p(y) = \frac{8\gamma H}{\pi^2} \cdot \frac{a}{g} \cdot \frac{\alpha}{\beta} \sum_{n=1}^{\infty} \frac{\Phi_n(y)}{(2n-1)^2} \; , \qquad (21)$$

$$P = \frac{16\gamma H^2}{\pi^3} \cdot \frac{a}{g} \cdot \frac{\alpha}{\beta} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \approx 0.543 \, \frac{\gamma H^2 a\alpha}{g\beta}, \qquad (22)$$

$$M = \frac{32\gamma H^3}{\pi^4} \cdot \frac{a}{g} \cdot \frac{\alpha}{\beta} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^4} \approx 0.325 \, \frac{\gamma H^3 a\alpha}{g\beta}. \qquad (23)$$

Backfill of finite length

Two cases are considered: backfill fully fixed at x = L, and backfill free at x = L; the boundary conditions being, respectively

$$u(L, y) = 0, \quad \text{and} \quad u_x(L, y) = 0 \qquad (24)$$

In both cases the solution can be expressed as

$$u(x, y) = \sum_{n=1}^{\infty} (B_n \cosh \lambda_n x + C_n \sinh \lambda_n x) \, \Phi_n(y) \qquad (25)$$

The coefficients B_n, C_n are determined in each case so as to satisfy the boundary condition at x = 0 and the pertinent boundary condition of Eqs. 24 at x = L.

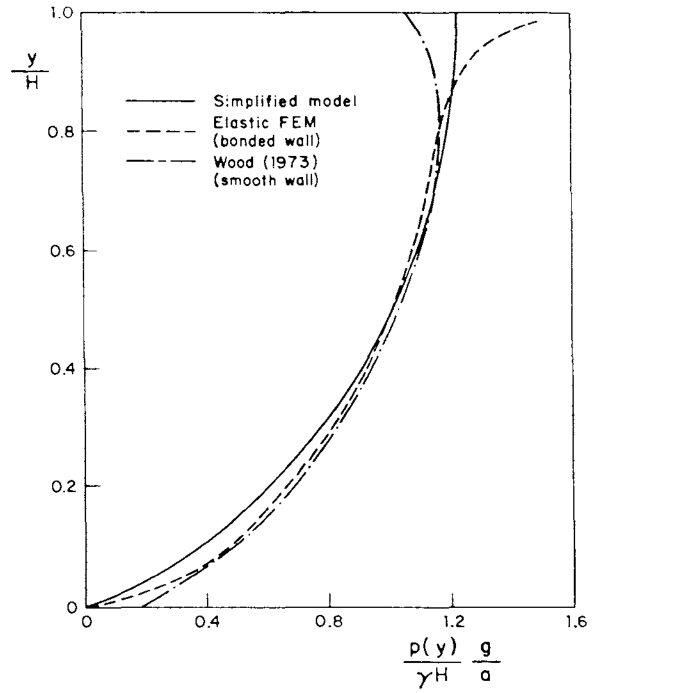

Fig. 3. Pressure distribution on a rigid wall. Horizontal body force on backfill with L/H = 5, and ν = 0.3. Comparison with classical elasticity solutions

Fig. 3 shows the pressure distribution for a length to height ratio of 5. Results are compared with data taken from Wood (1973) and from a FEM solution based in classical elastic theory. Thrusts and overturning moments are plotted in Fig. 4 together with Wood's results and FEM solutions. As it appears from these two figures, the agreement is very satisfactory.

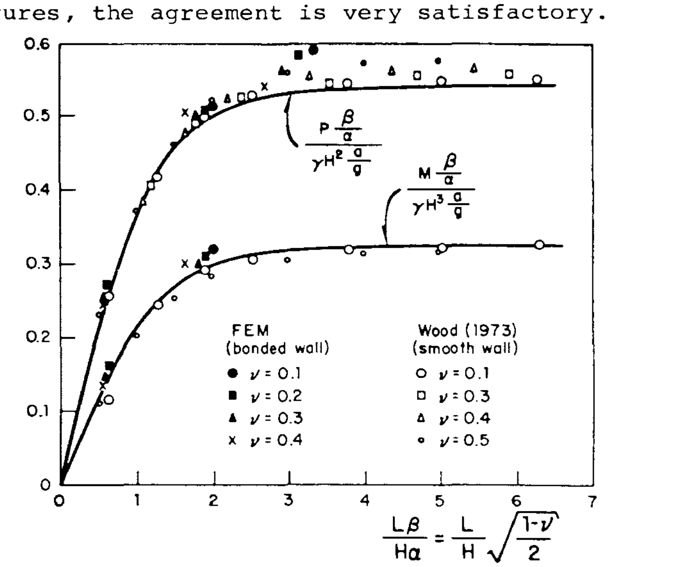

Fig. 4. Thrusts and overturning moments on a rigid wall. Horizontal body force. Comparison with results based on classical elastic theory

FIXED RIGID WALL. DYNAMIC RESPONSE

Seismic excitation

Assume that the base and wall of Fig. 2a move in the x-direction as a single rigid body with a given arbitrary acceleration $\ddot{s}(t)$. Let $u(x,y,t)$ be the resulting displacement of the backfill with respect to its base. Then $u(x,y,t)$ is the solution of the boundary value problem

$$u_{tt} = \alpha^2 u_{xx} + \beta^2 u_{yy} - \ddot{s}(t), \tag{26}$$

$$u(0,y,t) = 0,\ u(x,0,t) = 0,\ u_y(x,H,t) = 0 \tag{27}$$

It can be shown that the solution is

$$u(x,y,t) = \sum_{n=1}^{\infty} f_n(x,t)\ \Phi_n(y) \tag{28}$$

where

$$f_n(x,t) = -\frac{8}{\pi^2}\int_{-\infty}^{t}\ddot{s}(\tau)d\tau\int_0^{\infty}\frac{\sin\lambda x\sin[R_n(t-\tau)]\ d\lambda}{\lambda\ R_n(2n-1)} \tag{29}$$

with

$$R_n = \sqrt{\lambda^2\alpha^2 + \Omega_n^2}\ ,\quad \Omega_n = (2n-1)\pi\beta/(2H). \tag{30}$$

The pressure on the wall is therefore

$$p(y,t) = -\ \rho\alpha^2 u_x(0,y,t)$$
$$= \frac{8\rho\alpha^2}{\pi^2}\sum_{n=1}^{\infty}\frac{\Phi_n(y)}{(2n-1)}\int_{-\infty}^{t}\ddot{s}(\tau)d\tau\int_0^{\infty}\frac{\sin[R_n(t-\tau)]\ d\lambda}{R_n}\ . \tag{31}$$

Making use of the substitution

$$R_n = \mu\ \Omega_n \quad (\text{i.e., } \lambda = \frac{\Omega_n}{\alpha}\sqrt{\mu^2-1}\ ) \tag{32}$$

the inner integral becomes

$$\int_1^{\infty}\frac{\sin[\mu\,\Omega_n(t-\tau)]\ d\mu}{\alpha\ \sqrt{\mu^2-1}} = \frac{\pi}{2\alpha}\ J_o[\Omega_n(t-\tau)] \tag{33}$$

where $J_o[\cdot]$ stands for the Bessel function of the first kind and order zero (Gradshteyn and Ryzhik, 1965, 3.753.2, p. 419).

Setting

$$S(t,\Omega) = \Omega\int_{-\infty}^{t}\ddot{s}(\tau)\ J_o[\Omega(t-\tau)]\ d\tau \tag{34}$$

the following final results for pressure, $p(y,t)$, total thrust, $P(t)$, and overturning moment, $M(t)$, are obtained

$$p(y,t) = \frac{8\gamma H\alpha}{\pi^2\beta g}\sum_{n=1}^{\infty}\frac{\Phi_n(y)}{(2n-1)^2}\ S(t,\Omega_n), \tag{35}$$

$$P(t) = \frac{16\gamma H^2\alpha}{\pi^2\beta g}\sum_{n=1}^{\infty}\frac{S(t,\Omega_n)}{(2n-1)^3}\ , \tag{36}$$

$$M(t) = \frac{32\gamma H^3\alpha}{\pi^4\beta g}\sum_{n=1}^{\infty}\frac{S(t,\Omega_n)\,(-1)^{n+1}}{(2n-1)^4} \tag{37}$$

As far as the authors are aware, the function $S(t,\Omega)$ appears for the first time in earthquake engineering literature, in the results obtained by Kotsubo (1959) and by Ferrandon (1960) for the hydrodynamic seismic pressures on a vertical rigid wall.

By analogy with the theory of earthquake response of simple structures with zero damping, a response spectrum $B(\Omega,0)$ can be defined as

$$B(\Omega,0) = \sup_t\ \{|S(t,\Omega)|\} \tag{38}$$

We shall call $B(\Omega,0)$ the *Bessel acceleration spectrum* of the excitation for zero damping. It follows immediately that the absolute values of pressure on the wall, total thrust and overturning moment admit upper bounds that can be obtained from Eqs. 35-37 after replacing $S(t,\Omega_n)$ by $B(\Omega_n,0)$.

The theory can be extended to include (linear) internal damping in the backfill. With the specification of damping introduced in Eq. 9, the differential equation for u takes the form

$$u_{tt} = [1+\kappa\frac{\partial}{\partial t}][\alpha^2 u_{xx} + \beta^2 u_{yy}] - \ddot{s}(t), \tag{39}$$

the boundary conditions being the same as in the undamped case.

The final results are of the same form as those found in Eqs. 35-37 for the undamped case, with $S(t,\Omega)$ replaced by the function $S(t,\Omega;\zeta)$ defined by

$$S(t,\Omega;\zeta) = \frac{2\Omega}{\pi\sqrt{1-\zeta^2}}\int_{-\infty}^{t}\ddot{s}(\tau)\mathrm{Im}K_o[\Omega(t-\tau)e^{-i\arccos\zeta}]d\tau. \tag{40}$$

Here $K_o[\cdot]$ denotes the modified Bessel function of the second kind and order zero, and Im stands for 'imaginary part'. Let us give the *damped Bessel spectrum*, $B(\Omega,\zeta)$, defined as

$$B(\Omega,\zeta) = \sup_t\{|S(t,\ \Omega;\ \zeta)|\}. \tag{41}$$

Impulsive response

Let $u_i(x,y,t-\theta)$ be the displacement response of the backfill to a unit pulse of acceleration, $\ddot{s}(t) = \delta(t-\theta)$. For the undamped case it is found that

$$u_i(x,y,t-\theta) = -\frac{8U(t-\theta)}{\pi^2}\sum_{n=1}^{\infty}\frac{\Phi_n(y)}{2n-1}h_n(x,t-\theta) \tag{42}$$

where $U(\cdot)$ is the Heaviside's step function and

$$h_n(x,t-\theta) = \begin{cases}\frac{\pi}{2\alpha}\int_0^x J_0[\frac{\Omega_n}{\alpha}\sqrt{\alpha^2(t-\theta)^2-\xi^2}\ ]d\xi, \\ \qquad \text{if } 0 < x < \alpha(t-\theta) \\ \frac{\pi}{2\alpha}\sin[\Omega_n(t-\theta)],\ \text{if } x > \alpha(t-\theta) > 0.\end{cases} \tag{43}$$

This result shows that waves reflected at the vertical wall affect the behaviour of the backfill, up to instant t, only in the region $0 < x < \alpha(t-\theta)$, and have no influence on it for $x > \alpha(t-\theta)$. The wave front introduced by the presence of the wall is a vertical plane moving with velocity α. Furthermore, in the region that has not been reached by the front, the displacement of the fill does not depend on x, being the same as for a layer extending indefinitely in both senses of the x-axis. Similar results can be obtained for the damped case.

Frequency response

Let $p(y,\omega)e^{i\omega t}$, $P(\omega)e^{i\omega t}$, and $M(\omega)e^{i\omega t}$ be respectively the pressure distribution on the wall, the thrust, and the overturning moment caused by the excitation $\ddot{s}(t) = e^{i\omega t}$. It can be shown that

$$p(y,\omega) = \frac{8\gamma H\alpha}{\pi^2\beta g}\sum_{n=1}^{\infty}\frac{\Phi_n(y)}{(2n-1)^2}F_n(\omega,\xi) \qquad (44)$$

where

$$F_n(\omega,\xi) = \frac{2}{\pi}\int_0^{\pi/2}\frac{[(1-r^2\cos^2\phi)-i2\xi r\cos\phi]}{(1-r^2\cos^2\phi)^2+(2\xi r\cos\phi)^2}\,d\phi, \qquad (45)$$

and $r = \omega/\Omega_n$. $P(\omega)$ and $M(\omega)$ can be readily obtained by integration. In order to compare our results with those obtained by Wood (1973), Fig. 5 shows the frequency response of $P(\omega)/P(0)$, the normalized thrust for $\xi = 0.1$, calculated

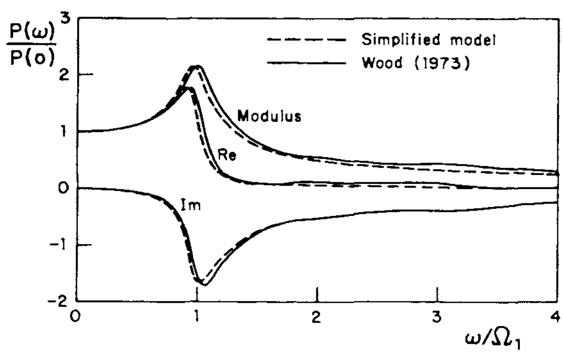

Fig. 5. Frequency response of normalized thrust. Comparison with Wood's results

from Eq. 44. Comparison is given with Wood's results for an elastic backfill with L/H = 50 and the same damping. Agreement is excellent. Similar results can be obtained for overturning moments and other dampings. Indeed, Fig. 6 shows the moduli of the normalized thrust for various dampings. It should be noted that the maxima of the normalized frequency responses decay with increasing ζ approximately as $(2\zeta)^{-1/2}$. This result differs from that obtained for the response of a single-degree-of-freedom-damped oscillator in which maxima decay as $(2\zeta)^{-1}$.

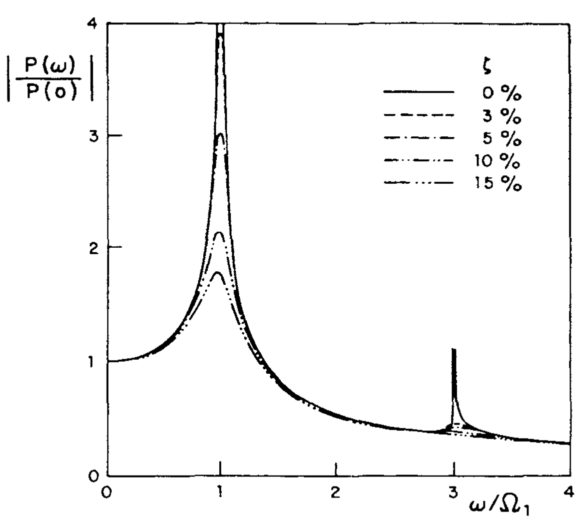

Fig. 6. Moduli of normalized thrust for various dampings

CONCLUSIONS

The simplified elastic model here presented appears to be a useful tool to deal with a variety of static and dynamic problems of retaining structures with small displacements. Mathematical simplicity is gained compared with the complexity of classical elasticity. On the other hand, accuracy and physical insight are retained with this model. The model can be extended to deal with three dimensional cases and non homogeneous fills. Moreover, an important advantage of the model is that it naturally includes horizontal radiation effects.

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