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Behavior of Interfaces Between Structural and Geologic Media

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SYNOPSIS The main objective of this paper is to identify and discuss the subject of the effect of interface behavior on the overall soil-structure interaction in building foundation systems. A brief review of the previous approaches based on the assumption of compatibility between the structure and soil is followed by a discussion of the recent efforts toward inclusion of relative slip, debonding and rebonding at interfaces. Here available models in the context of the lumped parameter and finite element approaches are reviewed. A number of models used in static and dynamic analyses are presented, and the difficulties associated with those based on relative displacement, particularly in relation to the (arbitrary) choices of normal and shear stiffness, are discussed. Some ideas toward a simple but potentially promising model based on the use of thin element of soil (or structural medium) as interface is presented.

The importance of appropriate laboratory tests is established and is followed by a review of available laboratory test devices for static and dynamic interfaces. Finally, a brief description of a new multi-degree-of-freedom device including testing of interface under vertical, horizontal, torsional and rocking modes is described together with preliminary test results.

INTRODUCTION

It is now well established that for any realistic evaluation of the behavior of a structural-soil system subjected to static or dynamic loads, it is essential to allow for the interaction or coupling between the structure and the geologic media. Many recent analyses for soil-structure interaction have included the coupled and mutual influences of deformation of the structure and geologic media, but usually by assuming compatibility at the interface between the two. It is realized, however, that the behavioral aspects such as relative slip, debonding and loss of contact, and rebonding of interfaces under various translational and rotational motions can influence the interaction behavior.

A number of constitutive or stress-strain models have been proposed in order to simulate the interface behavior, particularly for static behavior. No model has yet proved to be suitable for general applications. One of the deficiencies in the development of such a model lies in the general lack of appropriate laboratory test devices for determination of the constitutive parameters and for verification. The objectives of this paper are

1. To present a brief review and definition of interaction phenomenon and the methods for incorporating interaction by assuming compatibility,
2. To establish motivation for the study of interface behavior under various modes of deformation,
3. To present a historical review of the available models for static and dynamic analysis, and propose improvements and new concepts,
4. To identify importance of appropriate laboratory tests and present a review of static and dynamic test devices, and,
5. To describe a new test device together with typical preliminary test results.

INTERACTION BEHAVIOR

Importance of interaction phenomenon in static and dynamic soil-structure interaction has been recognized and studied by many investigators, and it is not intended to present a detailed review herein; comprehensive reviews on various aspects of soil-structure interaction are presented by Roesset, Whitman and Dobry (1973), Kausel and Roesset (1974), Desai (1977), Desai and Christian (1977), Idriss et al (1979), Isenberg, Vaughan and Sandler (1978), Kausel et al (1979), Desai (1979), Whitman and Bielak (1980) and Roesset and Scaletti (1980).

Whitman and Bielak (1980) explain soil-structure as follows:

> If the motion at any point on the soil-structure interface differs from the motion that would occur at this point in the free field if the structure were not present, there is soil-structure interaction. If the interface moves or distorts differently

than the corresponding surface in the free field, there is interaction. Average horizontal and vertical translation, rocking about a vertical axis are all included in the definition.

The motion of a point influenced by soil-structure interaction can include a component due to mutual deformations if compatibility is assumed, and another component due to relative generalized displacements under rotational and translational movements. Most previous analyses by using the lumped parameter or finite element approaches have considered only the first component. Study of the second component due to relative motion is of recent origin.

Compatibility Between Structure and Soil

In this approach, the analysis permits inclusion of deformation characteristics of both the structure and the foundation soil. However, complete compatibility is assumed at a point common between the soil and structure. A number of procedures have been proposed and used. Chief among these are: the lumped parameter models modified to account for soil response simulated by using spring-mass point simulations, continuum models and finite element model. As noted previously, a number of review papers are available for details of these approaches. Figure 1(a) to (d) shows schematic diagrams of some of these models.

MODES OF DEFORMATION

It is commonly assumed in many seismic analysis that the earthquake input motion is identical at all points beneath the structure, and very little experimental evidence is presently available to supplant this viewpoint. Scanlan (1976), and also, if the dimension of the foundation is large compared with wavelength of the input motion this assumption may be in error, Sun and Tang (1979). Thus a travelling wave may cause cancelling effects of the input motion and because the wave can reach different points in the foundation, it becomes necessary to consider both the translational and rotational motions at the structure-soil interface. Isenberg, Vaughan and Sandler (1978) noted that rocking is the principal effect of interest to the aseismic design of power plants, although vertical, horizontal and torsional effects also occur.

Within the context of compatible approaches, the importance of rotational motions such as torsional and rocking together with the translational motion has been identified and analyzed also by various investigators; Newmark (1969), Krizek, Gupta and Parmelee (1972), Urlich and Kuhlemeyer (19730, Lee and Wesley (1975), Luco (1976), Scanlan (1976), Kennedy (1976), Wolf (1976, 1977), Whitley et al. (1977), Dawson (1978), Sun and Tang (1979), Idriss et al. (1979), Kausel et al (1979), Byrne (1980), and Roesset and Scaletti (1980).

Relative Motion: Sliding, Debonding, Rebonding

As observed earlier, in addition to the effect of deformation characteristics of the structure and soil, interaction can be influenced by relative motions that occur in various translational and rotational modes. Sliding at interface, and debonding, and opening and closing of the interfaces are some of the major attributes of the relative motion; a schematic representation of these modes is depicted in Fig. 2. The main objective of this paper is concerned with the behavior and constitutive laws of interfaces when the structure and soil remain together, and the effects of relative motions.

The importance of such motions in dynamic analysis and design of structure-soil systems has been discussed and analyzed by Isenberg, Lee and Agbabian (1973), Kausel and Rosset (1974), Wolf (1976,1977), Isenberg, Vaughan and Sandler (1978), Idriss et al (1979), Idriss et al (1979), Kausel et al (1979), Roesset and Scaletti (1980), Aubry and Chouvet (1981), Salagado and Byrne (1981) and Isenberg and Vaughan (1981).

In Appendix A of the Report by the Ad Hoc Group on Soil-Structure Interaction of the Committee on Nuclear Structures and Materials, ASCE edited by Idriss et al (1979) it is observed "that relative displacements due to slip or separation are not tractable by linear or quasilinear analysis. And there are reasons to believe that such discontinuous displacements are not a major cause of error, although it is observed that such slips can induce high shear strains, drastic reduction in soil moduli, stress redistribution, large shear deformations and finite displacements across interfaces. Further, it was recommended that complete fixity be assumed between soil and structural elements in quasi-linear finite element analysis."

On the other hand, Kennedy (1976) and Wolf (1976) observed that (for soft geologic media) separation and sliding effects may cause substantial increases in the amplified response spectra in the high frequency range. Roesset and Scaletti (1980) performed two-dimensional plane-strain finite element analysis by using nonlinear soil response, and by modifying the finite element equations to allow for slip debonding and separation. They studied behavior of structures resting on the ground surface and of embedded structures. It was found that the effect of relative motion on the response of structure on soil may increase the maximum horizontal acceleration by about 15 percent due to separation; overall, this may not be significant from a design viewpoint. At the same time, there was a significant influence on the vertical forces due to separation and sliding. Moreover, the influence of the behavior of embedded structures was found to be substantial. It was also observed that sliding and separation can cause large increases in the soil stresses, and the behavior can be affected by the magnitudes of excitation and the frictional characteristics of the interfaces. It was also noted that improved nonlinear models for soil may indicate different behavior. The importance of rocking behavior and the possibility of cavitation in soil-structure interaction was identified by Isenberg, Vaughan and Sandler (1978).

From the foregoing, it appears that study of the influence of relative motion involving sliding and separation on soil-structure interaction have received only little attention. It is believed that additional research toward development of constitutive models for interfaces, determination of constitutive parameters from appropriate laboratory tests, and incorporation of the models in solution procedures in order to identify influence of relative motion are required for improved analysis and design.

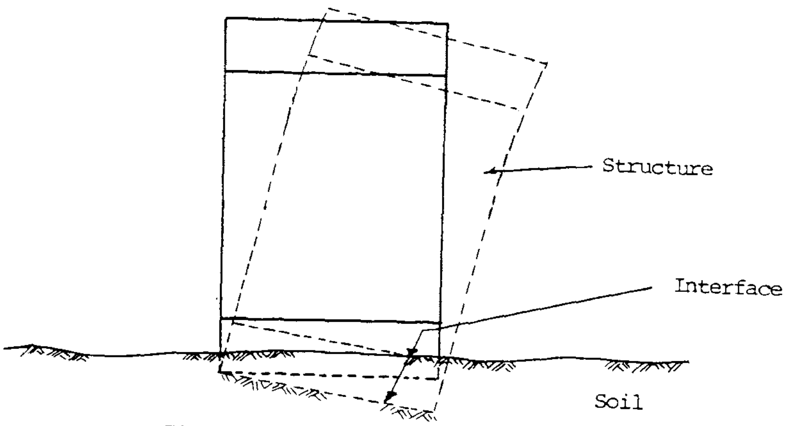


Fig. 1(a) Continuum Model

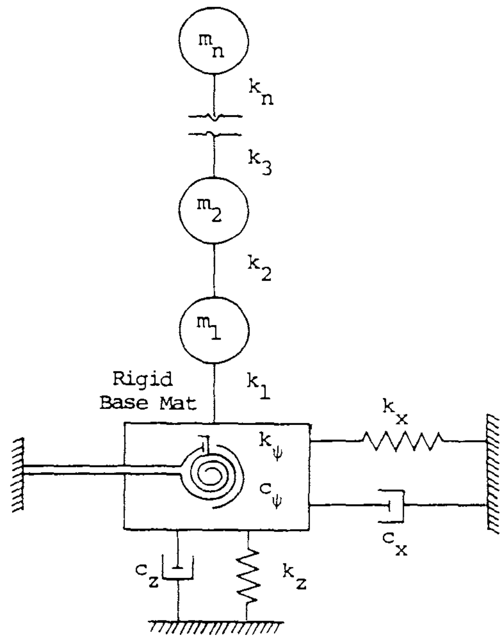


Fig. 1(b) Lumped Parameter Model; Idriss et al (1979).

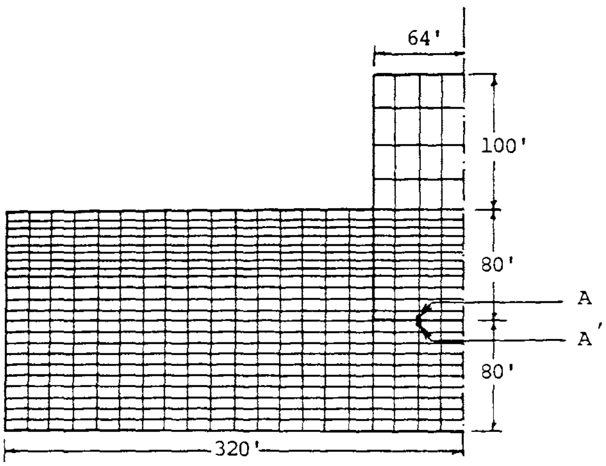


Fig. 1(c) Two-Dimensional Finite Element Model; Isenberg, Vaughan and Sandler (1978)

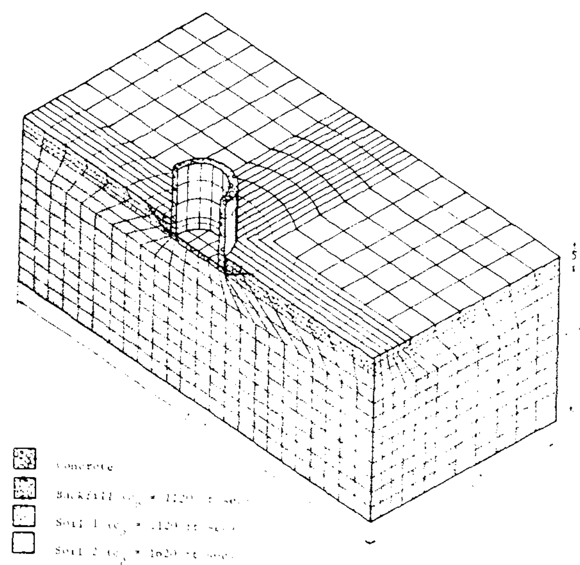


Fig. 1(d) Three-Dimensional Finite Element Model; Isenberg, Vaughan and Sandler (1978).

Fig. 1 Various Procedures for Soil-Structure Interaction

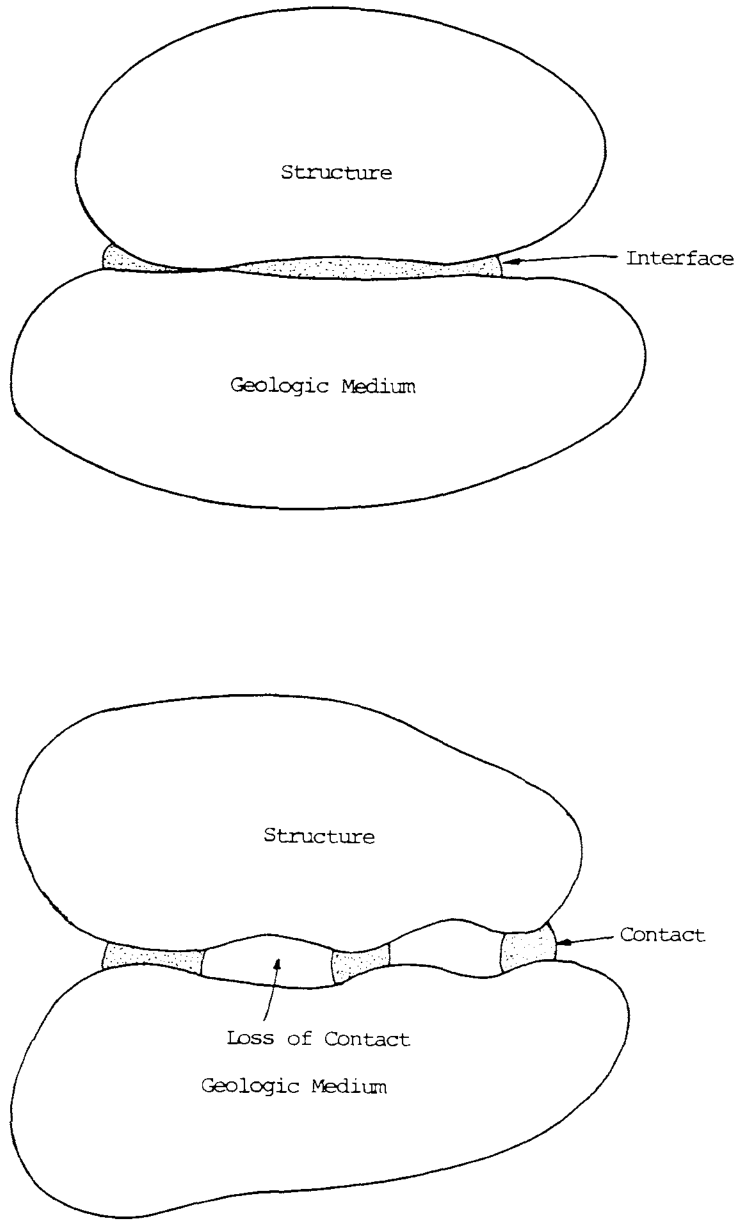


Fig. 2 Schematic of Contact and Debonding at Interface

INTERFACE MODELS

Junctions or interfaces between two dissimilar media having widely differing strength properties pose a different problem than deformation of a continuous medium. In the case of the latter, two adjacent points deform such that continuity of displacements at the points is maintained. On the other hand, two adjacent points A & A´ at the interface, Fig. 1(c), one in the structure and the other in the soil, may maintain continuity of displacement but only up to a certain load level. At higher loads, relative slip and debonding can occur and the two initially adjacent points may no longer have continuous displacements. Under certain types of loading and unloading, the interface may also experience separation or opening and then may close. Thus, the behavior at the interface renders the structure-soil system to deviate from being "continuous".

A variety of efforts have been made to account approximately for the foregoing special behavior at interfaces. These have included characterization of behavior of joints in rocks and interfaces in structure-soil systems.

Most of the studies towards development and application of models for interfaces and joints have involved static loading and use of such models for cyclic loading is of rather recent origin. Hence, for the sake of logical development and completeness, a review of the models for static analysis is first presented.

Models for Static Analysis

One of the earlier works in the context of rock-joint or fault behavior involved use of a model in which two intact masses were connected by using pin-ended element, Fig. 3(a), Anderson and Dodd (1966).

Ngo and Scordelis (1967) presented a linkage element for simulating cracks in concrete and described the behavior of a crack in the two-dimensional mass by using springs for normal and shear responses.

Goodman, Taylor and Brekke (1968) presented a rock joint element by expressing the relative displacement between the two-dimensional intact rock masses, and formulated the stiffness matrix for the joint in terms of normal and shear stiffness, Fig. 3(b). Zienkiewicz et al (1970) developed a similar joint element based on the isoparametric concept.

The element developed by Goodman, Taylor and Brekke (1968) has been formalized for application in linear and nonlinear interaction analysis by a number of investigators; for details see various Chapters in Desai and Christian (1977). For instance, Clough and Duncan (1971) used it for plane-strain problems of retaining walls. Here the shear stiffness, k_{ss}, for the interface is simulated by hyperbolic stress-strain model based on tests for direct shear apparatus. Desai (1972, 1974, 1977) extended the element for use in axi-symmetric problems for simulating interfaces in pile problems.

Ghaboussi, Wilson and Isenberg (1973) presented a model similar to above but used relative displacement as an independent degree-of-freedom. They also defined the behavior in terms of the normal and shear stiffness.

Desai and Appel (1976) and Phan (1979) and Desai, Phan and Perumpral (1980) have presented interface elements for three-dimensional linear and nonlinear analysis of soil-structure interaction problems, Fig. 3(c).

Herrmann (1978) presented an algorithm for interface element similar to the foregoing concepts with certain improvements through constraint conditions. He discussed various modes of interface behavior such as sliding and debonding and proposed a numerical algorithm that can provide convergent solutions. However, still the normal and shear stiffness during the various modesl were essentially chosen arbitrarily.

Reviews of foregoing models and related aspects are available in Goodman and St. John (1977), Desai (1977), Wilson (1977) and Desai (1979).

Theoretical Details

The foregoing models for interface element are usually based on the following constitutive or stress-strain relationship for a two-dimensional body

$$\{\sigma\} = [k_j]\{u_r\} \tag{1}$$

where $\{\sigma\}^T = [\sigma_{nn}\ \sigma_{ss}]$ is the vector of normal and shear stresses, $\{u_r\}^T = [u_{nr}\ u_{sr}]$ is the vector of relative displacements (strains) in the normal and shear modes, respectively, and $[k_j]$ = matrix containing stiffness of the interface element, which can be expressed as

$$[k_j] = \begin{bmatrix} k_{nn} & k_{ns} \\ k_{sn} & k_{ss} \end{bmatrix} \tag{2}$$

Very often the cross stiffness k_{ns} and k_{sn} are assumed to be zero, then

$$[k_j] = \begin{bmatrix} k_{nn} & 0 \\ 0 & k_{ss} \end{bmatrix} \tag{3}$$

In soil-structure interaction problems, it is usually assumed that the structural and the soil medium may not penetrate each other and hence, during the translational model, Fig. 4(b), the value of the normal stiffness, k_{nn} is assumed to be very high, of the order of 10^8-10^{12} (F/L^3). It is difficult to arrive at an appropriate high value of k_{nn} that would yield consistent and reliable results; it is often arrived at by performing a parametric study for a given problem.

Figure 4 shows various possible modes of deformation at an interface under static loading. They involve translation with compressive σ_{nn} with relative slip identified by using a criterion such as Mohr-Coulomb, debonding or opening of the inter face and its closing or rebonding.

As described subsequently, the value of k_{ss} is usually defined from stress-strain response expressed in terms of shear stress σ_{ss} vs relative translation (strain), u_{rs}, often obtained from direct shear tests. Here slip in translational motion is often assumed to occur when the induced shear stress σ_{ss} exceeds the Mohr-Coulomb strengt

$$\sigma_{nn} \geq c_a + \sigma_{nn}\tan\delta \tag{4}$$

where c_a = adhesion at the interface, and ϕ = angle of friction. After such slip has occurred, the value of and shear stiffness is arbitrarily reduced to a small value, say k_{ss} = 10 to 100 (F/L^3). In this case, the value of k_{nn} is often kept at the arbitrarily chosen high value. When debonding occurs with opening of the interface, the normal stiffness k_{nn} is reduced to a small value.

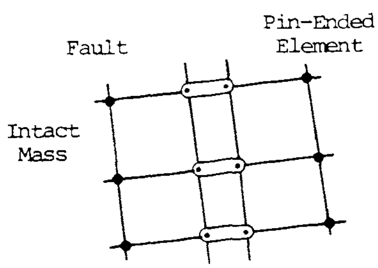


Fig. 3(a) Pin-Ended Model; Anderson and Dodd (1966)

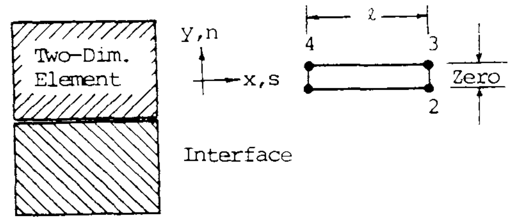


Fig. 3(b) Interface Element; Goodman, Taylor, and Brekke (1968)

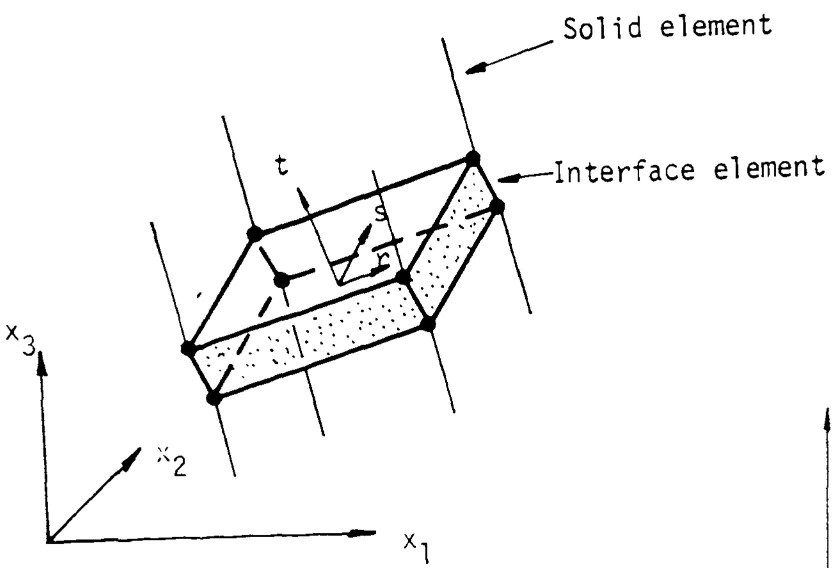


Fig. 3(c) Three-Dimensional Interface Element; Phan (1979), Desai, Phan, Perumpral (1980)

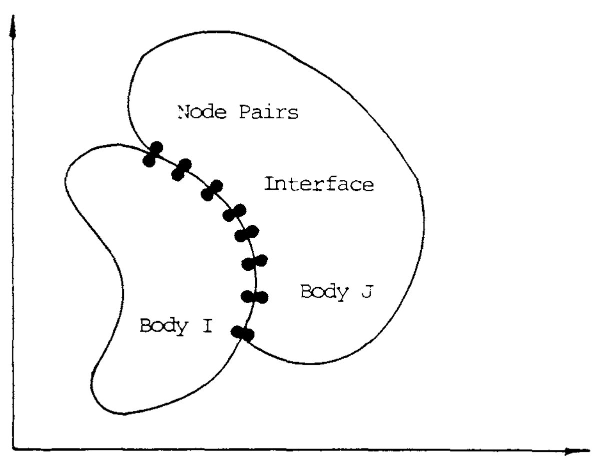


Fig. 3(d) Constraint-Interface Model; Katona et al (1976)

Fig. 3 Various Interface Models

Limitations

The computed behavior of an interface under the foregoing procedure and assumptions may work satisfactorily for translation up to the relative slip, but there appears no physical basis for adopting arbitrary values of k_{nn} and k_{ss} when relative slip and debonding has occurred. Because of this, very often, the above interface models involve considerable computational difficulties, and the results obtained cannot be always depended upon.

Katona et al (1976) have derived an interface model based on the virtual work principle modified by a special constraint condition, Fig. 3(d). This element can provide improved conditions at the interface as affected by the state of force (stress) induced during various modes, slip, debonding and rebonding, and can be considered to be an improvement over the other previous models. Hughes et al (1975) considered the contact-impact problem and proposed a contact model based on splitting of nodes. Peterson (1977) proposed a contact surface element to simulate the interface behavior and introduced multi-constraint relation to avoid numerical ill-conditioning.

Comment

Eventhough the foregoing interface models consider relative motions between structure and soil, still the displacements of two initially adjacent points after loading follow the requirement of continuity, Fig. 5(a). That is, these models, in reality, do not allow for discontinuities caused by relative slip and debonding. Such a formulation can be considered to allow essentially for the (large) differences in the deformation characteristics of the two media by introducing constraints through use of relative displacements. Since it can involve computational and other difficulties, and it is still based on continuity, it may be possible to consider the junction or interface as a "thin" solid element treated as soil or structural medium, and simulate the same effects due to the relative motion. Furthermore, as the relative motions at interfaces are caused mainly due to the fact that there is a (large) difference between the deformation characteristics of two media, it is felt that such an approach with 'thin' solid element, with appropriate modification, can prove to be as or more effective than the conventional models that are based on relative displacement and assumption of zero thickness for the interface.

Thus, it may be appropriate to investigate the use of a "thin" solid element at the interface assuming it to be either a soil or a structure element. Here the question of choosing an appropriate value of the thickness of the thin element may arise and need investigation in order to provide consistent, reliable and convergent solutions. However, this investigation need not be any more difficult than the problems encountered with the models of zero thickness based on relative displacements. Investigation of this concept is a subject of recent study by various investigators, including the author and his associates over the last three years: Lightner and Desai (1980) and Siriwardane and Desai (1980). Further details of this concept are given subsequently.

Pande and Sharma (1979) investigated the idea of using a thin interface element as an 8-node isoparametric element and compared it with the conventional model based on relative displacements. It was found that both approaches yield similar results for a wide range of problems. It was also shown that reliable results can be obtained with the 'thin' element for large values of aspect ratio defined as the ratio of length to thickness of the interface.

INTERFACE MODELS FOR DYNAMIC ANALYSIS

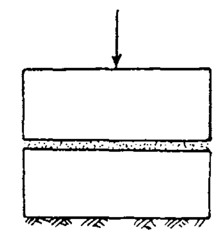
In dynamic analysis, rotational modes such as torsion and rocking need to be considered in addition to the translational mode, Fig. 6. Although a few studies have considered this subject, interface models for dynamic analysis have not received as much attention as those for static analysis.

Newmark (1965) considered interface behavior as sliding of a rigid block for analysis of dams and embankments; Crandall, Lee and Williams (1974) also assumed slip of a sliding rigid block. Seed (1976) noted that the shear strength to be used for sliding stability of structures on loose saturated soils should correspond to the shear stress level necessary to cause liquefaction in a given number of cycles of loading established by seismic design criteria.

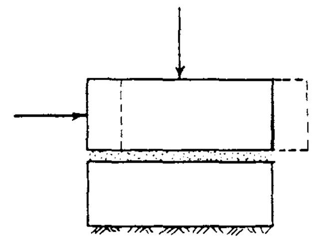
Isenberg, Lee and Agbabian (1973) used the interface element proposed by Ghaboussi et al. (1973) for three-dimensional analysis of a structure subjected to blast loadings; this essentially involved use of the same element as used for static analysis. Belytschko and Chiapatta (1973) used a model that allowed occurrence of slip in dynamic soil-structure interaction.

Wolf (1977) allowed for relative slip based on the co-efficient of friction and Mohr-Coulomb criterion; this was based on computation of displacement of a point on soil surface relative to the corresponding point on the foundation-disk. It was observed that the torsional effects due to travelling shear waves induce rocking perpendicular to the direction of excitation when liftoff or slip occurs.

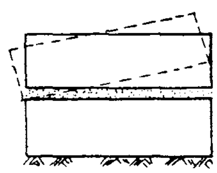
Kausel et al. (1979) gave a comprehensive consideration to interface behavior, discussed the importance of sliding or relative slip, and proposed a model to account for translational and rotational motions. It was observed that in most cases sliding will occur at interfaces and rarely in the soil mass and that the classical pseudo-elastic limit equilibrium analysis for sliding stability can give factors of safety against sliding unrealistically low in many cases and do not provide information on magnitudes of sliding motions. Lateral pressures on embedded structures will change due to soil and water pressures affected by magnitude and direction of ground acceleration and soil-structure interaction and with such unbalanced lateral pressures, stability against sliding may be reduced. Here on the active



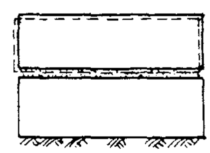
(a) Vertical



(b) Translation
and sliding



(c) Debonding



(d) Rebonding

Fig. 4 Various Modes of Deformation at Interface

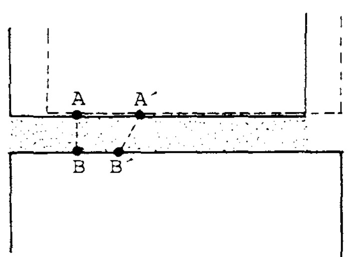


Fig. 5 Relative Motion and Continuity

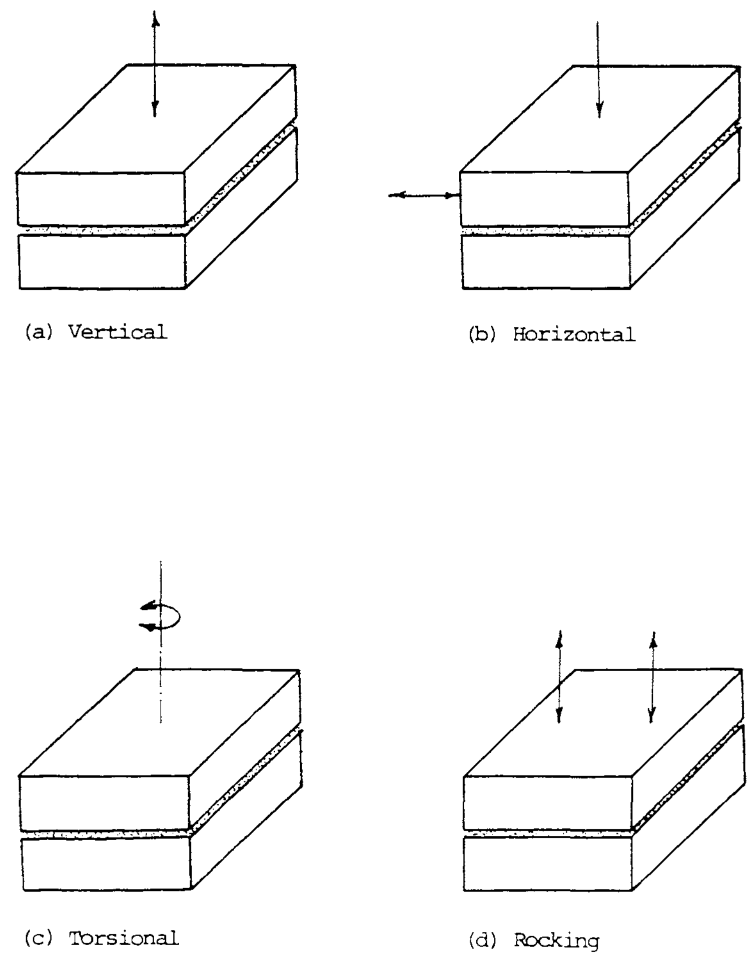


Fig. 6 Modes of Deformation at Interface in Cyclic Loading

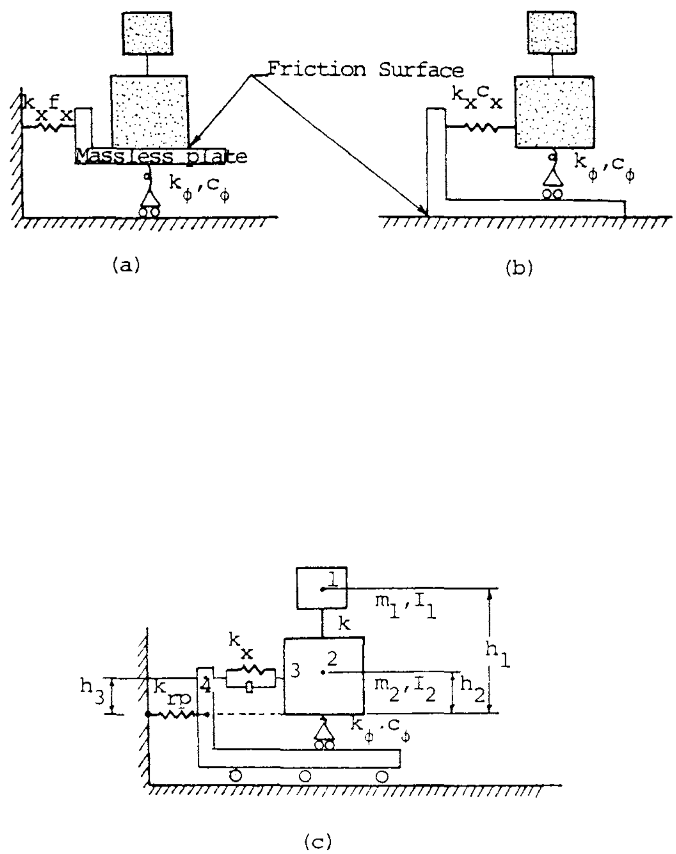


Fig. 7 Model Proposed by Kausel et al. (1979)

side, pressures will decrease resulting in separation of the structure and soil.

Figure 7 shows the model proposed by Kausel et al. (1979). The contact between the bottom mass (foundation) simulated by a massless plate and soil is defined by Coulomb friction. Although soil response is simulated by using springs and dashpots for translational and rotational motions, relative slip was permitted only in the translational motion. It was assumed to occur when the horizontal base force exceeded maximum frictional resistance between structure and soil. Once sliding occurred, the system was assumed to continue sliding until the relative sliding velocity reduced to zero regardless of contact forces.

Nazarian and Hadjian (1979) discussed three types of displacements, rigid body translation, rigid body rotation and flexure, for retaining walls and noted that damage to walls can be attributed to lateral pressures during earthquakes that can induce sliding or tilting on both of the structures. They proposed a model incorporating 'no tension' capability at the wall-soil interface, and observed that for improved analysis, allowance should be made for separation of walls from soils. As discussed before, Roesset and Scaletti (1979) considered sliding and separation with respect to two-dimensional plane strain analysis.

Aubry and Chouvet (1981) considered sliding, debonding and rebonding in cyclic motion by using Coulomb's law of friction. Salagado and Bryne (1981) used a lumped model in which the lumped mass representing the structure is connected to the free-field element in series representing the sliding of the structure. Isenberg and Vaughan (1981) gave a detailed consideration to interface behavior under dynamic loading and used the interface as a thin layer of solid soil element. They considered relative slip, debonding and rebonding with an interative procedure, and presented successful applications to a number of problems.

PROPOSED MODELS

A general model for interface behavior should include both the translational and rotational modes. Each mode should include provision for relative slip, debonding and rebonding. It is also desirable to develop such models for incorporation with available solution procedures for dynamic analysis such as lumped parameter and finite element methods.

Lumped Parameter Model

It is proposed to modify the previous lumped parameter model, Fig. 1(b), for relative slip, debonding and rebounding. The proposed modified model is shown in Fig. 8. It involves simulation of semi infinite soil medium by springs and dashpots for the translational, torsional and rocking modes. Special spring, sliding and debonding-rebonding element is inserted between the soil springs and dashpots for soil and the soil medium. The behavior of the structure is simulated by linear spingmass systems. The springs and dashpots simulating the soil response can be represented as linear or nonlinear; in the case of the latter the parameters can be made nonlinear functions of the states of stress and/or accumulated strain. The parameters can be evaluated as impedance functions based on the half-space theory or by appropriate modification of their static values, Idriss et al. (1979).

The spring-sliding mechanism for the interface behavior is assumed to be nonlinear function of the shear stress and/or relative displacements or rotations. Debonding can be approximately simulated by using a criterion based on the sign of the induced (vertical) force. If it becomes tensile, debonding is assumed to occur, and rebonding takes place when the force becomes compressive.

Finite Element Model

As discussed previously, the idea of the 'thin' (soil) element is proposed, Fig. 9. The important questions that should be addressed are the proper definition of normal and shear stiffness during those modes and proper definition of forces when debonding and rebonding occurs.

It has been found that one of the possible sources of difficulties can be due to inappropriate choice of arbitrary values of normal stiffness, k_{nn}. In order to reduce this difficulty, it is proposed to define normal stiffness based on the state of stress in the thin interface itself and/or the state of the surrounding structural and soil elements.

The constitutive matrix for the thin interface element is expressed as

$$\{\Delta\sigma\} = [C]\,\{\Delta\varepsilon_r\} \tag{5}$$

where $\{\Delta\sigma\}$= vector of stress components, $\{\Delta\varepsilon_r\}$ = vector of relative generalized displacements (strains) and $[C]$ is given by

$$[C] = \begin{matrix} [C_n] & [0] \\ [0] & [C_s] \end{matrix} \tag{6}$$

Here $[C_n]$ = portion related to normal behavior and is determined from appropriate linear or nonlinear parameters for the thin soil element. $[C_s]$ = portion related to the shear behavior and is essentially dependent on the shear modulus of evaluated from (direct) shear tests as follows:

$$G(\sigma,\gamma) = \frac{\sigma_{ss}}{\gamma} = \frac{\sigma_{ss}}{u_r/t} \tag{7}$$

where u_r = relative displacement, σ_{ss} = corresponding shear stress, and t = thickness of the interface element.

The normal portion $[C_n]$ can be defined on the basis of the state of stress and nonlinear material parameters of the adjoining structural and soil elements:

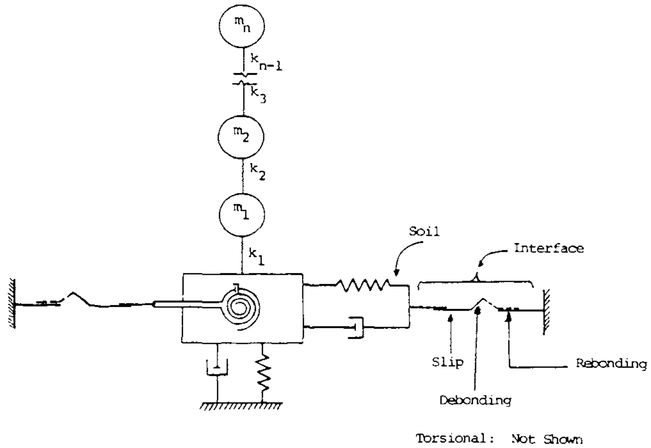


Fig. 8 Modified Lumped Parameter Model

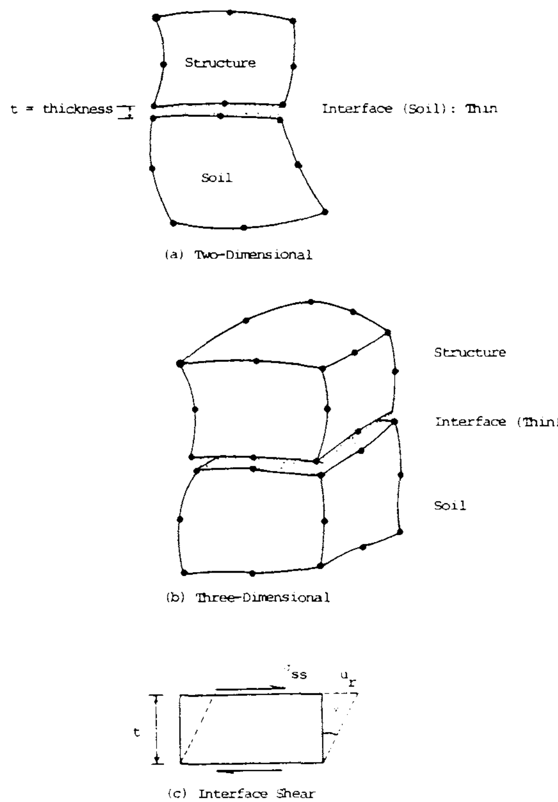
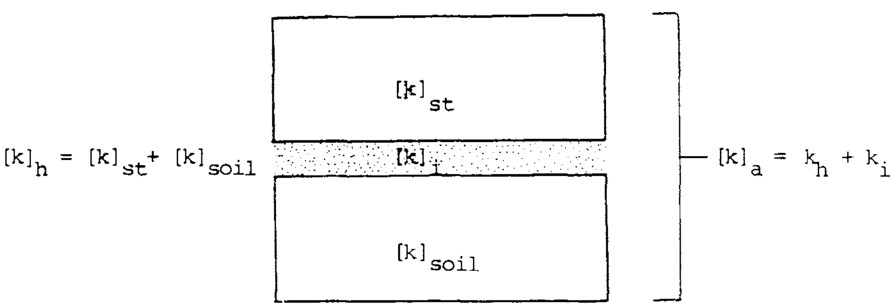


Fig. 9 Proposed 'Thin' Interfaces



$$[k]_i = \begin{bmatrix} [k_{nd}] & [0] \\ [0] & [k_s] \end{bmatrix}$$

Fig. 10 Decomposition of Interface Stiffness

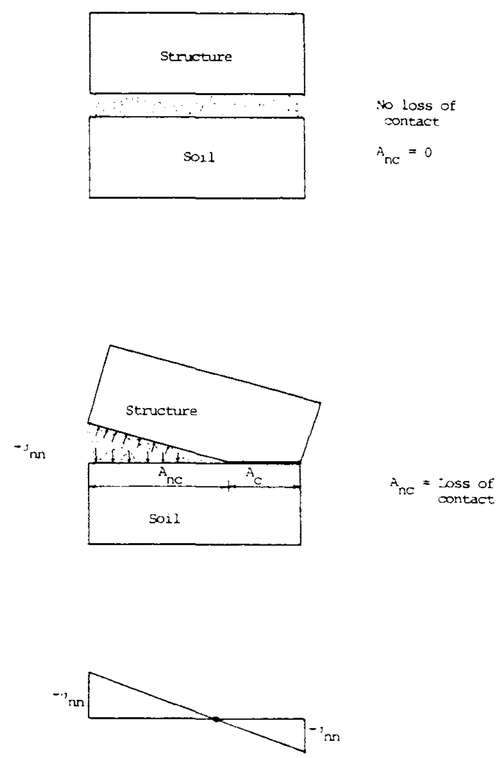


Fig. 11 Loss of Contact at Interface

$$[C_n] = [C_n\ (\alpha_i^{st}, \beta_i^{s})] \tag{8}$$

where α_i^{st} $(i=1,2,\ldots)$ = deformation parameters of the structural medium and β_i^{s}, $(i = 1,2,\ldots)$ = deformation parameters for the soil element.

The proposed concept can be expressed by decomposing the stiffness matrix $[k]_a$ for the system of structural, interface and soil elements, Fig. 10, as

$$[k]_a = [k]_h + [k]_i \tag{Aa}$$

$$[k]_a = [k]_h + ([k_{nd}] + [k_{si}]) \tag{Ab}$$

where $[k_h]$ = sum of stiffness matrices for the two solid elements, $[k]_{nd}$ = normal portion of the interface stiffness $[k]_i$, and $[k]_{si}$ = shear portion of $[k]_i$.

Debonding

Debonding can be identified by a criterion based on the sign of the induced (vertical) force; debonding can be assumed to occur when the force is tensile. The extent of debonding is determined by finding the point of zero normal stress, σ_{nn}, Fig. 11. When debonding occurs, an equivalent or residual load $\{Q_o\}$ is added to the element equations. The load vector $\{Q_o\}$ can be found as

$$\{Q_o\} = \iint [B]^T \{\sigma_{nn}\}_{nc} dAnc \tag{10}$$

where A_{nc} = area where loss of contact has occurred, $[B]$ = corresponding transformation matrix and $\{\sigma_{nn}^t\}$ = tensile normal stress.

Rebonding can be assumed to occur when the (vertical) force again becomes compressive. Then the equivalent load $\{Q_o\}$ is assumed to be $\{o\}$.

CONSTITUTIVE MODELS AND TESTING

In view of the complexity of the behavior of interfaces in static and cyclic loading, it is important to define appropriate constitutive models, and give special attention to determination of constitutive parameters from laboratory tests.

Constitutive Models

Figure 12 shows some of the commonly used models for interfaces. In the case of translational modes, the common scheme for lumped parameter models is to use a rigid-plastic type simulation, Fig. 12(a). Here, the sliding is assumed to occur when the induced yield stress equals or exceeds the Mohr-Coulomb strength, Eq. 4. It is possible to treat the behavior as (linear) elastic up to the yield and then sliding to occur according to the criterion in Eq. 4, Fig. 12(b).

In the context of the finite element method, as discussed previously, the normal stiffness, k_{nn}, is often assumed as follows

$k_{nn} = 10^8 - 10^{10}$	(F/L^3)	Before sliding
$k_{nn} = 10^8 - 10^{10}$		After sliding
$k_{nn} = 10^1 - 10^2$		For tensile conditions and debonding.

The shear stiffness k_{ss} is usually obtained on the basis of static direct shear tests, Figs. 12(c) and 13. With linear behavior it is assumed to be constant up to the yield stress. With nonlinear (elastic) assumption, it is evaluated as the tangent at a point on the shear stress vs relative displacement curve. Often, hyperbolic simulation is used for this purpose. After sliding occurs and for tensile conditions, the value of k_{ss} is arbitrarily set equal to a small value, say 10^2 (F/L^3).

If appropriate laboratory test data are available, it is possible to define interface behavior as elastic-plastic, Fig. 12(d), expressed as

$$\{\Delta\sigma\} = ([C]^e - [C^p])\ \{\Delta\epsilon_r\} \tag{11}$$

where $[C^e]$ = elastic constitutive matrix and $[C^p]$ = plastic constitutive matrix. The plasticity behavior can be defined through conventional models such as Mohr-Coulomb, and recently developed strain hardening cap type models, Fig. 12(d).

As discussed earlier, the foregoing constitutive models should be modified to include debonding and rebonding. This can be achieved by incorporating a residual load vector based on the tensile stresses induced in the interface or in a part of it together with an iterative scheme.

Laboratory Determination of Parameters

This is one of the most important phases in the development of models and has not received sufficient attention in the past. In this section, a review of the previous studies relevant to static and cyclic testing is first presented. Then a new and general device called dynamic multi-degree-of-freedom shear device is described together with preliminary test data and some projections on cyclic behavior under translational and rotational modes.

Static Direct Shear Testing, Fig. 14(a)

Potyondy (1961) performed a comprehensive series of direct shear tests for interfaces between concrete and soil (sand, clay). From a design viewpoint, the following results were reported:

$$f_c = c_a/c = 0.48 \text{ to } 0.81 \tag{12}$$

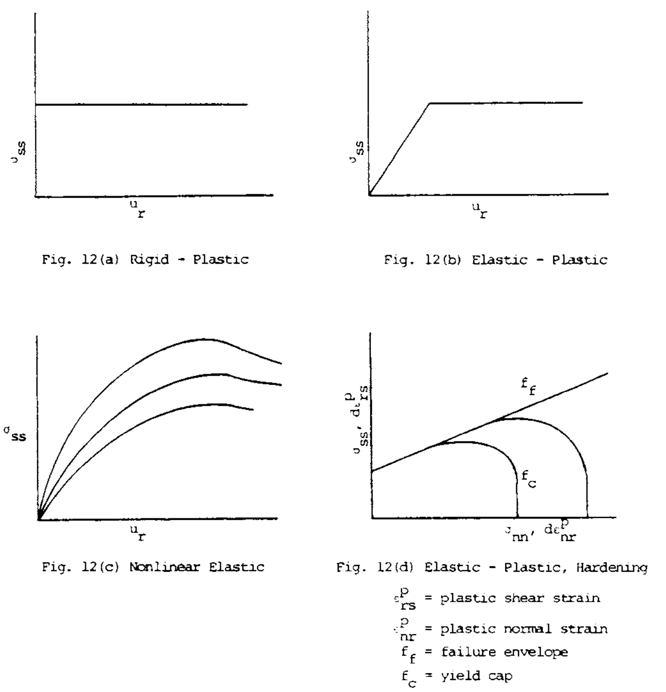


Fig. 12 Various Constitutive Models for Interface

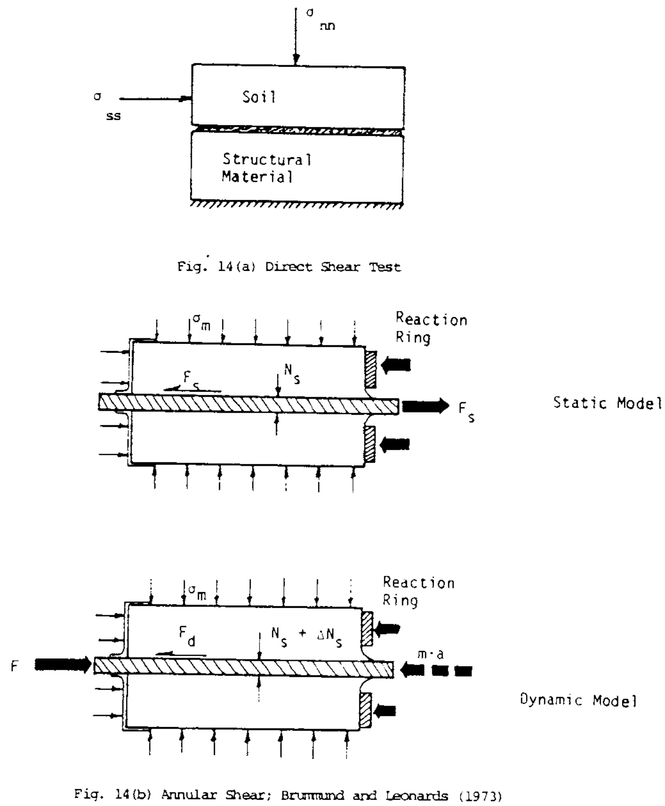


Fig. 14 Direct and Annular Shear Devices

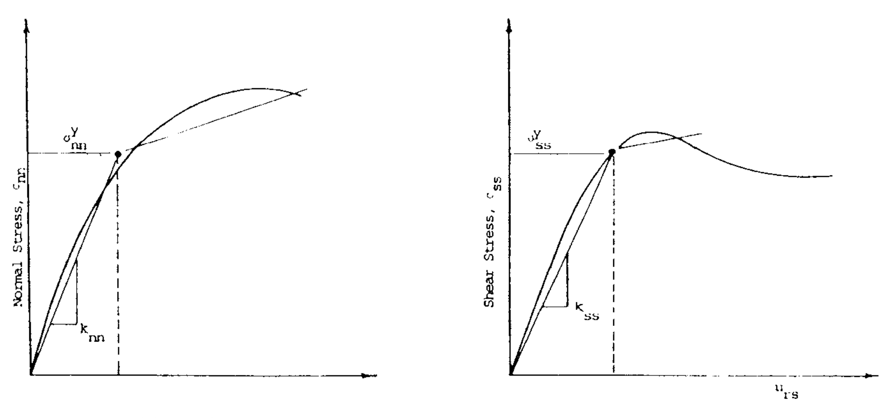


Fig. 13 Schematic Data for Interface Tests

$f = \delta/\phi = 0.82$ to 1.04

where c = cohesive strength of clay, and δ = angle of friction of sand. Tomlinson (1957), Mohan and Chandra (1961), Coyle and Suleiman (1967), Watt, Kurfurst and Zeman (1969), O'Neill and Reese (1972), Desai (1972, 1974) and others have reported direct shear test data to evaluate strength parameters c_a and δ for interfaces.

In the context of rock joints, Barton (1974) has given a comprehensive consideration to factors such as filled discontinuities, sliding along filled joints, influence of displacement and load history, dilatency and pore pressures; such results can be relevant to interfaces.

Clough and Duncan (1971), Desai and Holloway (1972), Desai (1974, 1975), Desai, Johnson and Hargett (1974) and others have reported direct shear test results for interface models and have used them in conjunction with finite element analysis of piles and retaining structures.

Cyclic Testing

Brummund and Leonards (1973) used a coaxial device, Fig. 14(b) with interface around a circular rod inserted along the center of a cylinder of sand enclosed in a light membrane. By using a vacuum, confining pressures up to 12.5 psi (8.6×10^4 N/cm^2) were induced. They found the following values of the ratio f_ϕ

	Static, f_ϕ	Dynamic, f_ϕ
Smooth concrete	= 0.76	= 0.84
Rough concrete	= 0.71	= 0.99

Consideration to interface behavior under dynamic loading has been given, among others, by Whitman and Healy (1962) and Goodman and Seed (1966).

A rational and theoretical consistent consideration to interface behavior between concrete and soil together with an advanced ring shear device, Fig. 15, has been given by Huck et al. (1974). However, it may be difficult to construct such a complex device for other modes, and the interface model can be difficult to implement.

Although, not directly related to laboratory tests, mention of the experimental work by Higgins et al. (1978) is appropriate here. On the basis of measurements of pressures on the sides and bottom of a cylindrical prototype structure subjected to earthquake type loads caused by explosives, they found that the lateral pressures (at interfaces) were functions of vertical acceleration.

DYNAMIC MULTI DEGREE-OF-FREEDOM SHEAR DEVICE (DMMDOFS)

As discussed before, in general, it is necessary to consider all motions at interfaces, namely vertical, horizontal, torsional and rocking. In order to develop constitutive models for interfaces subjected to these motions, it is desirable to develop a test device that can allow simulation of the translational and rotational modes. In order to identify the influence of cyclic loading in comparison to static behavior, such a device should include provision for tests for static (slow) and cyclic loading. Furthermore, the device should be capable of both strain and stress controlled tests. The dynamic multi-degree-of-freedom shear device has been designed and constructed to incorporate the foregoing characteristics, Desai et al. (1979, 1980), and Desai (1980).

A photograph of the device is shown in Fig. 16. Three different test boxes for translation, torsion and rocking modes, from integral parts of the device. At this time, only the test box with vertical and horizontal motions is operational and is briefly described herein.

The test box for the translational modes is depicted in Fig. 16. It is essentially a large direct shear assembly in which the bottom half consists of a square 16 x 16 x 9 inches (40 x 40 x 23 cms) sample of structural or geologic medium, and the top half, 12 x 12 x 9 inches (30.5 x 30.5 x 23 cms) can include geologic or structural medium. The interface is created at the junction of the two halves.

Loading

A maximum amplitude of load (vertical or horizontal equal to $\pm$ 12000 lbs (53.4 KN) can be applied; the frame, however, has been designed to withstand much higher loads. The frequency of load application can be up to about 5 Hz, although most are run for frequencies up to about 2 Hz.

For horizontal shear tests, the vertical load is applied and is kept constant, and the horizontal load is then applied, slowly to simulate static tests or cyclically to simulate earthquake type or repetitive loads. The latter can be applied in various forms, Fig. 17.

For strain controlled horizontal shear tests, the vertical load is kept constant whereas displacement amplitudes up to $\pm$ 1.5 inches (3.80 cm) can be applied with wave forms shown in Fig. 17.

Tests

Initially the tests are run for dry samples. In the case of stress controlled configuration, tests with various amplitudes of horizontal shear stress for given normal stress are run. The changing displacements are measured at the end of a given number of cycles, N, of load application. Based on the test results, relations between shear stress and relative displacements or strain are constructed for given N, normal stress σ_{nn} and frequency, f. These relations can provide evaluation of shear stiffness as a function of state of stress, number of cycles of loading, accumulated strain and frequency.

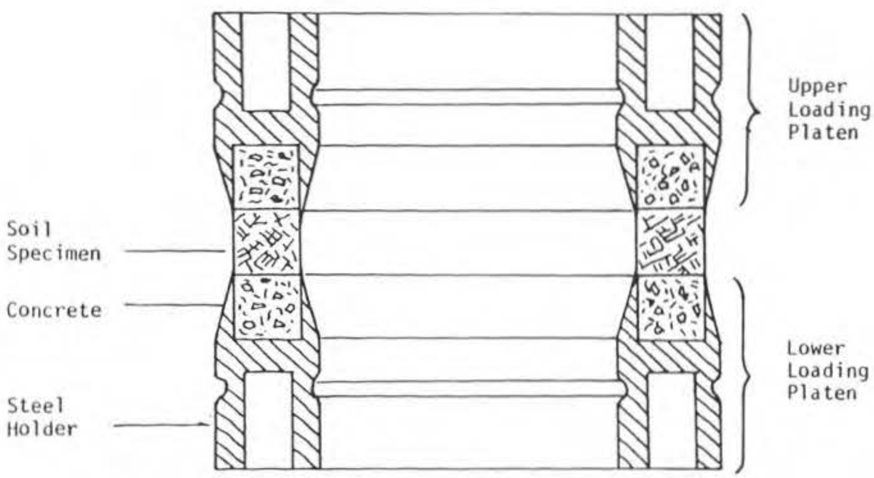


Fig. 15 Ring Shear Device, Huck et al. (1974)

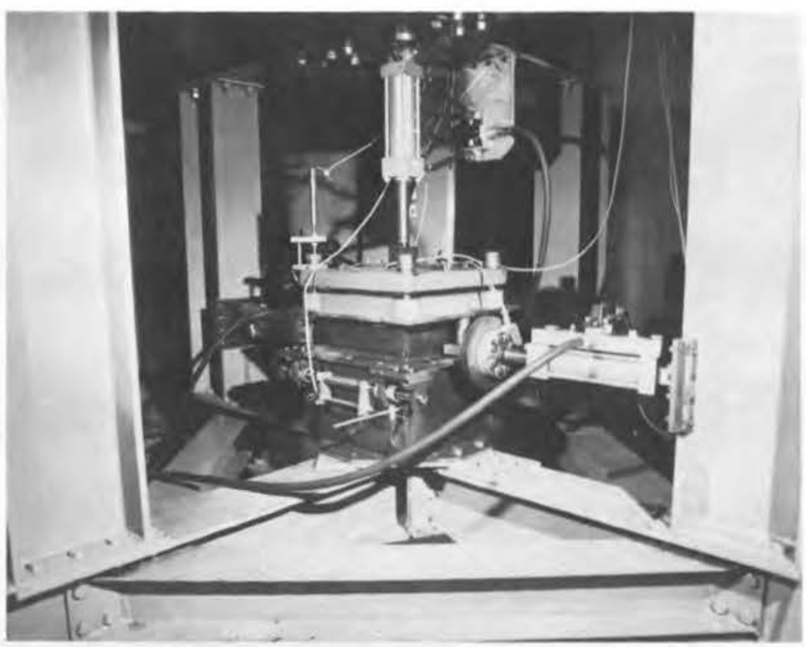


Fig. 16 Dynamic Multi Degree-of-Freedom Shear

For the strain controlled tests, results are obtained in terms of shear stress vs number of cycles for given frequency and normal stress. Stress-strain relations and Mohr-Coulomb plots for finding interface adhesion c_a and friction angle, δ, Eq. 4, are then obtained as function cycles of loading and frequency.

Test Results

Testing for interfaces such as those between concrete and soil, concrete and (railroad) ballast, wood in ties and ballast, concrete and water proofing membrane are in progress, Desai et al. (1979, 1980), Janardhanam (1980). Only a typical set of preliminary results for stress controlled tests for interfaces between concrete and ballast, Fig. 18. The tests were run at different normal stresses. For each normal stress, a number of horizontal stress amplitudes were applied. For the preliminary test results presented herein, the frequency of sinusoidal load application was 0.50 Hz and the initial density of ballast was 112 lbs/cft (1787 kg/m^3).

Figure 19(a) shows plots of horizontal stress, σ_{ss}, vs relative displacements for three different values of normal stresses. Figure 19(b) shows relation between horizontal displacement and number of cycles, N, for three ratios of shear to normal stress, and Fig. 19(c) shows the relation between initial shear stiffness, k_{si} vs normal stress, σ_{nn}, after various cycles of load applications, N, and Fig. 19(d) shows the relation between k_{si} and N for various normal stresses. It can be seen from Fig. 19(a) that the relation between shear stress and relative displacement is nonlinear at low σ_{nn} and essentially linear at higher σ_{nn}. The variation of k_{si} show, Fig. 19(c) essentially a nonlinear relation with σ_{nn}. Variation of k_{si} with N also is nonlinear, particularly at higher σ_{nn}. The initial stiffness decreases with number of shear stress applications, N, that is, with time, and at higher time levels and for higher normal stresses, it appears to stabilize, Fig. 19(d). Thus, the initial stiffness for f = 0.5 H_z can be expressed as

$$k_{si} = F_1(\sigma_{ss}, u_{rs}, \sigma_{nn}, N) \tag{14a}$$

If it were also dependent on f, then

$$k_{si} = F_2 (u_{rs}, \sigma_{nn}, N, f) \tag{14b}$$

Additional tests are in progress for various interfaces, and for translational (normal and shear) and rotational (torsion and rocking) modes, and different frequencies. Then the stiffnesses, in general, will be expressed as

$$[k] = [k\ (k_i, \{\sigma\}, \{u_r\}, N, f, \alpha_j) \tag{15}$$

where k_i = initial stiffness, α_j (i = 1,2,...N) = factors such as water content, initial density, and physical properties of interfaces.

CONCLUSIONS

A review of the importance of interaction between structure and geologic media indicated that there have been a number of studies to include (nonlinear) deformation characteristics of the two media based on the assumption of complete compatibility. It has been only recently that the effect of relative slip, debonding and rebonding at interfaces have been identified and analyzed. A review of various interface models for static and dynamic analyses indicate need for improved and rational models to account for the foregoing effects. Appropriate (laboratory) tests are needed to define constitutive models for interfaces, and there appears to be a general lack of testing devices; a new multi-degree-of-freedom shear device is described herein.

On the basis of this review, it appears that significant new research, analytical and experimental, will be needed in order to define and develop appropriate models for interfaces, and then delineate the effects of slip, debonding and rebonding under various translational and rotational modes on soil-structure interaction of systems subjected to dynamic loads.

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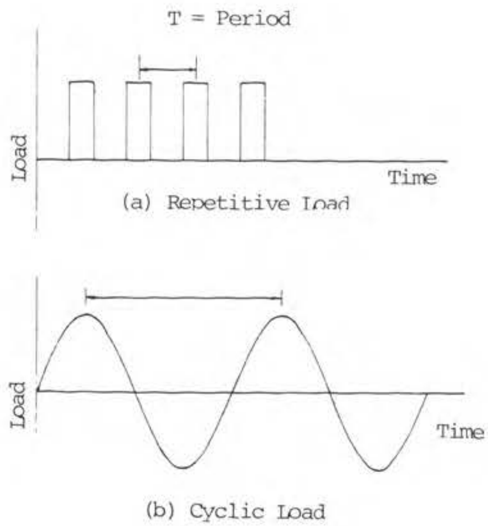

Fig. 17 Various Loading Forms

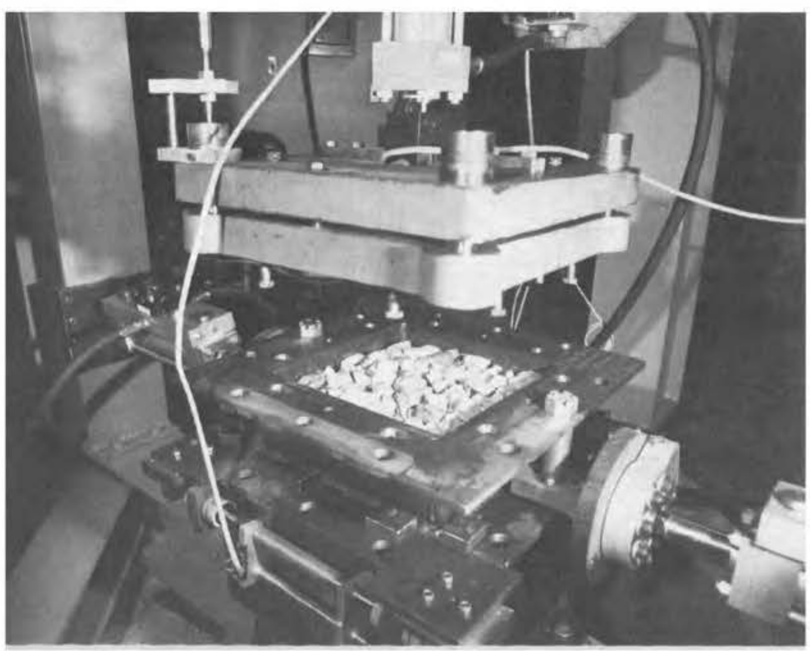

Fig. 18 Concrete-Ballast Interface

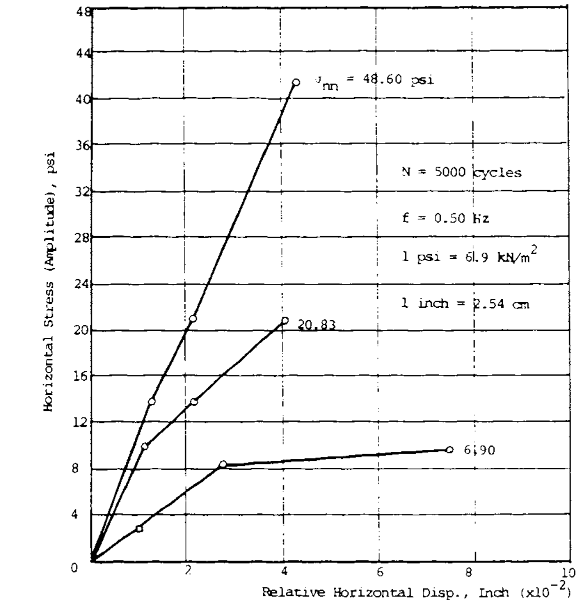

Fig. 19(a) σ_{ss} vs u_{rs}

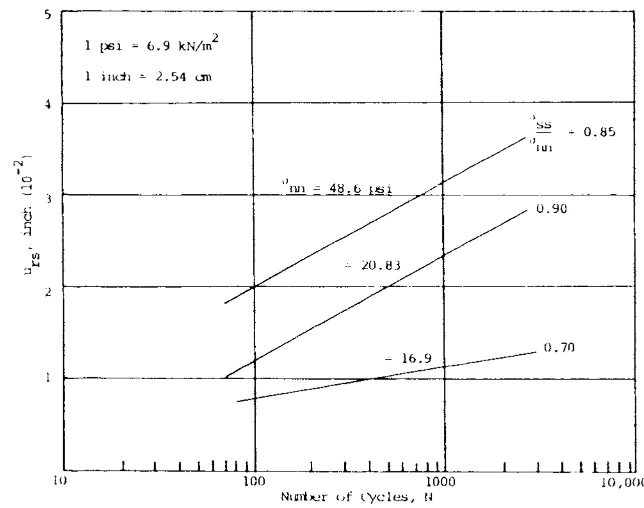

Fig. 19(b) u_{rs} vs N

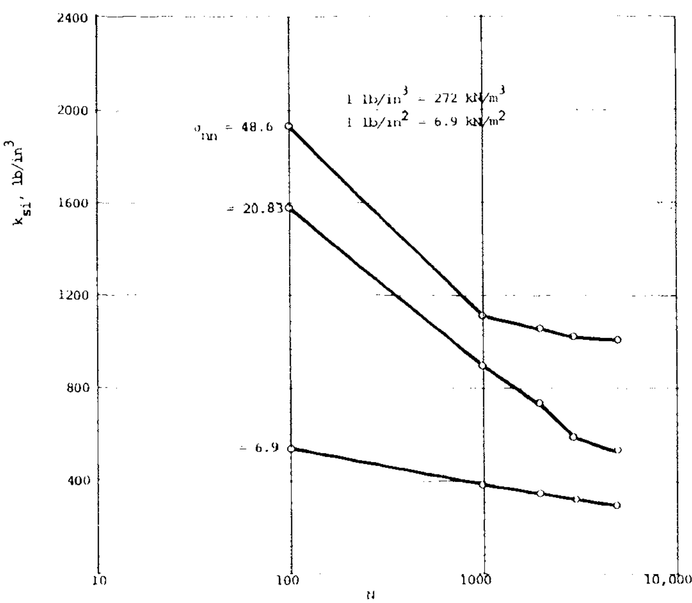

Fig. 19(c) k_{si} vs N

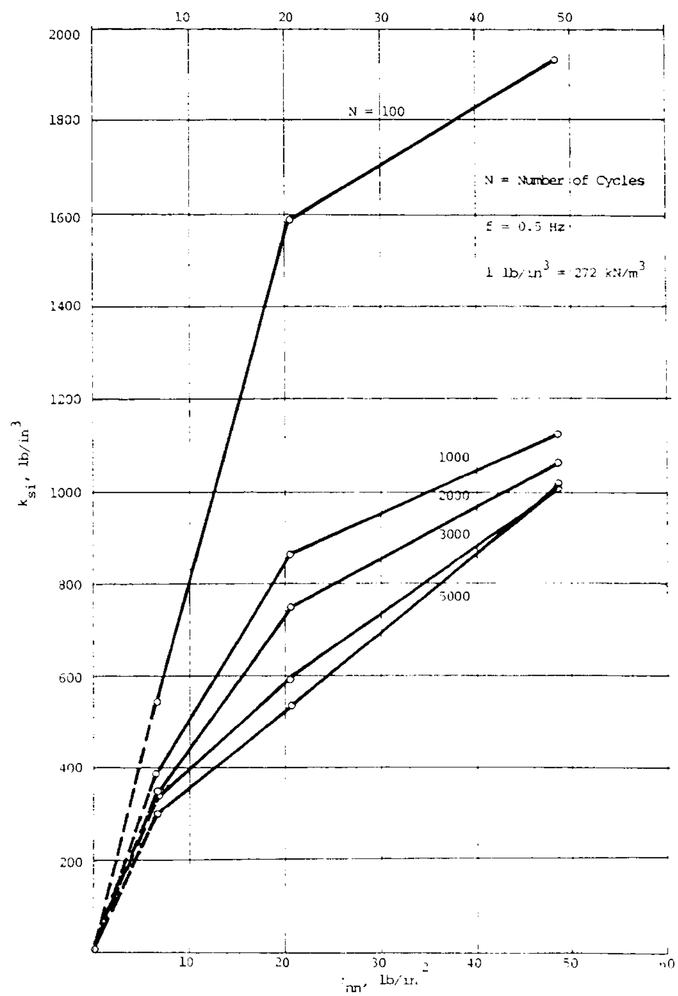

Fig. 19(d) k_{si} vs σ_{nn}

Fig. 19 Typical Cyclic Test Data for Concrete Ballast Interface

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