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SYNOPSIS The concept of the bounding surface in plasticity theory has been used to develop a general three-dimensional constitutive model for cohesive soils within the framework of critical state soil mechanics. The present work focuses on the response of the above model under cyclic loading conditions. It is shown mainly qualitatively and partially quantitatively, that the model predicts in detailed form a material response which does agree with observed experimental behavior under undrained and drained loading conditions at any overconsolidation ratio and for different cyclic deviatoric stress amplitudes.

INTRODUCTION

A common weakness of many stress-strain laws in soil mechanics is that they are pertinent to loading conditions of a very specific nature. If, however, the soil constitutive relations are to be of value for the analysis of earth structures under complex and interchangeable loadings, they must be equally applicable to monotonic or cyclic, drained or undrained or any other form of loading conditions, which necessitates their development within a more fundamental framework.

The classical mathematical theory of plasticity provides such a framework, but still some very important aspects of soil behavior such as the response to cyclic loading cannot be adequately described. The main reason for this deficiency is that plastic irreversible deformation cannot occur within the yield surface, which defines a purely elastic range of the material response, contrary to the observed behavior. For example, consider the cyclic deviatoric loading with fixed deviatoric stress amplitude under undrained conditions. A classical yield surface plasticity model will predict a pore-water pressure built-up only during the first half cycle as a result of the interchange between elastic and plastic volumetric strain, while the total volumetric strain remains constant. Subsequently, the cyclic stress oscillates within the expanded yield surface unable to cause any additional plastic strain and, therefore, any additional pore-water pressure, contrary to the observed experimental fact. If such important phenomena of the cyclic soil response are to be modeled, new concepts must supplement the classical approach.

The Bounding Surface in stress space (henceforth referred as B.S. for abbreviation) represents such a new concept; it was originally developed for cyclic metal plasticity by Dafalias (1975), Dafalias and Popov (1974,1975,1976) and independently by Krieg (1975). Its salient and novel feature is that plastic deformation can occur for a stress state within the surface by rendering the plastic modulus an increasing function of the "distance" between the actual stress point and an "image" stress point on the B.S., defined by a proper mapping rule. When the actual stress point reaches the B.S. (the "distance" is zero), it becomes indentical with its "image". Then, the B.S. plays the role of a classical yield surface. Upon unloading-reloading the plastic response is always defined by the above "distance" dependent value of the plastic modulus for any point within or on the B.S. A corresponding constitutive model for clays has been already formulated within the framework of critical state soil mechanics in a series of papers by Dafalias (1979a,1979b), Dafalias and Herrmann (1980a,in press,1980b) with the most recent formulation (1980b) being expressed in terms of the three stress invariants for general three dimensional loading. The model has been numerically implemented in computer codes. Its application to monotonic, drained and undrained, normally consolidated, lightly and heavily overconsolidated states of loading has shown very good agreement with corresponding experimental data.

This presentation focuses on the predictive capabilities of the same bounding surface model under cyclic loading conditions. More specifically, the model can predict:

a) The cyclic positive pore-water pressure built-up, axial strain accumulation and reduction of effective stress under undrained cyclic loading, and the cyclic volumetric and deviatoric strain accumulation under drained cyclic loading in compression, extension or both, for normally consolidated or lightly overconsolidated states.
b) The negative pore-water pressure build-up (undrained), dilative volumetric strain accumulation (drained), and axial strain accumulation under cyclic loading of heavily overconsolidated states.
c) The stabilization of the cyclic stress-strain loops if the cyclic deviatoric amplitude is small, or the progressive evolution of the material state towards the critical state, where failure is imminent, if the amplitude is large.

It is important to emphasize that the model predicts a detailed stress-strain history response as cyclic loading is applied, in contrast to an overall estimation of strain accumulation or pore-water pressure built-up versus number of cycles which traditionally, and most often empirically, has been used in soil dynamics. Cyclic loading is considered as nothing else but a sequence of monotonic steps for which the model has been proved so successful. This success is mainly due to the capability of the bounding surface formulation to describe realistically the material response at any overconsolidation ratio, combined with the fact that any cyclic loading brings normally consolidated samples to overconsolidated states. Most predictions are qualitative due to the lack of corresponding detailed experimental data. Particular emphasis is given to the concept of the "Elastic Nucleus" and the associated stabilization factor s which controls the development of cyclic response up to full stabilization before failure, if necessary. A few new material parameters are introduced, in addition to those of the critical state soil mechanics, which can be easily calibrated from conventional triaxial experiments. Future trends and improvements are finally discussed.

BRIEF PRESENTATION OF THE MODEL

Subsequently, effective stress components σ_{ij} are considered which are taken positive if compressive. A bar over a stress quantity implies a state on the bounding surface. The strain ε_{ij} is decomposed into an elastic and a plastic part, indicated by one and two primes, respectively. A dot indicates rate and the summation convention over repeated indices is employed. The deviatoric stress and strain components are denoted by s_{ij} and e_{ij}, respectively. For isotropic material, the elastic stress-strain rate relations are given as usual in terms of the bulk modulus K and the shear modulus G (or Poisson's ratio ν). The B.S. equation depends on the plastic rate of change of the total void ratio given by $\dot{e}''=-(1+e_o)\dot{\varepsilon}''_{kk}$ (e_o is the void ratio of the reference configuration), and three stress invariants defined by:

$$I = \sigma_{kk}, \quad J = (\tfrac{1}{2} s_{ij}s_{ij})^{1/2}, \quad S = (\tfrac{1}{3} s_{ij}s_{jk}s_{ki})^{1/3} \tag{1}$$

Instead of S, it is convenient to introduce the "Lode" angle α:

$$-\frac{\pi}{6} \le \alpha = \frac{1}{3}\sin^{-1}\left[\frac{3\sqrt{3}}{2}\left(\frac{S}{J}\right)^3\right] \le \frac{\pi}{6} \tag{2}$$

where $\alpha = \pm\pi/6$ corresponds to triaxial compression and extension, respectively.

Assuming that the origin lies always within a convex B.S., Dafalias (1979b) introduced a simple "radial" mapping rule defining the "image" stress point as the intersection of the B.S. with the straight line connecting the origin with the actual stress point. This can be expressed analytically by $\bar{\sigma}_{ij} = \beta(\sigma_{k\ell}, e'')\sigma_{ij}$ with the radial factor $1 \le \beta \le \infty$ determined from the equation of the B.S.:

$$F(\bar{I}, \bar{J}, \alpha, e'') = 0 \tag{3}$$

where we have $\bar{I} = \beta I$, $\bar{J} = \beta J$, $\bar{S} = \beta S$ and $\bar{\alpha} = \alpha$.

The plastic strain rate is now given by:

$$\dot{\varepsilon}''_{ij} = \langle L \rangle \frac{\partial F}{\partial \bar{\sigma}_{ij}}, \quad L = \frac{1}{K_p}\frac{\partial F}{\partial \bar{\sigma}_{k\ell}}\dot{\sigma}_{k\ell} = \frac{1}{\bar{K}_p}\frac{\partial F}{\partial \bar{\sigma}_{k\ell}}\dot{\bar{\sigma}}_{k\ell} \tag{4}$$

where K_p is the plastic modulus associated with the actual stress rate and $\bar{K}_p$ is the "bounding" plastic modulus associated with the "image" stress rate. The $\langle \ \rangle$ denotes the operation $\langle * \rangle = * \bar{H}(*)$ with $\bar{H}$ the heavyside step function. Plastic loading, unloading and neutral loading is defined by $L > 0$, $L < 0$ and $L = 0$ respectively, with L called the loading function. The inclusion of K_p or $\bar{K}_p$ in L treats simultaneously stable (hardening) response when the moduli are positive and unstable (softening) response when they are non-positive.

From the consistency condition $\dot{F} = 0$ one has:

$$\bar{K}_p = 3(1 + e_o)\ (\partial F/\partial e'')\ F_{,\bar{I}} \tag{5}$$

where a comma followed by the symbol of an invariant as a subscript indicates partial differentiation with respect to this invariant. Observe from Eq. (5) that with $\partial F/\partial e'' > 0$, the bounding plastic modulus $\bar{K}_p$ is positive (consolidation), negative (dilatation), or zero (unrestricted shear flow), according to the value of $F_{,\bar{I}}$. Correspondingly, the bounding surface expands, contracts, or does not harden.

The functional dependence of K_p on $\bar{K}_p$ and the distance δ between σ_{ij} and $\bar{\sigma}_{ij}$ in invariant stress space is assumed to be given by:

$$K_p = \bar{K}_p + H\frac{\delta}{\langle \bar{r} - s\delta \rangle} \tag{6}$$

where H is a proper material hardening shape function, $\bar{r}$ is the distance between the "image" stress point and the origin and s is a material constant called the "stabilization factor". When $\delta = 0 \rightarrow K_p = \bar{K}_p$ and for all $\delta \ge \bar{r}/s$ the $\langle \ \rangle$ becomes zero rendering $K_p = \infty$, thus defining indirectly a purely elastic domain within the B.S. which is called the "Elastic Nucleus" or E.N. for abbreviation. The E.N. is congruent to the B.S. and its size depends on s. For s = 1 the E.N. degenerates into a point, the origin. It must be emphasized that the concept of the E.N. as introduced by Dafalias (1980) and Dafalias and Herrmann (1980a) is not equivalent to that of a yield surface (no consistency condition required etc.) and the stress point can always cross the E.N. and move outside with a smooth elasto-plastic transition. The value of s is very important in relation to the cyclic response as it will be seen shortly. Undrained behavior can be obtained on the basis of the above equations by imposing the internal constraint $\dot{\varepsilon}_{kk} = 0$ on the total volumetric strain rate.

THE BOUNDING SURFACE

Extending the ideas of the critical state soil mechanics (Schofield and Wroth (1968)) from the triaxial to the invariant stress space, a meridional section of the bounding surface is eloquently shown in Fig. 1. The quantity N can be identified as the slope of the projection CSL of the critical state line in invariant stress space which intersects the bounding surface at C where $(\partial F/\partial \bar{I}) = 0$. For triaxial conditions N is related to the triaxial CSL slope M by $M = 3\sqrt{3}$ N. The radial rule associating the "image" stress point $\bar{I}$, $\bar{J}$ to the stress point I, J is illustrated. The projection of C on the I axis is denoted by I_1 and is given by $I_1 = I_o/R$ with I_o being the intersection of $F = 0$ with the hydrostatic axis I. The dependence of $F = 0$ on the third stress invariant is introduced through N, which is assumed to be a proper function of α (Dafalias and Herrmann (1980b)). The functional dependence of N on α requires the determination of the values of N in triaxial compression: N_c, and extension: N_e. The dependence of $F = 0$ on e'' is introduced through I_o, where $dI_o/de'' = -I_o/(\lambda - \omega\kappa)$, $\omega = I_o/(\langle I_o - I_\ell \rangle + I_\ell)$ with λ, κ representing the slopes of the normal consolidation and swelling-rebound lines in the e - ℓnp plot and I_ℓ the value of I = 3p at which this logarithmic relation changes to linear to avoid excessive elastic stiffness softening around I = 0.

For further reference let us introduce the quantities $\theta = J/I$ and $x = \theta/N$. For $0 \le \theta \le N$ the bounding surface is the ellipse 1, Fig. 1, defined by means of a single parameter R. Extension of the ellipse in the range $N \le \theta \le \infty$ did not yield satisfactory results for heavily overconsolidated states. Instead, a hyperbola is proposed, as shown in Fig. 1, defined by means of the parameter A, which positions the hyperbolic asymptote with respect to CSL by means of $D = AI_o$. A varies with α in the same way

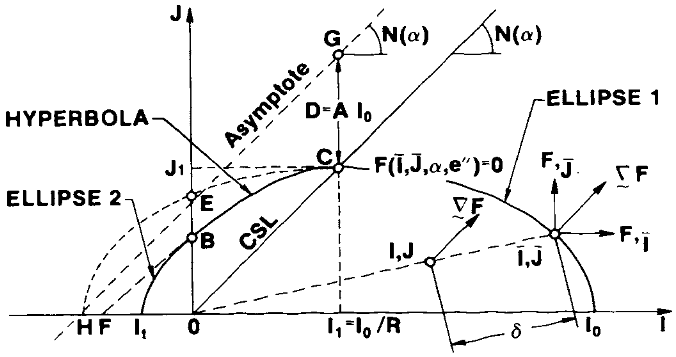

Fig. 1. Schematic illustration of the Bounding Surface and the "radial" mapping rule in invariant stress space.

as N does, with A_e and A_c being the values of A in triaxial extension and compression, respectively. Finally, extension in the tension regime for $-\infty < \theta \leq 0$ is obtained by an ellipse 2, Fig. 1, as discussed by Dafalias and Herrmann (1980a). The intersection I_t of ellipse 2 with the I axis measures the tensile strength of the soil.

Finally, the form of the shape hardening function H entering Eq. (6) is given by:

$$H = p_a \, h(\alpha) \left[1 + |x|^{-m}\right] \left[9 \, F_{,\bar{I}}^2 + \frac{1}{3} F_{,\bar{J}}^2\right] \tag{7}$$

where p_a is the atmospheric pressure providing the proper stress units, $m = 0.2$ is satisfactory for most clays rendering $K_p = \infty$ when $J = 0$ except when $\delta = 0$. The second bracket is introduced in association with a unit normal formulation in the triaxial space. The shape hardening parameter h is the most important and can be considered a function of α.

CYCLIC RESPONSE

Subsequently a sequence of different cyclic loading histories in triaxial compression and extension is applied, and the soil response according to the B.S. constitutive model is eloquently shown. For all histories the following material constants are used: $\lambda = 0.055$, $\kappa = 0.02$, $\nu = 0.3$, $M_c = 3\sqrt{3}N_c = 1.11$, $M_e = 3\sqrt{3}N_e = 1.0$, $R = 2$, $A_c = A_e = 0.10$, $m = 0.20$, $h = 30$, and $s = 1, 1.5$. The initial void ratio was taken to be $e_o = 1.6$. In order to appreciate the importance of the parameter s, consider first the response under undrained cyclic deviatoric loading in triaxial compression of a normally consolidated sample at $p_o = 57$ psi. The response for 14 cycles is shown with dashed lines for $s = 1$ and with solid lines for $s = 1.5$ in Figs. 2a, b, c, for the same amplitude of the deviatoric stress q. Observe that the undrained stress path loops in the p - q space move towards the CSL as the pore-water pressure u increases. A classical yield surface plasticity model would have only shown the first loop with immediate stabilization. If $s = 1$, the progress towards the CSL will continue for any amplitude of q. Recent experiments by Sangrey et al (1969) showed that this is not true and that the amplitude of q plays an important role. For small amplitude full stabilization

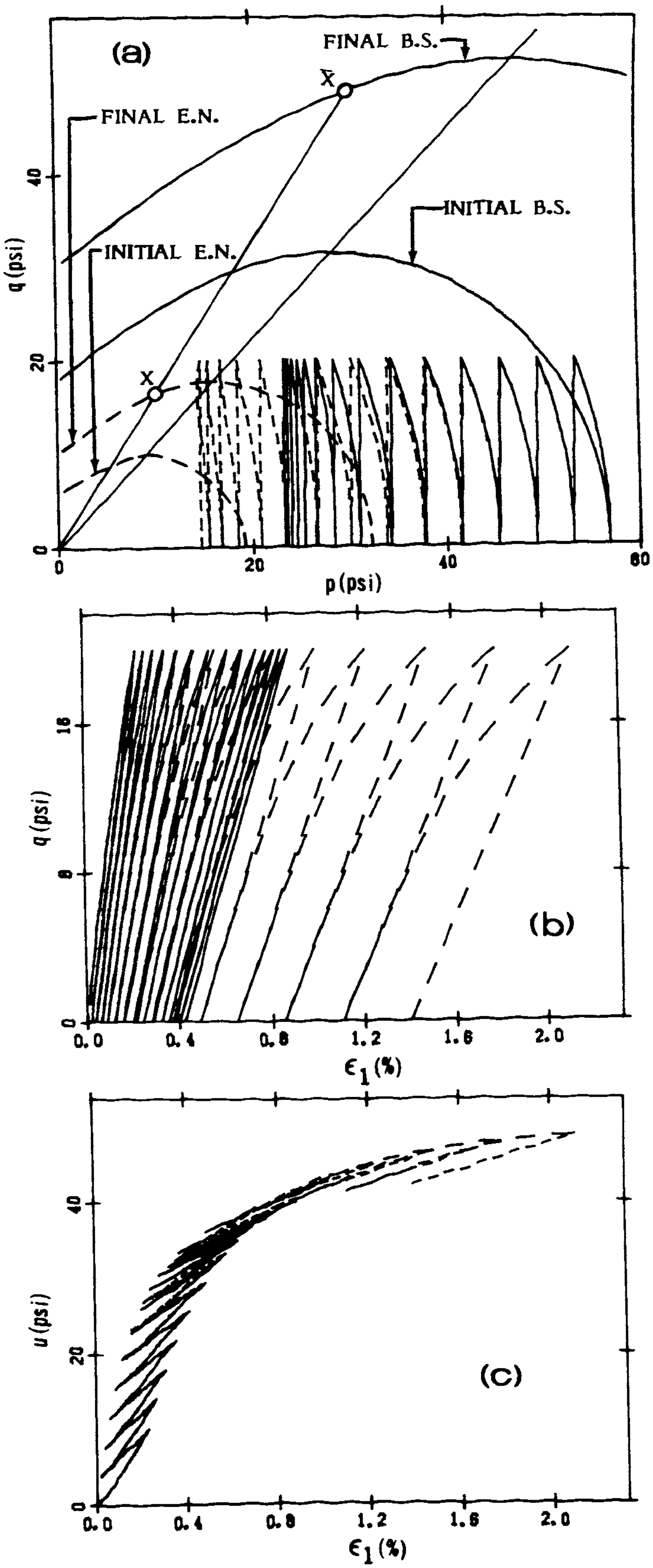

Fig. 2a,b,c. Cyclic undrained response for s = 1 (dashed line) and s = 1.5 (solid line) showing the effect of the Elastic Nucleus.

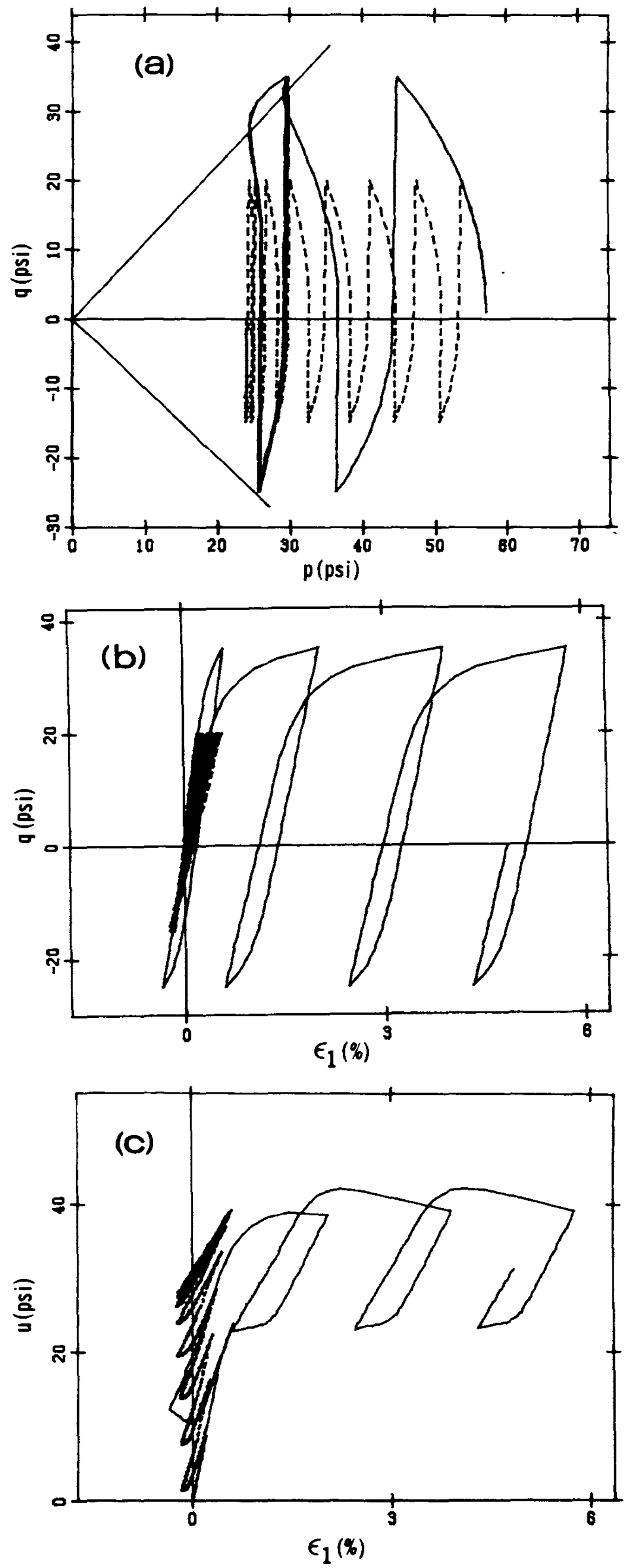

Fig. 3a,b,c. Cyclic undrained response of normally consolidated sample for s = 1.5 and two amplitudes of q.

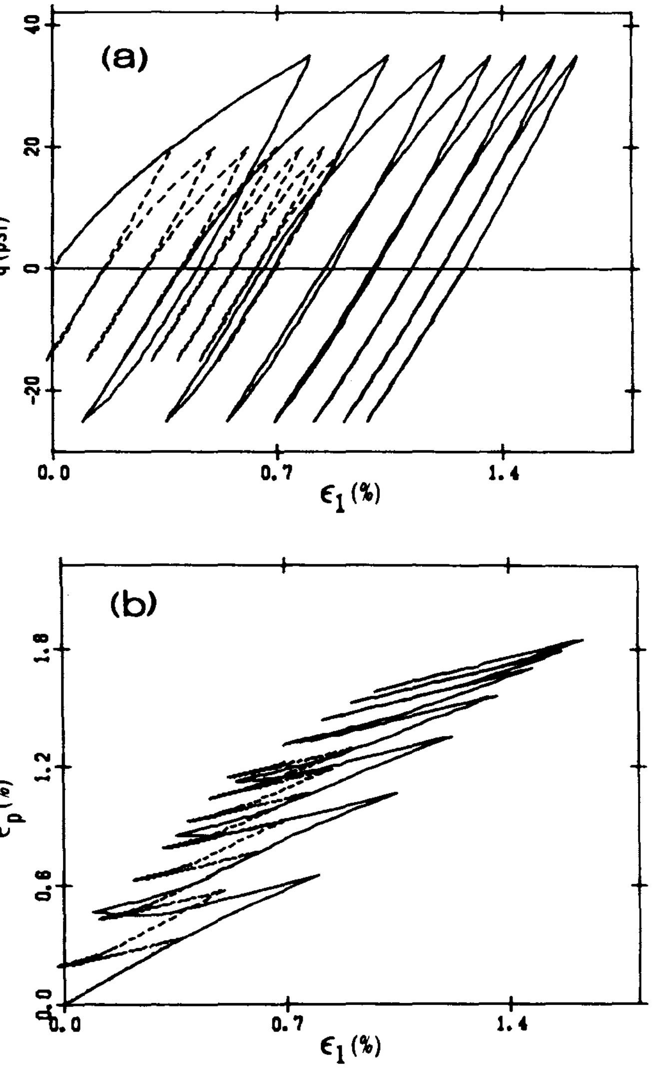

Fig. 4a,b. Cyclic drained response of normally consolidated sample for s = 1.5 and two amplitudes of q.

occurs before reaching the CSL which implies, in terms of the present formulation, that a fully elastic range has been developed. This is the Elastic Nucleus which is defined by means of s > 1. The initial and final positions of the B.S. and E.N. are shown in Fig. 2a for s = 1.5. Point X is such that $\delta = X\overline{X} = \overline{r}/s = 0\overline{X}/1.5$. It is evident that as the cyclic p - q loops enter progressively the E.N. fully elastic response is assumed and full stabilization is obtained (hence the name of s : stabilization factor) as shown by the solid lines in Figs. 2a,b,c.

A similar senario is repeated in the following. But now instead of using two values of s and the same q amplitude, the value s = 1.5 is fixed and comparison is made between responses for two cyclic loadings of different q amplitudes in both compression and extension. The larger q amplitude response is shown by a solid line and the smaller by a dashed. The smaller q amplitude always shows a tendency for full and faster stabilization as the stress moves inside the E.N., more so than for the larger q. Figs. 3a, b, c show the response under undrained conditions and Figs. 4a, b under drained for a normally consolidated soil at p_o = 57 psi. (ε_p represents volumetric strain).

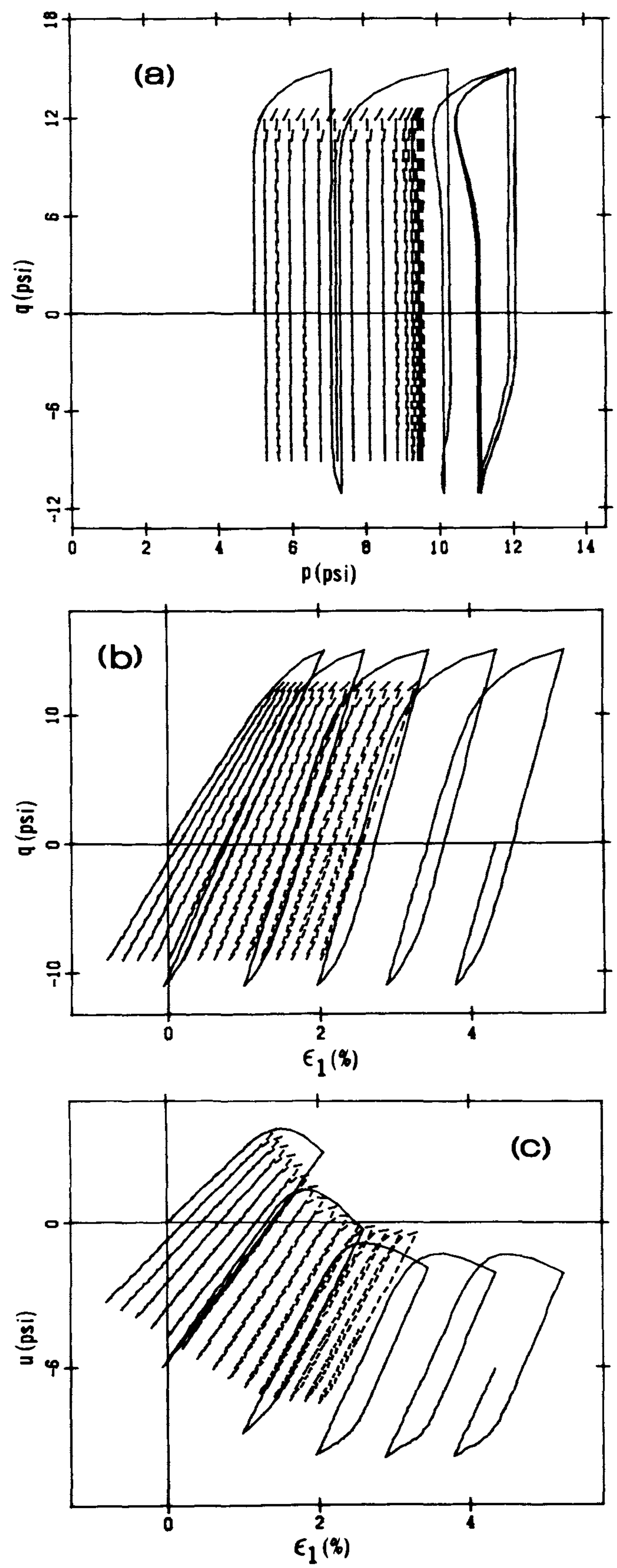

Fig. 5a,b,c. Cyclic undrained response of heavily overconsolidated sample for s = 1.5 and two amplitudes of q.

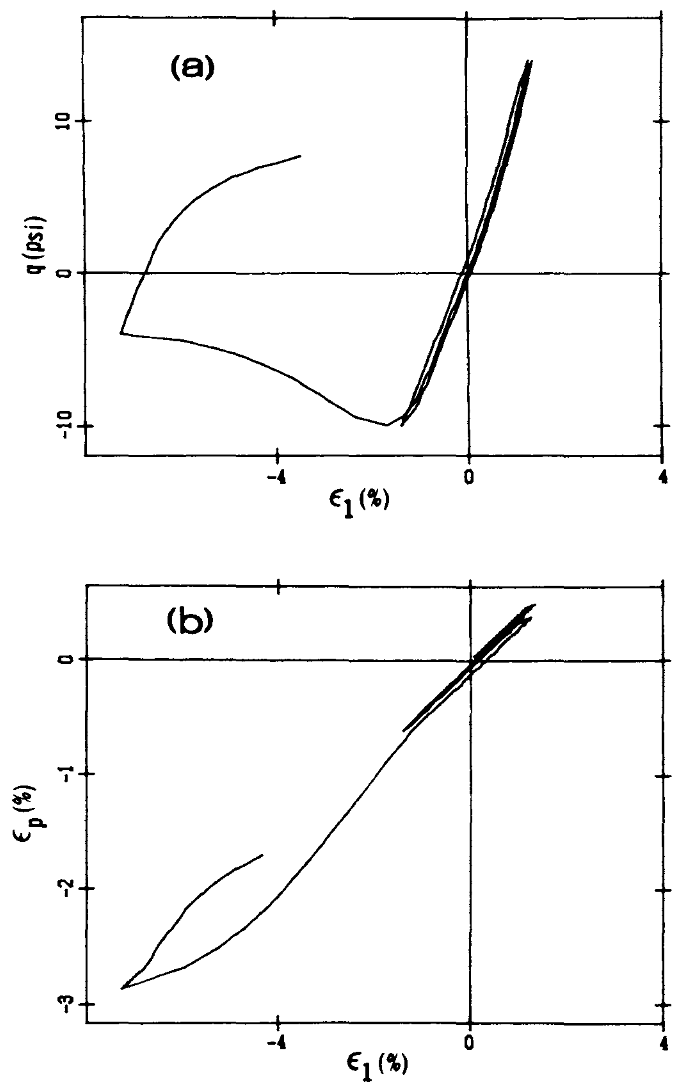

Fig. 6a,b. Cyclic drained response of heavily overconsolidated sample for s = 1.5 and one amplitude of q.

Figs. 5a, b, c show the results of cyclically loading under undrained conditions a heavily overconsolidated sample, with OCR = 11.40 and preconsolidation at p_o = 57 psi. Observe the negative cyclic pore-water pressure development and the stabilization for the smaller q amplitude again. Finally Figs. 6a, b show the response at OCR = 11.40 under drained conditions for one only q amplitude bringing the stress path beyond the CSL in both compression and extension and indicating after 2 cycles a large dilation and critical failure with large ε_1. It is interesting to note, for the normally consolidated sample, that although the stress reaches the CSL, the model shows a failure due to progressive accumulation of the strain ε_1 rather than an abrupt critical failure. The latter can be achieved only if the q amplitude is increased at the end of the cyclic loading.

Finally, a comparison with an actual experiment is shown in Fig. 7, with the material constants reported on this figure. The experimental data are taken from Wroth and Loudon (1967) and provide only the p - q undrained cyclic stress path. The stabilization factor s was taken equal to 1.

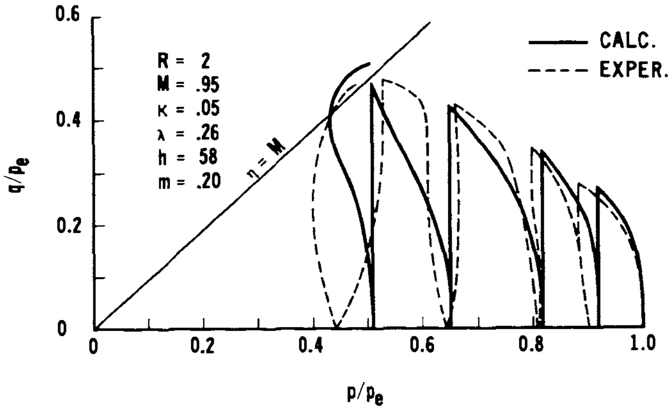

Fig. 7. Theory versus experiments for undrained cyclic loading. Experimental data from Wroth and Loudon (1967).

CONCLUSION

This presentation focuses on the cyclic response of cohesive soils, as predicted by a general bounding surface plastic constitutive model within the framework of critical state soil mechanics, which has been already proved very successful in predicting the soil response under monotonic loading. A cyclic loading is treated as a sequence of monotonic ones, which brings a normally consolidated state towards increasing overconsolidation and a heavily overconsolidated state towards decreasing overconsolidation within the bounding surface. The material response is obtained in detailed form during the cyclic loading. The concepts of the elastic nucleus and the stabilization factor s are defined and shown to play an important role in the stabilization of cyclic processes associated with different deviatoric stress amplitudes. Important features of the model are its fundamental character, the ease of its numerical implementation and its calibration in terms of the results of conventional triaxial experiments.

Because the model is equally valid for both cyclic and monotonic loading conditions and has been formulated for general three-dimensional behavior, it provides a powerful tool for characterizing the constitutive behavior of cohesive soils for stress analysis purposes. The predicted results do agree with qualitative observation of cyclic behavior of cohesive soils. Detailed experimental data are lacking, which are necessary to assert the quantitative cyclic predictive capability of the model and allow a further investigation of the cyclic parameter s (stabilization factor), especially its relation to the amplitude of q and its dependence on the past cyclic loading history as described by the accumulated volumetric and deviatoric plastic strains.

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