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Asymptotic Stability of Nonholonomic Mobile Robot Formations using Multilayer Neural Networks[1]

Travis Dierks and S. Jagannathan

Abstract**—*In this paper, a combined kinematic/torque control law is developed for leader-follower based formation control using backstepping in order to accommodate the dynamics of the robots and the formation in contrast with kinematic-based formation controllers that are widely reported in the literature. A multilayer neural network (NN) is introduced along with robust integral of the sign of the error (RISE) feedback to approximate the dynamics of the follower as well as its leader using online weight tuning. It is shown using Lyapunov theory that the errors for the entire formation are asymptotically stable and the NN weights are bounded as opposed to uniformly ultimately bounded (UUB) stability which is typical with most NN controllers. Simulation results are included.*

Index Terms —Neural network, formation control, Lyapunov method, kinematic/dynamic controller, RISE

I. Introduction

Over the past decade, the attention has shifted from the control of a single nonholonomic mobile robot [1] to the control of multiple mobile robots because of the advantages a team of robots offer for complex tasks like search and rescue operations, mapping unknown or hazardous environments, security and bomb sniffing besides increased efficiency.

Many formation control papers using kinematic controllers have appeared recently. However perfect velocity tracking assumption is used and the individual robot and the formation dynamics are ignored. Therefore, in [2], the follower robot dynamics are considered using a neural network (NN), however; the formation dynamics are ignored.

In this paper, the frame work developed for controlling single nonholonomic mobile robots is expanded to leader follower formation control by incorporating the dynamics of the robots as well as the formation in the controller design. Thus, the dynamical extension introduced here provides a more rigorous method of taking into account the specific vehicle dynamics to convert a steering system command into control inputs via backstepping. Both velocity feedback control inputs and velocity tracking control laws are presented to prove the formation is *asymptotically stable*. A multilayer NN is introduced to learn the dynamics of the each follower robot and its leader online, and is combined with a recently developed robust integral of sign of the error (RISE) feedback method originating in [4]. The asymptotic stability of the entire formation as well as the boundedness of the weights of the followers' NNs and the leader's NN is demonstrated using Lyapunov methods as opposed to uniform ultimately boundedness (UUB), a result common in the NN controls literature [2][3].

The RISE method [4] is designed to reject bounded disturbances, unmodeled dynamics, and NN functional reconstruction errors to yield asymptotic tracking. To accommodate the RISE technique, the NN must be constructed using desired trajectory, which is similar to the DCAL-based NN scheme [5]. An approach to blend a multilayer NN with RISE feedback for a single rigid robot control is taken in [6]. Boundedness of the actual NN weights is shown separately using projection algorithm and convergence of the tracking errors is then demonstrated by using constant controller gains. Selection of the predefined convex set for the projection algorithm both to prevent the NN weights from diverging and ensuring the initial weights be a part of the set is a challenging task.

By contrast, in this work the constant bounds and gains in [6] are replaced for formation control with time varying functions allowing bounds and gains to be determined with more certainty, and a novel weight tuning is used [6]. An additional advantage of using the proposed NN weight tuning as opposed to the projection algorithm is less decision making in the NN learning process, which can lead to reduced system delays. Further, Lyapunov analysis is presented to show the asymptotic convergence of the tracking errors and boundedness of the NN weights simultaneously. Finally, the bounds and gains developed here also applicable to single rigid robot control [6] besides formation control. No offline training is needed for the NNs. Simulation results are provided to demonstrate the theoretical results.

II. Leader-Follower Formation Control

The goal of separation-bearing formation control is to find a velocity control input such that [5]

$$\lim_{t\to\infty}(L_{ijd}-L_{ij})=0 \text{ and } \lim_{t\to\infty}(\Psi_{ijd}-\Psi_{ij})=0 \quad (1)$$

where L_{ij} and ψ_{ij} are the measured separation and bearing of the follower robot, and L_{ijd} and ψ_{ijd} represent desired distance and angles respectively [5]. To avoid collisions,

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separation distances are measured from the back of the leader to the front of the follower. The kinematic equations for the front of the j^{th} follower robot can be written as

$$\begin{bmatrix}\dot{x}_j\\ \dot{y}_j\\ \dot{\theta}_j\end{bmatrix}=\begin{bmatrix}\cos\theta_j & -d_j\sin\theta_j\\ \sin\theta_j & d_j\cos\theta_j\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_j\\ \omega_j\end{bmatrix} \tag{2}$$

where d_j is the distance from the rear axle to the to front of the robot, x_j, y_j, and θ_j are actual Cartesian position and orientation of the physical robot, and v_j, and ω_j are linear and angular velocities, respectively. Many robotic systems can be characterized as a robotic system having an n-dimensional configuration space $\mathcal{C}$ with generalized coordinates $(q_1,...q_n)$ and subject to m constraints described in detail in [1] and mathematically after applying the transformation described in [1] as

$$\overline{M}_j(q_j)\dot{v}_j+\overline{V}_{m_j}(q_j,\dot{q}_j)v_j+\overline{F}_j(v_j)+\overline{\tau}_{d_j}=\overline{B}_j(q_j)\tau_j \tag{3}$$

where $\overline{M}_j\in\Re^{rxr}$ is a symmetric positive definite inertia matrix, $\overline{V}_{mj}\in\Re^{rxr}$ is the centripetal and coriolis matrix, $\overline{F}_j\in\Re^{rx1}$ is the friction vector, $\overline{\tau}_{dj}$ represents unknown bounded disturbances, $\overline{\tau}_j=\overline{B}_j\tau\in\Re^{rx1}$ is the input vector, and $v_j=\begin{bmatrix}v_j & \omega_j\end{bmatrix}^T\in\Re^{rx1}$. The robotic system (3) satisfies the following properties:

1. Boundedness[6]: $\overline{M}_j$, the norm of $\overline{V}_{mj}$, and τ_{dj} are all bounded. Furthermore, $\forall y(t)\in\Re^r$, $\overline{m}_1\|y\|^2\le y^T\overline{M}_j(q_j)y\le\overline{m}_2(q_j)\|y\|^2$ where $\overline{m}_1$ is a known real positive constant, $\overline{m}_2(q_j)$ is a known real positive function, and $\|\bullet\|$ is the Frobenius vector norm [3].

A. Backstepping Design:

The description of the behavior of a mobile robot is described by (2) and (3). Standard approaches [2-5] to leader follower formation control deal only with (2) and ignore dynamics (3). To incorporate the dynamics of the mobile platform, it is desirable to convert a control velocity $v_{jc}(t)$ into a control torque, $\tau_j(t)$ for the physical robot. In our previous work [7], the dynamics of the robots and the formation are assumed to be known accurately. By contrast, our aim in this paper is to design augmented NN/RISE based torque controller such that (2) and (3) exhibit the desired behavior for a given control velocity $v_{jc}(t)$ thus removing perfect velocity tracking and relaxing that the dynamics are known.

B. Multilayer Neural Networks

A multilayer NN is considered here consisting of tunable weights $V\in\Re^{axL}$ in the input layer and tunable weights $W\in\Re^{Lxb}$ in the output layer with a inputs, b outputs, and L hidden neurons. The *universal approximation property* for NN's [19] states that for any smooth function $f(x)$, there exists a NN such that $f(x)=W^T\sigma(V^Tx)+\varepsilon$ where ε is the NN functional approximation error and $\sigma(\cdot):\Re^a\to\Re^L$ is the activation function in the hidden layers. The sigmoid activation function is considered here. For complete details of the NN and its properties, see [19].

Remark: $\|\cdot\|$ and $\|\cdot\|_F$ will be used interchangeably as the Frobenius vector and matrix norms [3].

To aid in future analysis, define the hidden layer output error for a given x as [3]

$$\tilde{\sigma}=\sigma-\hat{\sigma}\equiv\sigma(V^Tx)-\sigma(\hat{V}^Tx) \tag{4}$$

Then, using the Taylor series expansion for $\sigma(V^Tx)$ [3], (4) can be written as

$$\tilde{\sigma}=\sigma'(\hat{V}^Tx)\tilde{V}^Tx+O(\tilde{V}^Tx)^2=\hat{\sigma}'\tilde{V}^Tx+O(\tilde{V}^Tx)^2 \tag{5}$$

where
$$\sigma'(\hat{z})\equiv\left.\frac{\partial\sigma(z)}{\partial z}\right|_{z=\hat{z}}=\sigma(1-\sigma) \tag{6}$$

and
$$O(\tilde{V}^Tx)^2=[\sigma(V^Tx)-\sigma(\hat{V}^Tx)]-\hat{\sigma}'\tilde{V}^Tx. \tag{7}$$

The following mild assumptions will be used.

Assumption 1. Follower j is equipped with sensors capable of measuring the separation distance L_{ij} and bearing ψ_{ij} and both leader and follower are equipped with instrumentation to measure their linear and angular velocities as well as their orientations θ_i and θ_j.

Assumption 2. Wireless communication is available between follower j and leader i with communication delays being zero.

Assumption 3. Leader i communicates its linear and angular velocities v_i, ω_i as well as its orientation θ_i and control torque τ_i to its followers at each sampling instant.

Assumption 4. For the nonholonomic system of (2) and (3) with n generalized coordinates q, m independent constraints, and r actuators, the number of actuators is equal to the number of degrees of freedom ($r=n-m$).

Assumption 5. The reference linear and angular velocities measured from the leader i are bounded and $v_{jr}(t)\ge 0$ for all t.

Assumption 6. Let perfect velocity tracking hold such that $v_j=v_{jc}$ and $\dot{v}_j=\dot{v}_{jc}$ (this assumption is relaxed later).

Assumption 7. On any compact subset of $\Re^n$, the ideal NN weights are bounded by known positive values for all followers $j=1,2,...N$ and leader i such that $\|V_{i,j}\|_F\le V_M$ and $\|W_{i,j}\|_F\le W_M$ [3]. Furthermore, augmented weight matrices can be defined such that

$$Z_{i,j}\equiv\begin{bmatrix}W_{i,j} & 0\\ 0 & V_{i,j}\end{bmatrix} \text{ and } \hat{Z}_{i,j}\equiv\begin{bmatrix}\hat{W}_{i,j} & 0\\ 0 & \hat{V}_{i,j}\end{bmatrix}$$

and $\|Z_{i,j}\|_F\le Z_M$.

Assumption 8. Let the NN approximation property hold for a function $f(x)$ with accuracy ε_N for all followers j and leader i and for all $x_{di,j}$ in the compact set S [19] such that $\|\varepsilon_{i,j}\| < \varepsilon_N$, $j = 1,2,...N$. Furthermore, let $\|\dot{\varepsilon}_{i,j}\| < \varepsilon'_N$ and the disturbances and their derivatives be bounded such that $\|\bar{\tau}_{di,j}\| \le d_M$ and $\|[\dot{\bar{\tau}}_{di,j} \quad \ddot{\bar{\tau}}_{di,j}]\| \le d'_M$ [1][6].

Assumption 9. The formation leader follows no physical robots, but follows the virtual leader described in [1]. Furthermore, the virtual leader's velocity is defined by a time varying function that is twice differentiable.

Assumption 10. The formation leader is capable of measuring its absolute position via instrumentation like GPS so that tracking the virtual robot is possible.

Remark: It should be noted that the *Assumptions 1-10* are standard in single robot as well as formation control literature.

C. Kinematic Controller

Consider the two robot formation depicted in Fig. 1. In our previous work [7], we found the error dynamics for follower j to be

$$e_j = \begin{bmatrix} e_{j1} \\ e_{j2} \\ e_{j3} \end{bmatrix} = \begin{bmatrix} L_{ijd}\cos(\Psi_{ijd} + e_{j3}) - L_{ij}\cos(\Psi_{ij} + e_{j3}) \\ L_{ijd}\sin(\Psi_{ijd} + e_{j3}) - L_{ij}\sin(\Psi_{ij} + e_{j3}) \\ \theta_r - \theta_j \end{bmatrix} \quad (8)$$

and its derivative as

$$\dot{e}_j = \begin{bmatrix} \dot{e}_{j1} \\ \dot{e}_{j2} \\ \dot{e}_{j3} \end{bmatrix} = \begin{bmatrix} -v_j + v_i\cos e_3 + \omega_j e_2 - \omega_i L_{ijd}\sin(\Psi_{ijd} + e_3) \\ -\omega_j e_2 + v_i\sin e_3 - d_j\omega_j + \omega_i L_{ijd}\cos(\Psi_{ijd} + e_3) \\ \omega_i - \omega_j \end{bmatrix} \quad (9)$$

The following velocity control inputs were proposed [7] for follower robot j to achieve the desired position and orientation with respect to leader i as

$$v_{jc} = \begin{bmatrix} v_{jc} \\ \omega_{jc} \end{bmatrix} = \begin{bmatrix} v_i\cos e_{j3} + k_1 e_{j1} - \omega_i L_{ijd}\sin(\Psi_{ijd} + e_{j3}) \\ \omega_i + (v_i + k_v)k_2 e_{j2} + (v_i + k_v)k_3\sin e_{j3} + \gamma_{wjc} \end{bmatrix} \quad (10)$$

where
$$\gamma_{\omega jc} = -\frac{|e_{j2}|\left(\omega_i(d_j + L_{ijd}) + (v_i + k_v)k_3 d_j + k_v\right)}{1/k_2 + |e_{j2}|d_j} \quad (11)$$

Theorem 1 [7]*:* Given the nonholonomic system of (2) and (3) with *n* generalized coordinates *q, m* independent constraints, and *r* actuators, along with the leader follower criterion of (1), let *Assumptions 1* through *6* hold. Let a smooth velocity control input v_{jc} for the follower j given by (10) and (11). Then there exists a vector of positive constants $K = [k_1 \quad k_2 \quad k_3]^T$ such that the origin $e_j = 0$ consisting of the position and orientation error for the follower is asymptotically stable.

Proof: Consider the following Lyapunov function candidate

$$V_j = \frac{1}{2}(e_{j1}^2 + e_{j2}^2) + \frac{1 - \cos e_{j3}}{k_2} \quad (12)$$

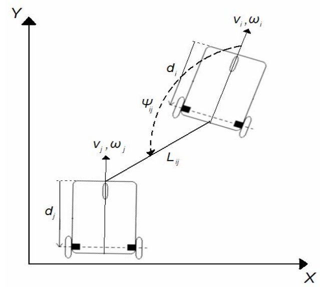

Fig. 1: Leader-follower formation control

It is shown in [7] that the velocity control (10) provides asymptotic stability for the error system (8) and (9).

D. Dynamical NN/RISE Controller

Now assume that the perfect velocity tracking assumption does not hold making *Assumption 6* invalid. Define velocity tracking and filtered tracking errors as

$$e_{jc} = v_{jc} - v_j \quad (13)$$

$$r_j = \dot{e}_{jc} + \alpha_j(t)e_{jc} \quad (14)$$

where $\alpha_j(t)$ is a real time varying function greater than zero defined as $\alpha_j(t) = \alpha_{j0} + \alpha_{j1}(t)$ where α_{j0} is a constant and $\alpha_{j1}(t)$ is a time varying term. Multiplying both sides of (14) by $\bar{M}_j$, adding and subtracting $\bar{V}_{m_j}v_{jc}$ and $\bar{F}_j(v_{jc})$, and substituting the robot dynamics (3) allows (14) to be rewritten as

$$\bar{M}_j r_j = f_{d_j} + T_j + \bar{\tau}_{d_j} - \bar{\tau}_j \quad (15)$$

where
$$f_{d_j} = \bar{M}_j\dot{v}_{jc} + \bar{V}_{m_j}v_{jc} + \bar{F}_j(v_{jc}) \quad (16)$$

and
$$T_j = e_{jc}(\alpha_j(t)\bar{M}_j - \bar{V}_{m_j}) + \bar{F}_j(v_j) - \bar{F}_j(v_{jc}) \quad (17)$$

Differentiating (17) then yields

$$\bar{M}_j\dot{r}_j = -\dot{\bar{M}}_j r_j + \dot{f}_{d_j} + \dot{T}_j + \dot{\bar{\tau}}_{d_j} - \dot{\bar{\tau}}_j \quad (18)$$

Defining the control torque as in [6] to be

$$\bar{\tau}_j = \hat{f}_{d_j} + \mu_j \quad (19)$$

where $\hat{f}_{d_j}$ is the estimate of f_{d_j} and μ_j is the RISE feedback term defined similarly to [4] and [6] as

$$\mu_j = (k_s+1)e_{jc}(t) - (k_s+1)e_{jc}(0) + \int_0^t [(k_s+1)\alpha_j(s)e_{jc}(s) + (\beta_{j1}(s) + \beta_{j2})\operatorname{sgn}(e_{jc}(s))]ds \quad (20)$$

such that $\dot{\mu}_j = (k_s+1)r_j + \beta_{j1}(t)\operatorname{sgn}(e_{jc}) + \beta_{j2}\operatorname{sgn}(\dot{e}_{jc})$ where β_{j2} and k_s are a real positive constants, $\beta_{j1}(t)$ is a real, positive, time varying gain function, and $\operatorname{sgn}(\bullet)$ is the signum function.

Remark: In [4] and [6], $\beta_{j1}(t)$ and $\alpha_j(t)$ are considered to be positive constants. We choose time varying functions here to facilitate in defining the upper bounds necessary for the RISE aspects of the NN/RISE controller which will be discussed in the proceeding development and in the Appendix. Also, the constant β_{j2}

is added to the RISE term to facilitate in the stability analysis of the system.

Using the *universal approximation property* for NN's [3], define

$$\dot{f}_{d_j} = W_j^T \sigma(V_j^T x_{dj}) + \varepsilon_j \quad (21)$$

and define the NN approximation of (21) to be

$$\dot{\hat{f}}_{d_j} = \hat{W}_j^T \sigma(\hat{V}_j^T x_{dj}) \quad (22)$$

where $\hat{W}_j^T$ is the NN estimate of the ideal weight matrix and $x_{d_j} = [1 \quad v_{jc} \quad \dot{v}_{jc} \quad \ddot{v}_{jc} \quad \theta_j]^T$. Noting $\hat{\sigma}_j \equiv \sigma(\hat{V}_j^T x_{dj})$, substituting the derivative of (20) into (18) and applying (22) gives

$$\overline{M}_j \dot{r}_j = -\dot{\overline{M}}_j r_j + W_j^T \sigma_j - \hat{W}_j^T \hat{\sigma}_j + \dot{T}_j - \dot{\mu}_j + \dot{\bar{\tau}}_{d_j} + \varepsilon_j \quad (23)$$

Adding and subtracting e_{jc} and $\hat{W}_j^T \tilde{\sigma}_j + W_j^T \hat{\sigma}_j$ as well as substituting the derivative of (20) and the Taylor series approximation (5) into (23) yields

$$\overline{M}_j \dot{r}_j = -\frac{1}{2}\dot{\overline{M}}_j r_j + \tilde{N}_j + N_{Bj1} + N_{Bj2} - e_{jc} - (K_s + 1) r_j - (\beta_{j1} + \beta_{j2}) \operatorname{sgn}(e_{jc}) \quad (24)$$

where

$$\tilde{N}_j = -\frac{1}{2}\dot{\overline{M}}_j r_j + \dot{T}_j + e_{jc} \quad (25)$$

$$N_{Bj1} = \tilde{W}_j^T \hat{\sigma}'_j V_j^T x_{dj} + W_j^T O(\tilde{V}_j^T x_{dj})^2 + \varepsilon_j + \dot{\bar{\tau}}_{d_j} \quad (26)$$

$$N_{Bj2} = \tilde{W}_j (\hat{\sigma}_j - \hat{\sigma}_j \hat{V}_j^T x_{dj}) + \hat{W}_j^T \hat{\sigma}'_j \tilde{V}_j^T x_{dj}. \quad (27)$$

and $\tilde{V}_j = V_j - \hat{V}_j$ and $\tilde{W}_j = W_j - \hat{W}_j$. An upper bound for $\tilde{N}_j$ can be obtained using the Mean Value Theorem as [4]

$$\|\tilde{N}_j\| \le \rho(\|z_j\|)\|z_j\| \quad (28)$$

where $z_j = [e_{jc}^T \quad r_j^T]^T$ and $\rho(\|z_j\|)$ is a positive, globally invertible, non-decreasing function.

Lemma 1: The terms of (26) and (27) and their derivatives can be bounded by computable positive time varying functions as

$$\|N_{Bj1}\| \le B_{Nj1}(t) \qquad \|\dot{N}_{Bj1}\| \le B'_{Nj1}(t) \quad (29)$$

$$\|N_{Bj2}\| \le B_{Nj2}(t) \qquad \|\dot{N}_{Bj2}\| \le B'_{Nj2}(t) \quad (30)$$

Proof: See *Appendix*. For convenience, define $B_{Nj3}(t) = B_{Nj1}(t) + B_{Nj2}(t)$ and $B'_{Nj3}(t) = B'_{Nj1}(t) + B'_{Nj2}(t)$.

It should be noted at this point that $\dot{v}_{jc} = f_j(\dot{v}_i, \dot{\omega}_i, v_i, \omega_i, e_j, \dot{e}_j)$. The dynamics of leader i written in the form of (3) and rewritten as

$$\dot{v}_i = \overline{M}_i^{-1}(q_i)[\overline{B}_i(q_i)\tau_i - \overline{V}_{m_i}(q_i, \dot{q}_i)v_i - \overline{F}_i(v_i) - \overline{\tau}_{d_i}] \quad (31)$$

Substituting (31) and (9) into $\dot{v}_{jc}$ results in the dynamics of leader i robot to become apart of $\dot{v}_{jc}$ as

$$\dot{v}_{jc} = f_j(v_i, \omega_i, \theta_i, \tau_i, e_j) \quad (32)$$

It is assumed that the leader and follower robots' dynamics are sufficiently smooth such that $\dot{v}_i$, v_{jc}, and $\dot{v}_{jc}$ are also smooth functions. Under these assumptions $\ddot{v}_{jc}$ can be approximated with relatively small error by the standard second order backwards difference equation for a small sample period Δt as

$$\ddot{v}_{jc} = v_{jc}(t) - 2v_{jc}(t - \Delta t) + v_{jc}(t - 2\Delta t) \quad (33)$$

Using (33) and forming $\dot{v}_{jc}$ under the assumption that $\dot{v}_i = 0$ and then including the terms of the function defined in (32), the NN input vector x_{d_j} takes the form of

$$x_{d_j} = [1 \ v_{jc} \ \dot{v}_{jc}|_{\dot{v}_i=0} \ \ddot{v}_{jc} \ \theta_j \ v_i \ w_i \ \tau_i \ \theta_i \ e_j]^T \quad (34)$$

so that the dynamics of the leader i can be estimated by the NN, and the terms of $\dot{v}_{jc}$ omitted by assuming $\dot{v}_i = 0$ can be accounted for.

Theorem 2: Let Assumptions *1-5* and *7-8* hold, and let k_s be sufficiently large positive constant. Let a smooth velocity control input $v_{jc}(t)$ for follower j be defined by (10), and let the torque control for follower j given by (19) be applied to (3). Let the weight tuning laws be defined as

$$\dot{\hat{W}}_j = F\hat{\sigma}_j e_{jc}^T - F\hat{\sigma}'_j \hat{V}_j^T x_{dj} e_{jc}^T - \kappa F\|e_{jc}\|\hat{W}_j \quad (35)$$

$$\dot{\hat{V}}_j = G x_{dj} (\hat{\sigma}'^T_j \hat{W}_j e_{jc})^T - \kappa G\|e_{jc}\|\hat{V}_j \quad (36)$$

where $F = F^T > 0$, $G = G^T > 0$, and $\kappa > 0$. Then the position, orientation, and velocity tracking errors e_j and e_{jc} are asymptotically stable, and the neural network weight estimate errors $\tilde{W}_j$ and $\tilde{V}_j$ are bounded for follower j provided that β_{j1} is selected as

$$\beta_{j1} \ge B_{Nj3}(t) + \frac{1}{\alpha_{j0}} B'_{Nj3}(t) + \kappa(Z_M + \|\hat{Z}_j\|_F)\|\hat{Z}_j\|_F \quad (37)$$

Proof: Consider the following Lyapunov candidate

$$V'_j = V_j + V_{jNN} \quad (38)$$

where V_j is defined as (12) and

$$V_{jNN} = \frac{1}{2} e_{jc}^T e_{jc} + \frac{1}{2} r_j^T \overline{M}_j r_j + P_j + Q_j \quad (39)$$

where

$$P_j = \beta_{j1}(0)\|e_{jc}(0)\| - e_{jc}(0)^T N_{Bj3}(0) - \int_0^t L_j(s)ds \quad (40)$$

$$L_j = r_j^T (N_{Bj1} + N_{Bj1} - \beta_{j1} \operatorname{sgn}(e_{jc})) + e_{jc}^T N_{Bj2} - \dot{\beta}_{j1}\|e_{jc}\| + \kappa tr\{\tilde{Z}_j \hat{Z}_j\} \quad (41)$$

and

$$Q_j = \frac{1}{2}\left(tr\{\tilde{W}_j^T F^{-1} \tilde{W}_j\} + tr\{\tilde{V}_j^T G^{-1} \tilde{V}_j\}\right) \quad (42)$$

If β_{j1} is chosen according to (37), the following inequality can be obtained (this claim is proved in the Appendix)

$$\int_0^t L_j(s)ds \le \beta_{j1}(0)\|e_{jc}(0)\| - e_{jc}(0)^T N_{Bj3}(0) \quad (43)$$

Therefore, it can be concluded that $P \ge 0$ and noted that $\dot{P} = -L$. Taking the time derivative of (38) yields $\dot{V}'_j = \dot{V}_j + \dot{V}_{jNN}$, and it was stated in *Theorem 1* and proved in [7] that $\dot{V}_j < 0$, so our efforts will focus on V_{jNN}. Before proceeding, it is important to note there exists $U_1(y)$ and $U_2(y)$ such that

$$U_1(y_j) \le V_{jNN} \le U_2(y_j) \qquad (44)$$

where $y_j = [z_j^T \;\; \sqrt{P_j} \;\; \sqrt{Q}_j] \in \Re^{2r+2}$. $U_1(y)$ and $U_2(y)$ are defined in [6] to be $U_1(y) = \lambda_1 \|y\|^2$ and $U_2(y) = \lambda_2(q)\|y\|^2$ where $\lambda_1 = \frac{1}{2}\min\{1, m_1\}$, $\lambda_2 = \max\{\frac{1}{2}\overline{m}(q), 1\}$ and m_1 and $\overline{m}(q)$ are defined in the *Boundedness* property for robotic systems described above.

Differentiating (39) yields

$$\dot{V}_{jNN} = e_{jc}^T \dot{e}_{jc} + r_j^T \overline{M}_j \dot{r}_j + \frac{1}{2} r_j^T \dot{\overline{M}}_j r_j + \dot{P}_j + \dot{Q}_j \qquad (45)$$

Making use of (14), (24), and the derivatives of (40) and (42) yields

$$\begin{aligned} \dot{V}_{jNN} &= -\alpha_j e_{jc}^T e_{jc} - r_j^T (k_s + 1) r_j - r_j^T \beta_{j2} \operatorname{sgn}(e_{jc}) + r_j^T \tilde{N}_j \\ &\quad + e_{jc}^T N_{Bj2} + \dot{\beta}_{j1} \|e_{jc}\| + tr\{\tilde{W}_j F^{-1} \dot{\tilde{W}}_j\} + tr\{\tilde{V}_j G^{-1} \dot{\tilde{V}}_j\} \\ &\quad - \kappa \|e_{jc}\| tr\{\tilde{Z}_j \hat{Z}_j\} \end{aligned} \qquad (46)$$

Substitution of (27) and simplifying allows (46) to be rewritten as

$$\begin{aligned} \dot{V}_{jNN} &= -\alpha_j \|e_{jc}\|^2 - (k_s + 1)\|r_j\|^2 - \dot{e}_{jc}^T \beta_{j2} \operatorname{sgn}(e_{jc}) + \dot{\beta}_{j1} \|e_{jc}\| \\ &\quad - \alpha_j \beta_{j2} \|e_{jc}\| + r_j^T \tilde{N}_j - \kappa \|e_{jc}\| tr\{\tilde{Z}_j \hat{Z}_j\} \\ &\quad + tr\{\tilde{W}_j (F^{-1} \dot{\tilde{W}}_j + \hat{\sigma}_j e_{jc}^T - \hat{\sigma}'_j \hat{V}_j^T x_{dj} e_{jc}^T)\} \\ &\quad + tr\{\tilde{V}_j (G^{-1} \dot{\tilde{V}}_j + x_{dj} e_{jc}^T \hat{W}_j^T \hat{\sigma}'_j)\} \end{aligned} \qquad (47)$$

Applying the weight adaptation laws (35) and (36), equation (47) takes the form of

$$\begin{aligned} \dot{V}_{jNN} &= -\alpha_j \|e_{jc}\|^2 - (k_s + 1)\|r_j\|^2 - \dot{e}_{jc}^T \beta_{j2} \operatorname{sgn}(e_{jc}) \\ &\quad + \|e_{jc}\| (\dot{\beta}_{j1} - \alpha_j \beta_{j2}) + r_j^T \tilde{N}_j \end{aligned} \qquad (48)$$

Recalling that $\alpha_j(t) = \alpha_{j0} + \alpha_{j1}(t)$, and selecting $\alpha_{j1}(t) \ge |\dot{B}_{j1}| / \beta_{j2}$, (48) can be rewritten as

$$\begin{aligned} \dot{V}_{jNN} &\le -\alpha_j \|e_{jc}\|^2 - (k_s + 1)\|r_j\|^2 + \|\dot{e}_{jc}\| \beta_{j2} \operatorname{sgn}(e_{jc}) \\ &\quad - \alpha_{j0} \beta_{j2} \|e_{jc}\| + r_j^T \tilde{N}_j \end{aligned} \qquad (49)$$

However, calculation of $\dot{\beta}_{j1}$ is not only difficult, but also only an upper bound of $\dot{\beta}_{j1}$ can be accurately calculated because of derivative chain rules. Therefore, will $\dot{\beta}_{j1}$ be estimated as $\dot{\beta}_{j1} = \hat{\dot{\beta}}_{j1} + \varepsilon_\beta$ where ε_β is the estimation error and $\hat{\dot{\beta}}_{j1}$ is the estimate of $\dot{\beta}_{j1}$ using a standard backwards difference equation written as $\hat{\dot{\beta}}_{j1} = \beta_{j1}(t) - \beta_{j1}(t - \Delta t)$ where Δt is a small sampling period.

Now, selecting $\alpha_{j1}(t) \ge |\hat{\dot{B}}_{j1}| / \beta_{j2}$, $\alpha_{j0} \ge \varepsilon_\beta / \beta_{j2}$, and defining $\overline{\alpha}_{j0} = \alpha_{j0} - \varepsilon_\beta / \beta_{j2}$, equation (49) can be rewritten as

$$\begin{aligned} \dot{V}_{jNN} &\le -\alpha_j \|e_{jc}\|^2 - (k_s + 1)\|r_j\|^2 + \|\dot{e}_{jc}\| \beta_{j2} \operatorname{sgn}(e_{jc}) \\ &\quad - \overline{\alpha}_{j0} \beta_{j2} \|e_{jc}\| + r_j^T \tilde{N}_j \end{aligned} \qquad (50)$$

Noting that $\|\dot{e}_{jc}\| - \overline{\alpha}_{j0} \|e_{jc}\| < \|r_j\|$ yields

$$\dot{V}_{jNN} \le -\alpha_j \|e_{jc}\|^2 - (k_s + 1)\|r_j\|^2 + r_j^T (\tilde{N}_j + \beta_{j2}) \qquad (51)$$

Based on (28), define a second bounding function as $\rho'(\|z_j\|)\|z_j\| = \rho(\|z_j\|)\|z_j\| + \beta_{j2}$ so that

$$\|\tilde{N}_j\| + \beta_{j2} \le \rho'(\|z_j\|)\|z_j\| \qquad (52)$$

Now, using the bound in (52), and completing the square with respect to $\|r_j\|$, we arrive at

$$\dot{V}_{jNN} \le -\lambda_j \|z_j\|^2 - \frac{k_s}{2}\left(\|r_j\| - \frac{\rho'(\|z_j\|)\|z_j\|}{k_s}\right)^2 + \frac{\rho'(\|z_j\|)^2 \|z_j\|^2}{2k_s} \qquad (53)$$

where $\lambda_j = \min\{\alpha_{j0}, k_s/2 + 1\}$. The second term is always less than or equal to zero, so considering the first and third terms, a continuous positive-semi-definite function $U(y_j) = c\|z_j\|^2$, for some real positive constant *c*, can be defined on the domain *D* such that

$$\dot{V}_{jNN} \le -U(y_j) \text{ for } D = \left\{ y_j \in \Re^{2r+2} \mid \|y_j\| \le \rho'^{-1}\left(\sqrt{2\lambda_j k_s}\right) \right\} \qquad (54)$$

The inequalities in (44) and (54) can be used to show that $V_{jNN} < \infty$ and bounded in *D*, and therefore e_{jc}, r_j, P_j, and Q_j are also bounded in *D*. Continuing this way after observing the boundedness of e_{jc} and r_j in *D*, standard linear analysis methods can be used to prove that all of the quantities in (20) and (24) are also bounded in *D*. Therefore, the definitions for $U(y_j)$ and z_j can be used to prove that $U(y_j)$ is uniformly continuous. For complete details of the steps to draw this conclusion, see [6].

Let $S \subset D$ denote a region of attraction such that

$$S = \left\{ y_j(t) \subset D \mid U(y_j(t)) < \lambda_1 (\rho'^{-1}(\sqrt{2\lambda_j k_s}))^2 \right\} \qquad (55)$$

Applying Theorem 8.4 of [8], it can be concluded $c\|z_j\|^2 \to 0$ as $t \to \infty$ $\forall$ $y_j(0) \in S$. From the definition of z_j, it is clear that $\|e_{jc}\| \to 0$ as $t \to \infty$ $\forall$ $y_j(0) \in S$ thus illustrating the asymptotic stability of the tracking error and the boundedness of the neural network weight estimates.

Remark: The region of attraction (55) can be made arbitrarily large to include a larger set of initial conditions by increasing the gain k_s.

E. Leader Control Structure

The kinematics and dynamics of formation leader i can be described similarly to (2) and (3). From [1], the leader tracks a virtual reference robot with the kinematic constraints of $\dot{x}_r = v_r \sin\theta \quad \dot{y}_r = v_r \sin\theta_r \quad \dot{\theta}_r = \omega_r$, and the leader's tracking and its derivative are found to be

$$\begin{bmatrix} e_{i1} \\ e_{i2} \\ e_{i3} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_i \\ y_r - y_i \\ \theta_r - \theta_i \end{bmatrix} \quad (56)$$

$$\begin{bmatrix} \dot{e}_{i1} \\ \dot{e}_{i2} \\ \dot{e}_{i3} \end{bmatrix} = \begin{bmatrix} -v_i + v_{ir}\cos e_{i3} + \omega_i e_{i2} \\ -\omega_i e_{i2} + v_{ir}\sin e_{i3} \\ \omega_{ir} - \omega_i \end{bmatrix} \quad (57)$$

In order to stabilize the leader's kinematic error system, the control velocity proposed in [1] is written as

$$v_{ic} = \begin{bmatrix} v_{ic} \\ \omega_{ic} \end{bmatrix} = \begin{bmatrix} v_{ir}\cos e_{i3} + k_{i1}e_{i1} \\ \omega_{ir} + k_{i2}v_{ir}e_{i2} + k_{i3}v_{ir}\sin e_{i3} \end{bmatrix} \quad (58)$$

In order to define the dynamical NN/RISE controller for the leader i, define the velocity tracking and filtered tracking errors as $e_{ic} = v_{ic} - v_i$ and $r_i = \dot{e}_{ic} + \alpha_i(t)e_{ic}$.

Using similar steps and justifications used to form (15) for follower j, define the error system for leader i to be

$$\overline{M}_i r_i = f_{d_i} + T_i + \bar{\tau}_{d_i} - \bar{\tau}_i \quad (59)$$

where f_{d_i} and T_i are defined similarly to (16) and (17), respectively, and $\bar{\tau}_{d_i}$ represents the unknown, bounded disturbances subject to the bounds described in *Assumption 8*. The control torque, $\bar{\tau}_i$, for leader i can be defined similarly to follower $j's$ as

$$\bar{\tau}_i = \hat{f}_{d_i} + \mu_i \quad (60)$$

where $\hat{f}_{d_i}$ is the estimate of f_{d_i} and μ_i is the RISE feedback term defined similarly to the follower's controllers in (20).

Using the same steps and justifications used to form (24), the closed loop error system for the for the lead robot i can be written as

$$\overline{M}_i \dot{r}_i = -\frac{1}{2}\dot{\overline{M}}_i r_i + \widetilde{N}_i + N_{Bi1} + N_{Bi2} - e_{ic} - (k_{si}+1)r_i - \beta_{i1}(t)\,\mathrm{sgn}(e_{ic}) \quad (61)$$

where k_{si} is a positive control gain parameter, and $\widetilde{N}_i$, N_{Bi1} and N_{Bi2} are defined similarly to (25), (26), and (27), respectively, and are bounded similarly to the bounds defined in (28), (29), and (30), respectively.

Assumption 9 and the fact that the virtual robot does not have its own dynamics allows two derivatives of the reference velocity v_{ir} to be calculated. Therefore, when calculating $\ddot{v}_{ic}$, the only unknown quantity encountered is $\dot{v}_i$ which can be written as in (31). Using similar steps and justifications used to form the NN input vector for follower j in (34), the NN input vector for leader i can be defined as $x_{d_i} = [1 \;\; v_{ic}^T \;\; \dot{v}_{ic}^T \;\; \ddot{v}_{ic}^T\big|_{\dot{v}_i=0} \;\; \theta_i \;\; v_i \;\; \omega_i \;\; \tau_i^T]^T$. The NN weight updates for the leader i are defined similarly to follower j's shown in (35) and (36).

Theorem 3: Let *Assumptions 1-5* and *7-10* hold for leader *i,* and let $K_i = [k_{i1} \;\; k_{i2} \;\; k_{i3}]^T$ be a vector of positive constants, and let k_{si} be a sufficiently large positive constant. Let there be a smooth velocity control input $v_{ic}(t)$ for the leader i given by (58), and let the torque control input for the lead robot i defined by (60) be applied to the mobile robot system in the form of (3). Then leader's position, orientation, and velocity tracking errors are asymptotically stable and the NN weight estimates are bounded.

Proof: Due to space constraints, proof of *Theorem 3* is not included. However, the steps are similar to those used in *Theorem 2* and choosing the Lyapunov candidate $V_i' = V_i + V_{iNN}$ where

$$V_i = \frac{1}{2}(e_{i1}^2 + e_{i2}^2) + \frac{1-\cos e_{i3}}{k_{i2}} \quad (62)$$

and

$$V_{iNN} = \frac{1}{2}e_{ic}^T e_{ic} + \frac{1}{2}r_i^T \overline{M}_i r_i + P_i + Q_i \quad (63)$$

where P_i and Q_i are defined similarly to (40) and (42), respectively.

F. Formation Stability

The stability of the formation can be demonstrated by using the individual Lyapunov functions as given in the following theorem.

Theorem 4: Consider a formation of *N+1* robots consisting a leader i and N followers. Let *Assumptions 1-5* and *7-10* hold. Let there be a smooth velocity control input $v_{ic}(t)$ given by (58) and torque control from (60) for the lead robot i be applied. Let there be a smooth velocity control input $v_{jc}(t)$ given by (10) and torque control given by (19) for the j^{th} follower robot be applied. Then there exists vectors of positive constants, $K = [k_1 \;\; k_2 \;\; k_3]^T$ and $K_i = [k_{i1} \;\; k_{i2} \;\; k_{i3}]^T$, and sufficiently large positive constants, k_s and k_{si} such that the origin, $e_{ij} = [e_i^T \;\; e_{ic}^T \;\; e_j^T \;\; e_{jc}^T]^T = 0$ where $e_{ij} \in \Re^{(n+r)(1+N)x1}$ represents the augmented position, orientation and velocity tracking error systems for the leader i and N followers, respectively is asymptotically stable, and $\widetilde{Z}_{ij} = [\widetilde{Z}_i \;\; \widetilde{Z}_j] = 0$ is the augmented NN weight estimation error matrix for the leader i and N followers, respectively, is bounded. Let the NN weight updates for leader i be defined similarly to (35) and (36) and the NN weight updates for follower j be given by (35) and (36).

Proof: Due to space constraints, proof of *Theorem 4* is not included here. However, the theorem follows by selecting the Lyapunov candidate

$$V_{ij} = \sum_{1}^{N} V_j + V_i + V_{NN} \tag{64}$$

where V_j is defined by (12), V_i is defined by (62), and V_{NN} is defined as

$$V_{NN} = \frac{1}{2} e_c^T e_c + \frac{1}{2} r^T \overline{M}\ r\ + P\ + Q > 0 \tag{65}$$

where $e_c^T = [e_{ic}^T \quad e_{jc}^T]^T \in \Re^{1xr(N+1)}$, $r = [r_i^T \quad r_j^T]^T \in \Re^{r(N+1)x1}$, $\overline{M} = diag\ (\overline{M}_i, \overline{M}_j) \in \Re^{r(N+1)xr(N+1)}$ for $j = 1,2,...N$, $P = P_i + \sum_{1}^{N} P_j$, and $Q = Q_i + \sum_{1}^{N} Q_j$.

*Remark: T*he stability of the entire formation for the case when follower *j* becomes a leader to follower *j+1* follows directly from *Theorem 2* and the Lyapunov candidate

$$V_j'' = \sum_{j}^{j+1} V_j' \tag{66}$$

where V_j' is defined in (38).

III. Simulation Results

A wedge formation of five identical nonholonomic mobile robots is considered where the leader's trajectory is the desired formation trajectory, and simulations are carried out in MATLAB. The NN controller gains are set as $F = 5$, $G = 5$ and $\kappa = .1$, and the following gains were utilized.

Leader	k_{is}=daig{65}	$K_{i1}=10$	$K_{i2}=5$	$K_{i3}=4$
Follower *j*	k_s=diag{65}	$k_1=7$	$K_2=15$	$k_3=.01$

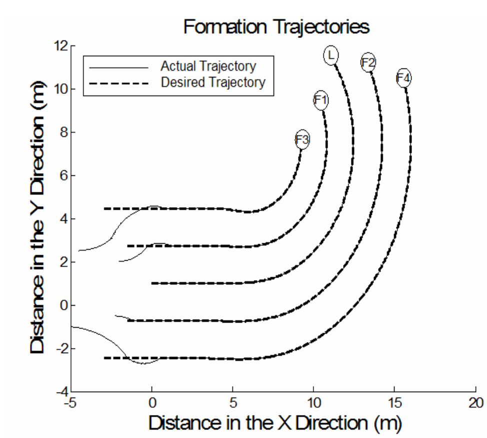

Fig. 2: Formation Trajectories

The following gain parameters are selected for the NN/RISE controller: $\beta_{i,j1} = B_{Ni,j3}(t) + \frac{1}{\alpha_{i,j0}} B'_{Ni,j3}(t) + 0.05 + \left\|\hat{Z}_{i,j}\right\|_F^2$ with the values of $B_{Ni,j3}(t)$ and $B'_{Ni,j3}(t)$ defined in the Appendix, $\alpha_{i,j0} = 10$, and $\beta_{i,j2} = 20$.

Also, the following robotic parameters are considered for the leader and its followers in both scenarios: m=5 kg, I = 3 kg^2, R=.175 m, r = 0.08 m, and d=0.45 m.

Figure 2 displays the formation trajectories taken by each robot as well as the desired formation path. Examining the plot, it is clear that the robots quickly converge to and track the desired formation trajectory, and the formation errors converge to zero as the developed theory suggests.

IV. Conclusions

An asymptotically stable multilayer NN tracking controller for leader-follower based formation control was presented that considers the dynamics of the leader and the follower using backstepping with RISE feedback. The feedback control scheme is valid even when the dynamics of the followers and their leader are unknown since the NN learns them all online. RISE feedback-based controller design allows the asymptotic stability compared to the uniform ultimately boundedness, a result common in the NN control literature. Simulation results were provided to illustrate the effectiveness of the control.

V. References

[1] R. Fierro and F. L. Lewis, "Control of a Nonholonomic Mobile Robot Using Neural Networks," IEEE Trans. on Neural Networks, Vol. 8, pp. 589-600, 1998.

[2] Y. Li, and X. Chen, " Dynamic Control of Multi-robot Formation," Proc. of the IEEE Int. Conf. on Mechatronics, Taiwan, pp 352-357, July 2005.

[3] F.L. Lewis, S. Jagannathan, and A. Yesilderek, "Neural Network Control of Robot Manipulators and Nonlinear Systems," Taylor & Francis, London, 1999.

[4] B. Xian, M.S. de Queiroz, and D. M. Dawson, *A Continuous Control Mechanism for Uncertain Nonlinear Systems in Optimal Control, Stabilization and Nonsmooth Analysis,* pp. 251-262, Heidelberg, Germany: Springer-Verlag, 2004

[5] C. M. Kwan, D. M. Dawson, and F. L. Lewis, "Robust Adaptive Control of Robots Using Neural Networks: Global Tracking Stability," Proc. of the IEEE Conference of Decision and Control, New Orleans, Louisiana, pp. 1846-1851, 1995.

[6] P. M. Patre, W. E. Dixon, K. Kaiser, and W. MacKunis, "Asymptotic Tracking for Uncertain Dynamic Systems via a Multilayer NN Feedforward and RISE Feedback Control Structure," American Control Conference , New York City, New York, 2007.

[7] T. Dierks, and S. Jagannathan, "Control of Nonholonomic Mobile Robot Formations: Backstepping Kinematics into Dynamics", IEEE Transactions on Robotics and Automation, *submitted.* Available via the internet (URL: http://www.umr.edu/~tad5x4/RFC_paper1.pdf) 2007.

[8] H. K. Khalil, *Nonlinear Systems*, 3rd ed, New Jersey: Prentice-Hall, 2002.

VI. Appendix

Due to space constraints, the Appendix has been posted on: (URL: http://www.umr.edu/~tad5x4/CDC_APP.pdf)