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Constitutive Equations for Sands and Overconsolidated Clays under Dynamic Loads Based on Elasto-Plasticity

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SYNOPSIS This paper is concerned with the constitutive equations of the sands and overconsolidated clays under cyclic loads. The constitutive equations are derived, based on the theory of plasticity and real stress-strain behavior of soils. The non-associated flow rule is applied to the derivation of the equations. The derived equations can explain the mechanical behavior of overconsolidated clays and sands under cyclic stresses and have nine soil parameters, and are applicable to liquefaction analysis.

INTRODUCTION

This paper is concerned with the stress-strain relation of soil under cyclic loads. Hardin(1978) experimentally reported that the yield surface for most soils enclosed the single point, because the inelastic strain occurred under unloadings. Pender(1978) considered that whenever the stress ratio changed even inside of the boundary surface(e.g., critical state energy theory),the plastic yielding occurred. Dafalias et al.(1977) also discussed the vanishing of elastic range. From the above works, we assume that the material is elastic in the overconsolidated region only when the stress ratio is constant.

The concept of the bounding surface has been proposed by Dafalias et al., and Dafalias et al. (1980) and Mroz et al.(1979) developed a bounding surface model for clay. In this paper, the similar concept of bounding surface in the stress space are used in conjunction with the critical state soil plasticity.

Following to Pender's idea and the concept of the bounding surface, we will derive a constitutive model which can describe the mechanical behavior of overconsolidated calys and sands under cyclic loads, based on the theory of elasto-plasticity.

DERIVATION OF CONSTITUTIVE EQUATIONS

The boundary between normally consolidated state and overconsolidated state is assumed to be given by the following O.C. boundary surface as shown in Fig.1.

$$f_b = \bar{\eta}^*_{(0)} + M^*_m \ln(\sigma'_m/\sigma'_{me}) = 0 \qquad (1)$$

where σ'_m is a mean effective stress, σ'_{me} is preconsolidation pressure and $\bar{\eta}^*$ in Eq.(1) is a stress parameter for representation of anisotrpic consolidation introduced by Sekiguchi et al. (1977), and is defined by

$$\bar{\eta}^*_{(0)} = [(\eta^*_{ij} - \eta^*_{ij(0)})(\eta^*_{ij} - \eta^*_{ij(0)})]^{1/2} \qquad (2)$$

in which

$$\eta^* = s_{ij}/\sigma'_m \qquad (3)$$

$$\eta^*_{ij(0)} = (s_{ij}/\sigma'_m)_{(0)} \qquad (4)$$

where $\eta^*_{ij(0)}$ is the value of η^*_{ij} at the end of anisotropic consolidation and s_{ij} is a deviatoric stress tensor.

M^*_m in Eq.(1) is the value of $(\eta^*_{ij}\eta^*_{ij})^{1/2}$ when the maximum compression of the material takes place.

Since there is no elastic domain, the plastic strain incremental tensor $d\varepsilon^p_{ij}$ is given by the following non-associated flow rule, i.e.,

$$d\varepsilon^p_{ij} = \Lambda \frac{\partial f_p}{\partial \sigma_{ij}} df \qquad (5)$$

where f_p is a plastic potential function, f is a plastic yield function, σ_{ij} is the stress tensor and Λ is a hardening parameter.

The plastic potential function f_p is assumed to be given by

$$f_p = \bar{\eta}^* + \tilde{M}^* \ln(\sigma'_m/\sigma'_{m(n)}) = 0 \qquad (6)$$

In this equation, $\bar{\eta}^*$ is a relative stress parameter and in this case we define as follows.

$$\bar{\eta}^* = [(\eta^*_{ij} - \eta^*_{ij(n)})(\eta^*_{ij} - \eta^*_{ij(n)})]^{1/2} \qquad (7)$$

In Eqs.(6) and (7), $\eta^*_{ij(n)}$ and $\sigma'_{m(n)}$ are the values of η^*_{ij} and σ'_m at the n-th times turning over point of the loading direction. Furthermore, the parameter $\tilde{M}^*$ in Eq.(6) is a variable which is defined by

$$\tilde{M}^* = -\frac{\eta^*}{\ln(\sigma'_m/\sigma'_{mc})} \qquad (8)$$

$$\eta^* = (\eta^*_{ij}\eta^*_{ij})^{1/2} \qquad (9)$$

In Eq.(8), σ'_{mc} is a value of $\sigma'_m \exp(\eta^*/M^*_m)$ at the end of consolidation.

It is easily understood that $\tilde{M}^*$ is automatically determined once the current stress state and consolidation history are provided.

On the other hand, the yield function is given by

$$f = \bar{\eta}^* \qquad (10)$$

Figs.1 and 2 schematically illustrate the plastic potential curve $f_p = 0$ and stress paths under the undrained triaxial test condition.

Inside of the state boundary surface, namely in overconsolidated state, we keep $\tilde{M}^* = M^*_m$ after $\tilde{M}^*$ attains to the value of M^*_m. This assumption coincides with the concept of so-called phase transformation line defined by Ishihara et al. (1975).
In order to obtain the hardening function, we introduce a strain-hardening parameter $\bar{\gamma}^*$ corresponding to the parameter $\bar{\eta}^*$. $\bar{\gamma}^*$ can be called "relative plastic deviatoric strain".

$$\bar{\gamma}^* = [(e^p_{ij} - e^p_{ij(n)})(e^p_{ij} - e^p_{ij(n)})]^{1/2} \quad (11)$$

where e^p_{ij} is the plastic deviatoric strain component and $e^p_{ij(n)}$ is the value of e^p_{ij} at the state when the n-th times reverse of loading takes place as shown in Fig.1.

In addition, we assume that the following relationship between the hardening parameter $\bar{\gamma}^*$ and $\bar{\eta}^*$ is expressed by a hyperbolic curve as given in Fig.3.

$$\bar{\gamma}^* = \frac{\bar{\eta}^*[M^*_f + \eta^*_{(n)}]}{G'[M^*_f + \eta^*_{(n)} - \bar{\eta}^*]} \quad (12)$$

where G' is the initial tangent modulus, M^*_f is the value of η^* at the failure state and $\eta^*_{(n)}$ is defined by

$$\eta^*_{(n)} = [\eta^*_{ij(n)}\eta^*_{ij(n)}]^{1/2} \quad (13)$$

This hyperbolic strain-hardening function is a generalization of Nishi and Esashi's hardening function(1978).

From Eqs.(5),(6),(8),(10) and (12), we obtain the parameter Λ as follows.

$$\Lambda = \frac{\bar{\gamma}^*\,\bar{\eta}^*}{(e^p_{ij} - e^p_{ij(n)})(\eta^*_{ij} - \eta^*_{ij(n)})}\;\frac{(M^*_f + \eta^*_{(n)})^2\sigma'_m}{G'(M^*_f + \eta^*_{(n)} - \bar{\eta}^*)^2} \quad (14)$$

Finally, the constitutive equations for overconsolidated clays and sands are obtained from Eqs.(5),(6),(10) and (14). Taking into account the elastic component, they can be written by

$$d\varepsilon_{ij} = \frac{1}{2G}ds_{ij} + \frac{\kappa}{(1+e)}\frac{d\sigma'_m}{\sigma'_m}\frac{1}{3}\delta_{ij} + \Lambda[\frac{\eta^*_{ij} - \eta^*_{ij(n)}}{\bar{\eta}^*}\frac{1}{\sigma'_m} + \frac{\tilde{M}^*}{3\sigma'_m}\delta_{ij} - \frac{s_{kl}}{3\bar{\eta}^*}(\eta^*_{kl} - \eta^*_{kl(n)})\frac{\delta_{ij}}{\sigma'^2_m}]df \quad (15)$$

There are ten material parameters in the constitutive equations, namely G, κ, λ, σ'_{me}, G', e, M^*_m, M^*_f, η^*_0 and $\tilde{M}^*$. However, $\tilde{M}^*$ can be obtained when η^*_0, σ'_{me} and the current stress state are provided and bounded by the value of M^*_m. Thus the following nine parameters have to be given to complete the constitutive equations.

λ; consolidation index, κ; swelling index, ε; void ratio, G; elastic shear modulus, G'; parameter for strain-hardening function,

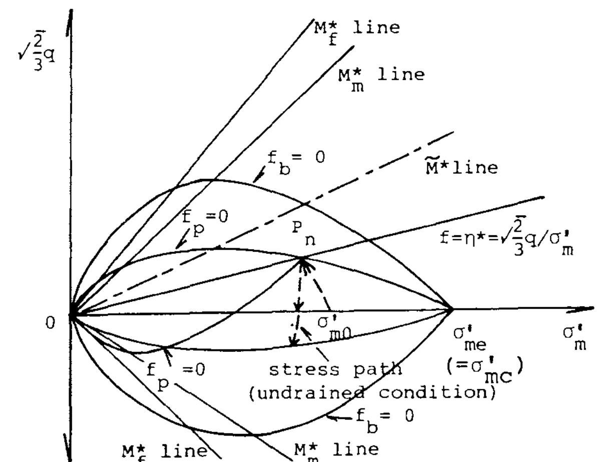

Fig. 1 Schematic diagram of the potential surfaces and yield line.

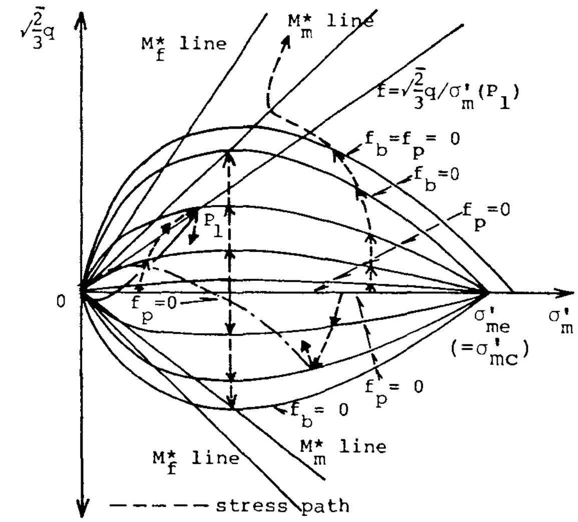

Fig. 2 Schematic diagram of stress paths under undrained condition

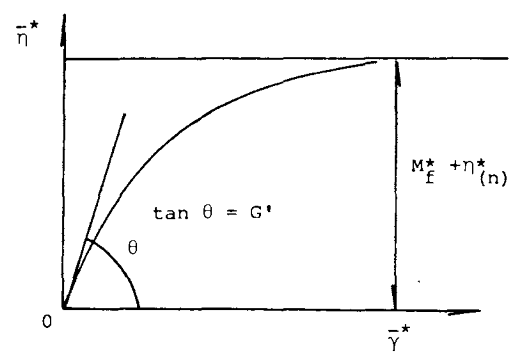

Fig 3 Hyperbolic representation of strain-hardening function

η^*_0 ; the value of η^* at the end of consolidation, M^*_m; the value of stress ratio η^* at the maximum compression, M^*_f; the value of stress ratio η^* at the failure state, σ'_{me} ; preconsolidation pressure.

COMPARISON WITH EXPERIMENTAL RESULTS

In the normally consolidated region of sand, $\tilde{M}^*$ is always equal to M^*_m. For clay, in the

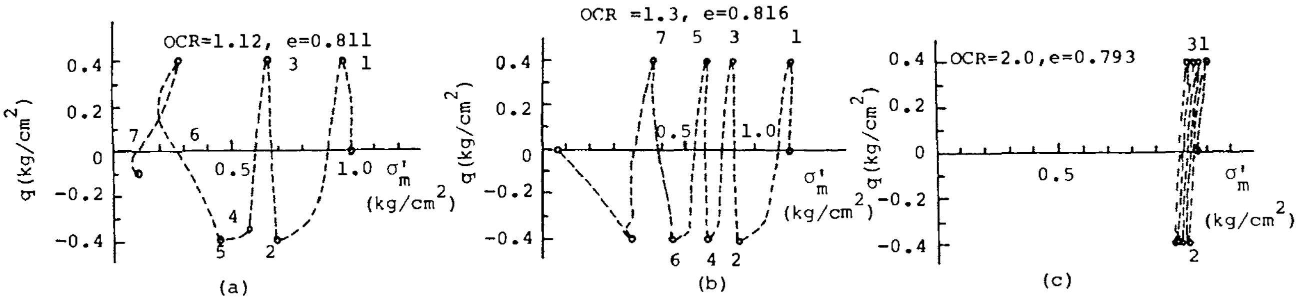

Fig. 4 Stress path of overconsolidated sand under cyclic stresses (after Ishihara & Okada, 1978)

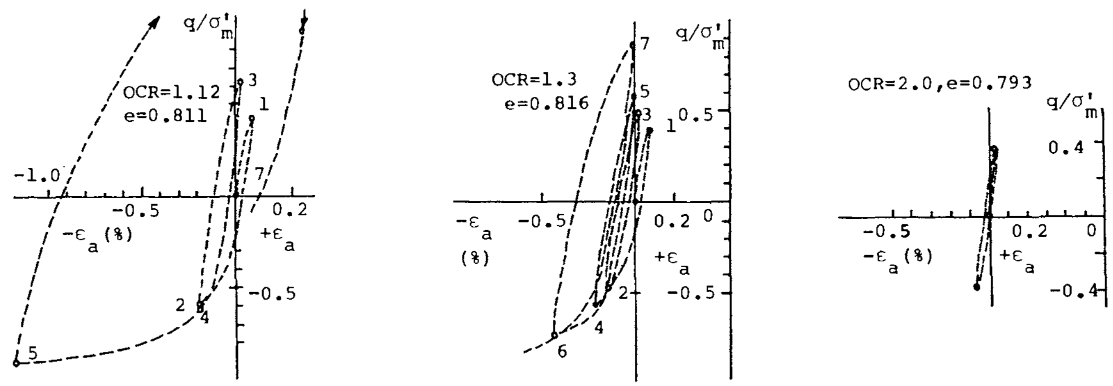

Fig. 5 Stress ratio vs. axial strain relations of overconsolidated sand under cyclic stresses (after Ishihara & Okada, 1978)

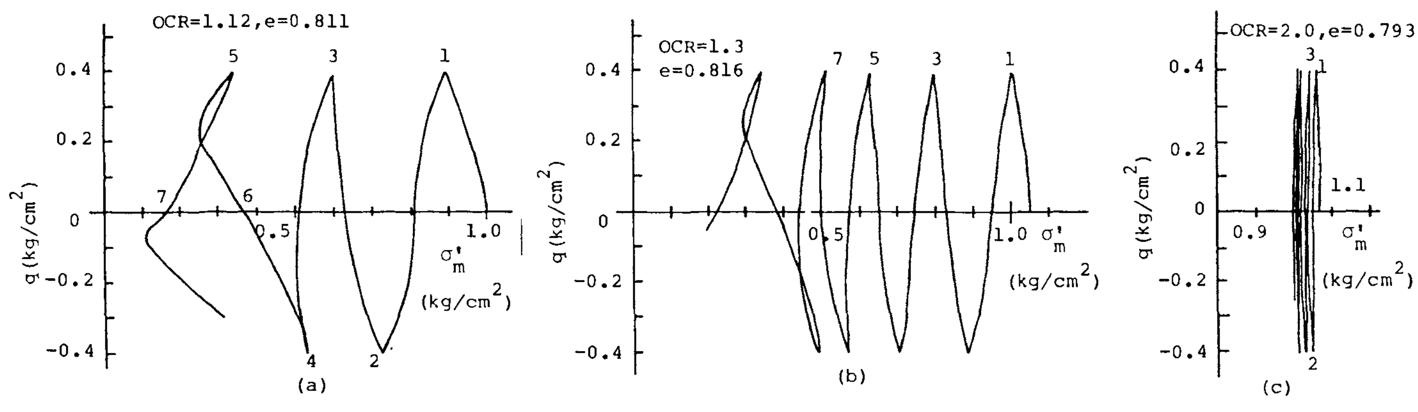

Fig.6 Stress path of overconsolidated sand under cyclic stresses (calculated results)

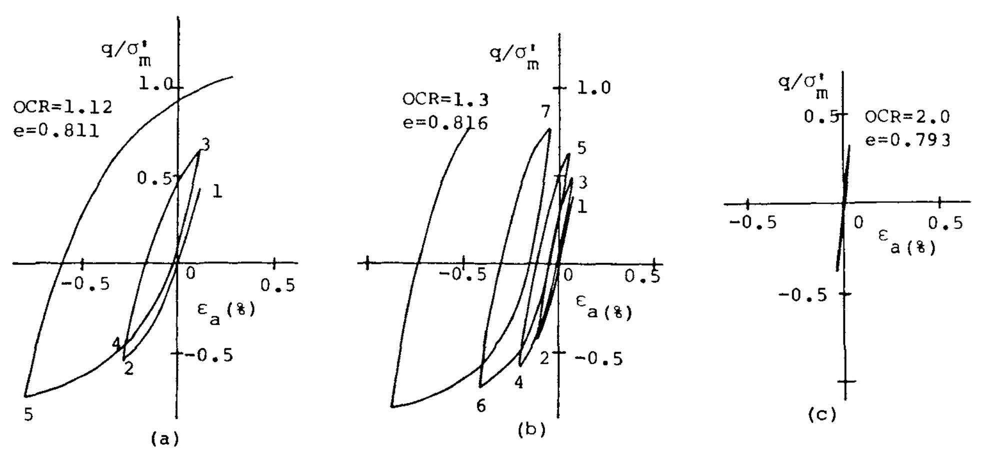

Table I Soil parameters

$M_{m(c)}$	=1.3	(E=3.0G)
$M_{m(E)}$	=1.05	
$M_{f(c)}$	=1.5	
$M_{f(E)}$	=1.06	
κ	=0.008	
λ	=0.017	
E	=1400 kg/cm^2	
G'=5.22xexp(4.24xOCR)		

Fig.7 Stress ratio vs. axial strain relations of overconsolidated sand under cyclic stresses (calculated results)

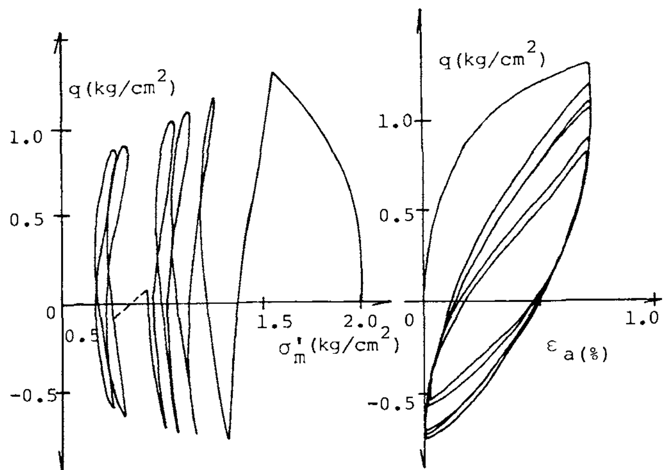

Fig.8 Stress-strain relation and stress path of normally consolidated clay (after Akai et al. 1979)

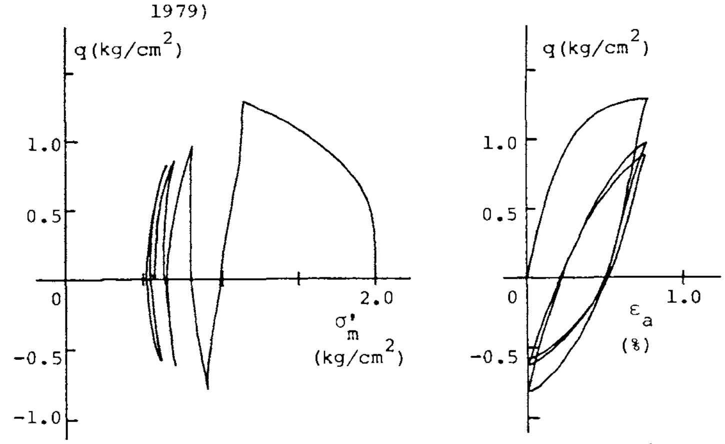

Fig.9 Stress-strain relation and stress path of normally consolidated clay (calculated result)

Table II Soil parameters of clay and test condition

ε_{max}	$\dot{\varepsilon}$	E	G'	$M_{m(c)}$	$M_{m(E)}$
0.78 %	0.12 %/min	600 kg/cm^2	1200	1.3	1.05
e_0	$M_{f(c)}$	$M_{f(E)}$	κ	λ	
0.8	1.95	1.575	0.0087	0.091	

normally consolidated region, elasto/visco-plastic constitutive relations proposed by one of the authors (F.Oka, 1981) are used. The prameters shown in Tables I and II are used to predict the undrained responses of sand and clay under cyclic loading. In Tables, the indices (c) and (E) denote the compression state and extention state respectively, and $M_f=\sqrt{3/2}M_f^*$ and $M_m=\sqrt{3/2}M_m^*$. In triaxial test conditions, σ'_m is equal to $(\sigma'_{11}+2\sigma'_{33})/3$ and q is given by $\sigma'_{11}-\sigma'_{33}$. Figs. 4 and 5 show the cyclic triaxial test results for overconsolidated Fuji River sand obtained by Ishihara et al.(1978). Figs.6 and 7 show the predicted results by the proposed theory, corresponding to Figs.4 and 5. The undrained cyclic triaxial test result for normally consolidated clay, reported by Akai et al.(1979) is shown in Fig.8. Fig.9 show the predicted result for clay. From Fig.9, it is found that the proposed theory can finely explain the stabilization of stress-strain loop after some cycles of load application. It may be concluded that the predicted results can fairly describe the undrained cyclic behavior of sand and clay.

CONCLUSIONS

The constitutive relation that can describe the cyclic behavior of sands and overconsolidated clays are proposed, based on the elasto-plasticity and the concept of the boundary surface. The relative stress ratio and the newly defined prameter "relative deviatoric strain" are used to formulate the hardening function. It must be emphasized that the derived theory also explain the behavior after the stress ratio attains to the value defined by the phase transformation angle. Then, the proposed theory is applicable to the liquefaction analysis.

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