Displacement of Bridge Abutment Under Earthquake Loading

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ABSTRACT

Bridge abutments may experience significant displacements during earthquakes. A two-dimensional model has been developed considering sliding and overturning displacement and 1) strain dependent soil stiffness and soil damping and 2) horizontal and vertical time dependent seismic load. The displacements are computed from the static equilibrium position where the seismic backfill force increments are considered for determining the active earth force acting behind the abutment wall. This means that the permanent displacement increment occurred if the acceleration acts towards the backfill and the abutment wall moves away from the backfill. The total displacement at the top of bridge abutment is calculated by adding the sliding and overturning displacements. An application of this model is presented, for a real abutment.

Keywords; *Displacement, Bridge abutment, Earthquake.*

INTRODUCTION

Traditional Analysis

In seismically active regions, traditional analysis of a bridge abutment is based on limit design method, where the seismic earth force acting behind the wall is calculated by Mononobe-Okabe method for a peak ground acceleration. The abutment dimensions are calculated to obtain a defined factor of safety against sliding, overturning and bearing capacity. In this method, no displacements have been specified. This method does not provide an estimate of abutment displacements. Collapses of bridge, due to large displacement of their abutment have been reported.

Displacement-Based Design

Richards and Elms (1979) proposed a simplified displacement method for dynamic design of rigid retaining walls. This method is based on Newmark's sliding block model (1965). The wall dimensions and its weight are determined to maintain stability against a permissible sliding displacement. However, Richards and Elms (1979) did not suggest how to determine a permissible displacement for the wall. It has been shown by Wu (1999) that the realistic displacements of rigid walls are greater than assumed by Richards and Elms. Only sliding displacement is considered and all displacements before cut-off acceleration are neglected in their method. In several earthquakes, the displacement of bridge abutment occurred in sliding and overturning. This means that, the Richard and Elm solution becomes unrealistic.

Chaudhry (1999) performed seismic displacement analysis of gravity type bridge abutment supported on piles. His model is capable for conducting linear, geometric nonlinear and material nonlinear analysis. The soil pile system and also the backfill soil are presented as spring and dashpot model. The stiffness and damping factor and their group efficiency computed based on Gazetas (1991) model. A parametric study has been conducted to investigate the effect of soil non-linearity. The study shows that maximum shear modulus of backfill soil has insignificant effect on maximum displacement response of the abutment as the force required to compress the backfill soil is too large to permit abutment motion into the backfill. However the shear modulus of foundation soil is a very important factor governing the maximum abutment response.

DISPLACEMENT MODEL

Figure 1 shows a typical highway bridge abutment on piles. The displacements of abutment are modeled by considering its displacements as a two-degree of freedom with sliding and overturning displacements (Fig. 2). The resistances from the soil pile system on this model are represented by equivalent spring and dashpot.

Proposed Displacement Model

In this model, the seismic displacements occur due to time dependent seismic load calculated as function of ground acceleration divided by gravity acceleration (a/g). The backfill soil behind the wall was considered as a seismic force acting to the wall. The stiffness value of spring and the damping value of dashpot are directly dependent on dimension of pile, the shear modulus of the soil, elastic modulus of pile and interaction between soil and pile Novak (1974). However, to obtain initial shear modulus and shear modulus ratio of soil, other factors such as Poisson’s ratio, soil density, void ratio and plasticity index, are needed Seed et al (1970). Group efficiency factors of Poulos (1972) are employed. Displacements of the bridge abutment are computed by solving the time dependent seismic equilibrium equation. The increments of displacement were evaluated from the static equilibrium position. Only the seismic backfill force increments are used for determining the active earth force acting on the wall. This means that, the permanent displacement occurred if the acceleration acts towards the backfill and the wall moves away from the backfill. The total permanent displacements were determined as cumulative permanent displacement for all of active state condition.

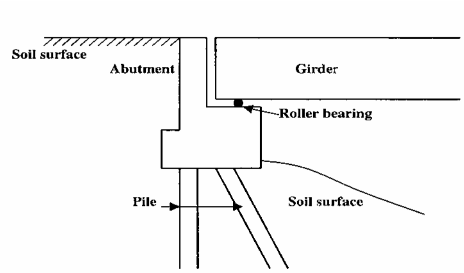
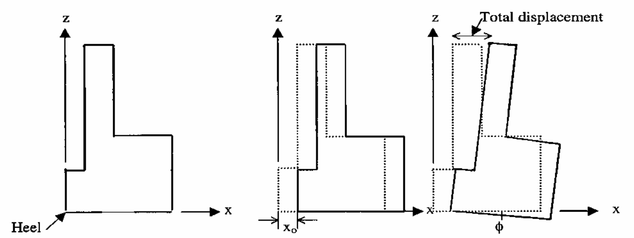


Figure 1 The Typical Bridges Abutment



a. Initial Condition b. Sliding c. Sliding and Rotation

Sign Convention

Figure 2 Movement of Abutment

Equation of Seismic Equilibrium

In general, the two-dimensional equation of seismic equilibrium was presented as

$$\begin{bmatrix} m & 0 \\ 0 & mM \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} c_x & -c_{x\phi} \\ -c_{\phi x} & c_\phi \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{\phi} \end{Bmatrix} + \begin{bmatrix} k_x & -k_{x\phi} \\ -k_{\phi x} & k_\phi \end{bmatrix} \begin{Bmatrix} X \\ \phi \end{Bmatrix} = \begin{Bmatrix} Px(t) \\ M_\phi(t) \end{Bmatrix} \quad (1)$$

where,

m = mass of the bridge abutment.
mM = the mass moment of inertia.
P_x = driving forces force,
M_ϕ = driving moment at the rotational point.
X = sliding displacement.
ϕ = rotational displacement.

k and c are stiffness and damping factors. Those values depend on mode of displacement.

The set of coordinates shown in Figure 2 used in this model. Since the rotational point is assumed at the heel of abutment, the equations of motion (equation 1) above will modify as below;

$$\begin{bmatrix} m & mH_e \\ mH_e & mH_e^2+\bar{I} \end{bmatrix} \begin{Bmatrix} \ddot{x}_r \\ \phi \end{Bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_{x\phi} \end{bmatrix} \begin{Bmatrix} x_r \\ \phi \end{Bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_{x\phi} \end{bmatrix} \begin{Bmatrix} x_r \\ \phi \end{Bmatrix} = \begin{Bmatrix} \Delta P_x(t) \\ mH_e\ddot{x}_g + \Delta M_\phi(t) \end{Bmatrix} \quad (2)$$

where,

H_e = the distance to the center of gravity of wall from its heel.
H = bridge abutment height.
δ = internal friction angle between backfill soil and wall face.
I = moment inertia of bridge abutment.
ΔP_x = seismic force increment
ΔM_ϕ= driving moment increment

Detailed expressions for computing driving forces ΔP_x and ΔM_ϕ are presented latter.

Stiffness and Damping Factors

Novak (1974) has proposed stiffness and damping factors of soil-pile system due to dynamic loading condition. His model mainly used to evaluate displacement of machine foundation. Novak's stiffness and damping factor has been adapted for non-linear soil behavior and are presented below (Munaf and Prakash, 2002 a,b):

Sliding

$$k_x = \left[\frac{E_p I_p}{r_o^3}\right] f_{x1} \quad \text{and} \quad c_x = \left[\frac{E_p I_p}{r_o^2 V_s}\right] f_{x2} \quad (3)$$

Rotation

$$k_\phi = \left[\frac{E_p I_p}{r_o^2}\right] f_{\phi 1} \quad \text{and} \quad c_\phi = \left[\frac{E_p I_p}{r_o^2 V_s}\right] f_{\phi 2} \quad (4)$$

Cross couple

$$k_{x\phi} = \left[\frac{E_p I_p}{r_o^2}\right] f_{x\phi 1} \quad \text{and} \quad c_{x\phi} = \left[\frac{E_p I_p}{r_o V_s}\right] f_{x\phi 2} \quad (5)$$

where,

I_p = Inertia moment of pile.
r_o = radius of single pile.
f_{x1} = $9.5/((E_p/G_{soil})+210)+0.001$.
f_{x2} = $34/((E_p/G_{soil})+200)+0.0033$.
$f_{\phi 1}$ = $325/((E_p/G_{soil})+1050)+0.153$.
$f_{\phi 2}$ = $270/((E_p/G_{soil})+990)+0.12$.
$f_{x\phi 1}$ = $-(43/((E_p/G_{soil})+450)+0.0112)$.
$f_{x\phi 2}$ = $-(64/((E_p/G_{soil})+300)+0.0225)$.
E_p = modulus elastic of pile.
G_{soll} = shear modulus of soil.

The $f_{x1}, f_{x2}, f_{\phi 1}, f_{\phi 2}, f_{x\phi 1}$ *and* $f_{x\phi 2}$ are modification of Novak's interaction factors, because, the soil modulus reduces with increasing strain, resulting in strain dependent stiffness and damping factors (Munaf and Prakash 2002 a,b)

Group Interaction Factors

To consider group effect, Paulos (1968) assumed a pile in the group as reference pile. In the illustration Figure 3, pile No. 1 is assumed as a reference pile and distance '*S*' is measured from the center of other pile to center of the reference pile.

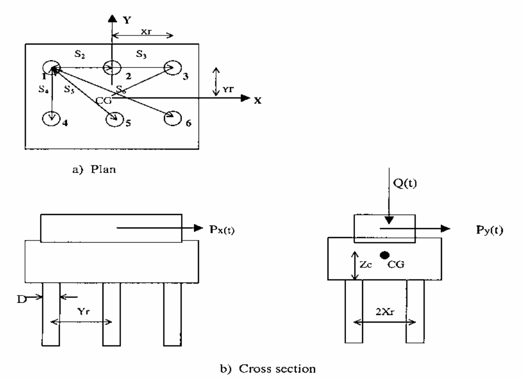


Figure 3 Section of Pile Group (Munaf and Prakash, 2002).

Use Figure 4 (Poulos, 1972), to obtain α_L for each pile in the horizontal x-direction, considering departure angle β (degree). α_L's are a function of L, r_o and flexibility K_R as defined in Figure 4 and departure angle (β). The group interaction factor ($\Sigma\alpha_L$) is the summation α_L for all the piles. Note that, the group interaction factor in horizontal x-direction depends on number and spacing of piles in this direction.

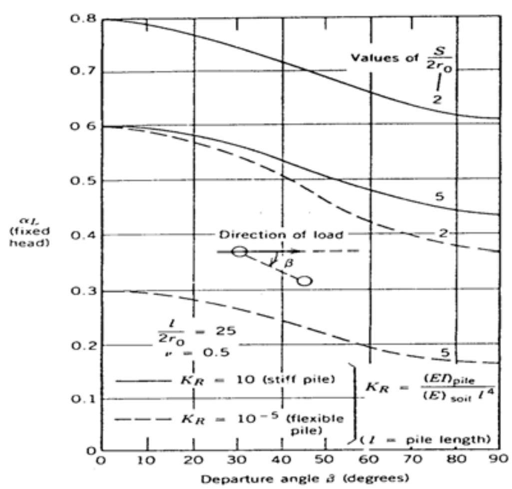


Figure 4 Graphical Solution of α_L (Poulus, 1972)

Group Stiffness and Damping Factors

Figure 3 shows schematically the plan and cross sections of an arbitrary pile group foundation. This figure will be used to explain and to obtain the stiffness and damping factors group of pile. They are presented as follows:

Sliding

$$k_x^g = \frac{\sum k_x}{\sum \alpha_{Lx}} \quad \text{and} \quad c_x^g = \frac{\sum c_x}{\sum \alpha_{Lx}} \tag{6}$$

Rotation

$$k_\phi^g = \frac{1}{\sum \alpha_{Lx}}\left[k_\phi + k_z x_r^2 + k_x z_c^2 - 2z_c k_{x\phi}\right] \quad \text{and}$$

$$c_\phi^g = \frac{1}{\sum \alpha_{Lx}}\left[c_\phi + c_z x_r^2 + c_x z_c^2 - 2z_c c_{x\phi}\right] \tag{7}$$

Cross couple

$$k_{x\phi}^g = \frac{1}{\alpha_{Lx}}\sum\left(k_{x\phi} - k_x z_c\right) \quad \text{and}$$

$$c_{x\phi}^g = \frac{1}{\alpha_{Lx}}\sum\left(c_{x\phi} - c_x z_c\right) \tag{8}$$

Forces Acting on Abutment

The forces acting on the abutment that cause the walls to move away include inertia forces of wall and dynamic active thrust. The dynamic active thrust includes backfill forces and external forces acting on the abutment. All of forces used in this analysis are time dependent.

Figure 5 shows the forces acting on the bridge abutment. These forces consist of:

1. The vertical seismic force increment (V_1) is

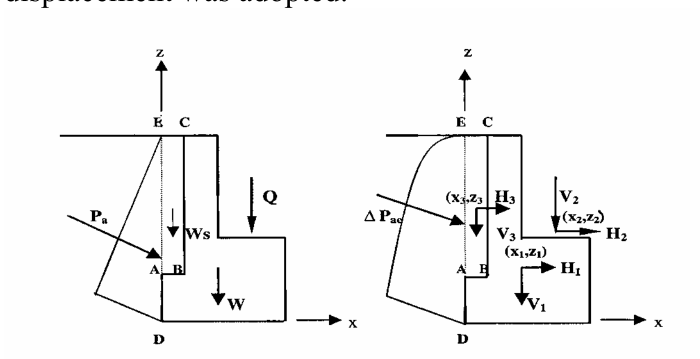
$$V_1 = k_v W \tag{9a}$$

where,

k_v = vertical seismic coefficient.

W = weight of the abutment.

Vertical force may act in the positive (+) or negative (-) direction. The case that gives maximum displacement was adopted.



a) Static forces b) Dynamic force

Figure 5 Forces acting on the bridge abutment

The point of application of V_1 is the center of gravity of the abutment and the horizontal distance from this point to the heel of the abutment is expressed as x_1 in Figure 5.

The horizontal force (H_1) due to weight (W) of the abutment is computed as

$$H_1 = k_h W \tag{9b}$$

where,

k_h = horizontal seismic coefficient

The height of the line of action of H_1 is at the centroid of the abutment, z_1 from the bottom.

2. The vertical seismic force increment, V_2, applied to the abutment is

$$V_2 = k_v Q \tag{10a}$$

where,

Q = Weight of the girder and traffic load acting on the bearing

The vertical force may act in the positive (+) or negative (-) direction. The case that gives the maximum displacement was adopted. The point of application of V_2 is the center of the bearing and the horizontal distance from this point to the heel of the abutment is expressed as x_2.

The horizontal seismic force H_2 of the girder is

$$H_2 = k_h Q \quad (10b)$$

The height of the line of action of H_2 is assumed to be coincident with the upper surface of the bearing and at a distance z_2 from the bottom of the abutment.

3. The seismic force due to the weight of earth (Ws) ABCE (Fig. 3) is given below with the point of application at the centroid (x_3,z_3) of the earth mass:

$$V_3 = k_v W_s \quad (11a)$$
$$H_3 = k_h W_s \quad (11b)$$

The dynamic force increment acting on the abutment is the sum of the earth force increment acting on the vertical line DE and the weight of soil mass ABCE and the seismic force. The earth pressure increment acting on the vertical line DE is calculated by modified the Mononobe-Okabe method. Its point of application is proposed at *1/2* of the height of the line ED and the direction is inclined δ to normal on ED. The total horizontal force increment (ΔP_x) and moment increment (ΔM_ϕ) about the heel (D) are,

$$\Delta P_x = H_1 + H_2 + H_3 + \Delta P_{ae} \cos(\delta) \quad \text{and}$$
$$\Delta M_{\phi} = V_1 . x_1 + V_2 . x_2 + V_3 x_3 + H_1 . z_1 + H_2 . z_2 + H_3 . z_3 + \Delta P_{ae} \cos(\delta) . 1/2H \quad (12)$$

NUMERICAL FORMULATION

Newmark's method (Dhatt and Touzot, 1984) is employed here for solving equation of motion. This method uses the governing equation evaluated at time t + Δt and the following truncated expressions for velocity and displacement $\{\underline{u}_{t+\tau}\}$ and $\{u_{t+\tau}\}$:

$$\{\dot{u}_{t+\tau}\} = \{\dot{u}_t\} + \tau\left((1-a)\{\ddot{u}_t\} + a\{\ddot{u}_{t+\tau}\}\right) \quad (13)$$

$$\{u_{t+\tau}\} = \{u_t\} + \tau\{\dot{u}_t\} + \frac{\tau^2}{2}\left((1-b)\{\ddot{u}_t\} + b\{\ddot{u}_{t+\tau}\}\right) \quad (14)$$

The general matrix form for the equations is

$$[M]\{\ddot{u}\} + [C]\{\underline{u}\} + [K]\{u\} = \{F(t)\} \quad (15)$$

For time (t + τ) Equation 15 can then be written as

$$[\overline{K}]\{u_{t+\tau}\} = \{R_{t+\tau}\} \quad (16)$$

where

$$[\overline{K}] = [M] + \tau a[C] + \frac{\tau^2}{2} b[K] \quad (17)$$

$$\{R_{t+\tau}\} = \frac{\tau^2}{2} b\{F_{t+\tau}\} + [M]\left(\{u_t\} + \tau\{\dot{u}_t\} + \frac{\tau^2}{2}(1-b)\{\ddot{u}_t\}\right) + [C]\left(\tau a\{u_t\} + \frac{\tau^2}{2}(2a-b)\{\dot{u}_t\} + \frac{\tau^3}{2}(a-b)\{\ddot{u}_t\}\right) \quad (18)$$

when τ = Δt, Newmark's method is unconditionally stable if

$$a \geq \frac{1}{2}; \quad b \geq \frac{1}{2}\left(a + \frac{1}{2}\right)^2$$

The values used in this investigation are $a = b = ½$. Thus, the value of $u_{t+\Delta t}$ at each time step is solved. The value of $\{\ddot{u}_{t+\Delta t}\}$ and $\{\underline{u}_{t+\Delta t}\}$ are computed.

STRAIN-DISPLACEMENT RELATIONSHIPS

The shear strain and displacement relationship is not well defined in many practical problems reasonable expressions must be assumed and used as the basis for evaluating the shear strain in each particular case. One such relationship has been recommended by Prakash and Puri (1988) as,

$$\gamma = \frac{\text{Amplitude of foundation vibration}}{\text{Average width of foundation}} \quad (19)$$

Because evaluation of shear strain in the field is, in many cases, not clear, Kagawa and Kraft (1980) used a following relationship for horizontal displacement.

$$\gamma_x = \frac{(1+\nu)X}{2.5D} \quad (20)$$

where,

ν = Poisson's ratio

X = horizontal displacement in x-direction

D = diameter of pile

Rafnsson (1992) stated that, the shear strain due to rocking can be reasonably determined as

$$\gamma_{\phi} = \frac{\phi}{3} \tag{21}$$

where, ϕ = rotation of foundation about y axis

The shear strain- displacement relationship for couple sliding and rocking can be determined as:

$$\gamma_{x\phi} = \frac{(1+\nu)X}{2.5D} + \frac{\phi}{3} \tag{22}$$

After the bridge abutment displacements occur, the soil strain increases. The soil modulus will be reduced because the soil modulus is a function of strain. This means that, the stiffness and damping factor will also be changed.

CASE STUDIES

Two bridges abutments were analyzed. They are Old St. Francis River and Old Wahite Ditch Bridge abutments. The Old St. Francis River Bridge abutment (13.0 m x 2.1 m) is supported on 8 vertical piles and 8 batter piles. All of piles are a cylindrical concrete piles with 0.506 m (20 inch) diameter and 10.67 m (35 ft) length. Plan and cross section of bridge abutment are shown in Figure 6.

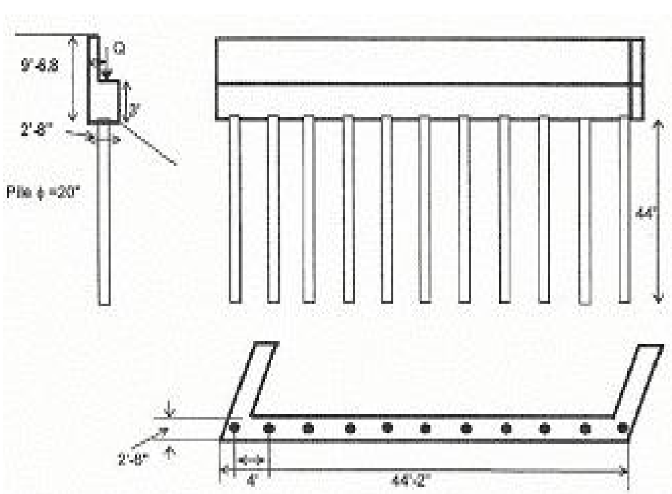


Figure **6** Old St. Francis River Pile Layout – Bridge Abutment

The stiffness of spring and damping factors are calculated with pile length 10.67 m (35 ft), pile radius 0.203 m (8 inch), elastic modulus of pile material 2.15×10^7kN-m^2. Stiffness and damping factors of a single batter piles are 0.8 times that of a vertical pile. (Prakash and Subramanayam, 1964).

Geotechnical field investigation data was collected for the subsurface condition of the site.(See Munaf et al 2003 for details) The subsurface soil consists of up to 25 feet of medium to stiff clay underlain by about 30 ft of medium dense sand underlain by a dense sand to a depth of upto 192.0 ft. For shake analysis its depth has been assumed up to 200 ft. Nonlinear soil modulus and strain-dependant material damping used in this analysis for sand and clay are shown in Figure 7 respectively. The values of G/G_{max} and damping ratio ξ for silt were obtained from the mean value of sand and clay. These values will be used to evaluate the time histories of earthquake at the base of bridge abutment.

Nonlinear soil shear modulus case is shown in Fig. 7. The values of G/G_{max} for silt were obtained from the mean value of sand and clay (PI=30). The vertical load acting on the top of bridge abutment is obtained from bridge structure analysis. Accordingly, a vertical load as Q = 100 kN (22481 lb) per m length of abutment is used in this case. The self-weight of bridge abutment was calculated by multiply its area with unit weight of bridge abutment material γ =3.58 kN/m^3 (150.19 pcf). The earth pressure behind the bridge abutment is calculated with soil data as unit weight of soil 19.54 kN/m^3 (122 pcf), internal friction angle 33^o and friction angle between soil and abutment 33^o. Due to seismic condition, all of loads were modified by a time dependent seismic coefficient.

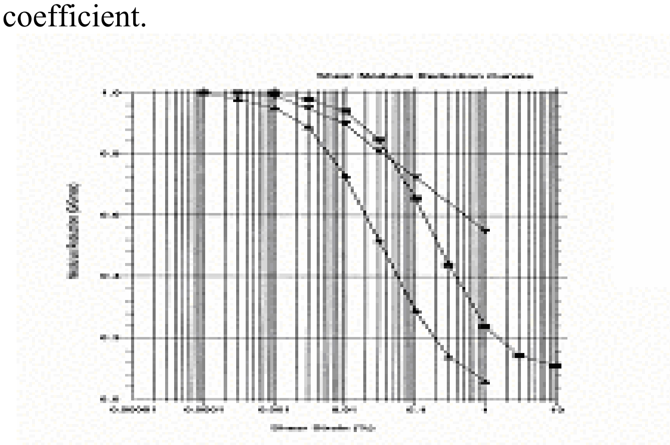
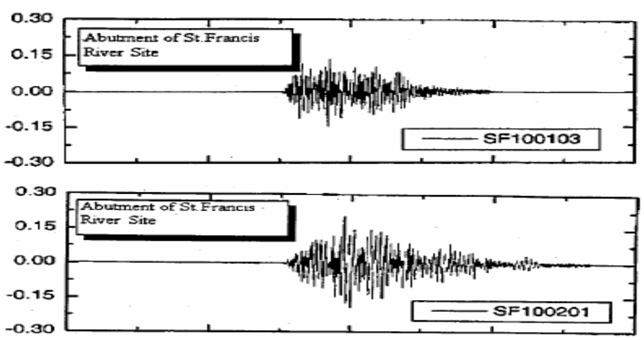


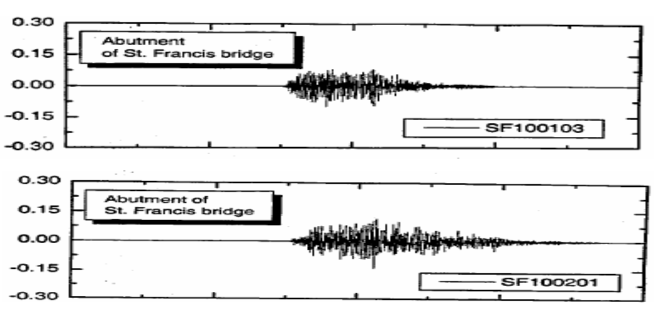
Figure 7 values of G/G_{max} versus shear strain (γ) (Seed and Idriss 1970, for sand, and Seed et al, 1986, for gravel)

To compare effect of peak acceleration, frequency or magnitude of earthquake (M), forty seismic accelerations time histories combination, with variation of peak acceleration, frequency and magnitude, were used in this study. Figure 8a and b show two of time histories of horizontal and vertical abutment base accelerations used for this analysis. Those accelerations are obtained based on wave propagation analysis of base rock motion at that site for horizontal acceleration.

To obtain vertical abutment base acceleration, the elastic modulus is changed as for shear wave and wave propagation analyses was conducted as for horizontal case. Peak vertical acceleration (α_v) was adjusted to 2/3 peak horizontal acceleration of (α_h) as per AASHTO recommendation and all the vertical motion was adjusted accordingly. For both horizontal and vertical accelerations, the peak magnitudes, as well as frequencies are different. Also peaks do occur at the same time.



a) Horizontal



b) Vertical

Figure 8 Acceleration time Histories used in this analysis, SF100103 for Peak horizontal acceleration o.106g,, M6.2 and SF100201 for Peak horizontal acceleration 0.113g,, M7.2

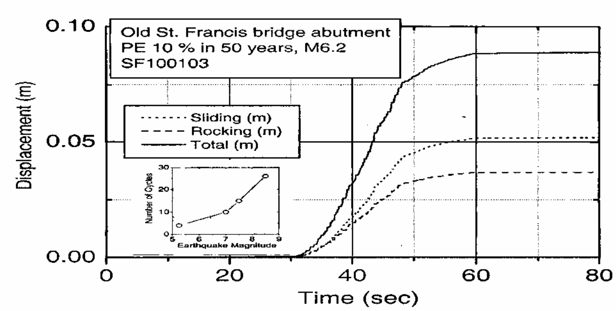
Figure 9 a and b show time histories of permanent displacement of bridge abutment for PE 10% in 50 years M6.2 and M7.2, respectively. Table 1 shows the sliding, rocking and total displacement at top of bridge abutment for different magnitude of earthquake (M), and PE of 10% and 2% in 50 years. Similar analysis was performed for Old Wahite Ditch Bridge site and the results are shown in Table 2.

The computed permanent displacement for peak acceleration 0.113g, M7.2 was about 50% higher than that for peak acceleration 0.106g M 6.2. These results lead to the following,

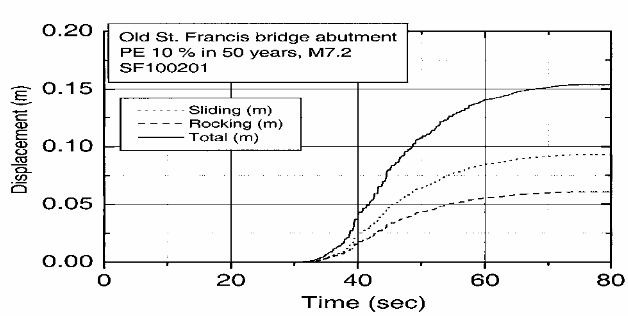
1. Frequency for M7.2 is much higher than that for M6.2.
2. Magnitude of acceleration for M7.2 is higher than that for M6.2.

DISCUSSION

It will be seen that these abutments may experience a displacement of 52 cm to 24.2 cm (Table 1 and Table 2). This is upper bound displacements. However, due to dynamic soil structure interaction effects of the two abutments and the connecting superstructure, there will be reduction in these displacements significantly.



a) Peak acceleration 0.106g,, M6.2.



b) Peak acceleration0.113g, , M7.2

Figure 9 Time Histories of Sliding, Rocking and Total Displacement

Table 1 Displacement of Old St. Francis River Bridge abutment

Displacement at top of abutment	PE 10% in 50 years		PE 2% in 50 years	
	M6.2	M7.2	M6.4	M8.0
Sliding (m)	0.052	0.093	0.096	0.31
Rocking (m)	0.037	0.061	0.069	0.21
Total (m)	0.089	0.154	0.165	0.52
Significant Cycles	8	11	9	20
Displacement in 1-cycle	0.011	0.014	0.018	0.026

Table 2 Displacement of Old Wahite Ditch bridge abutment

Displacement at top of abutment	PE 10% in 50 years		PE 2% in 50 years	
	M6.4	M7.0	M7.8	M8.0
Sliding (m)	0.037	0.028	0.139	0.178
Rocking (m)	0.018	0.053	0.0513	0.064
Total (m)	0.056	0.080	0.190	0.242
Significant Cycles	9	10	18	20
Displacement in 1-cycle	0.007	0.008	0.011	0.012

If it has been assumed that final displacement may not exceed that in one cycle, then the maximum displacements of the abutment may not exceed 2.6 cm to 1.2 cm (Table 1 and Table 2), which is quite acceptable. This, however, may be considered as the lower bound.

However, the assumption of real displacement corresponding to 1-cycle is subject to some serious question and examination at this time (2003).

CONCLUSIONS

The following conclusion are drawn:

1. A realistic displacement model for bridge abutment under earthquake condition has been developed.
2. The model can consider non-linear soil properties.
3. The computed displacements are not only controlled by peak ground acceleration, where the peak ground acceleration is commonly used in current earthquake design regulation, but by frequency of ground motion.
4. To evaluate bridge abutment stability, the displacement analysis of bridge abutment should be conducted.
5. The predictions of displacements represent a considerable advance over the existing solutions. However dynamic soil structure effects are still not clear and known; and further work is needed.

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