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A VARIABLE-ORDER FLOM ALGORITHM FOR HEAVY-TAILED CLUTTER SUPPRESSION

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Abstract—The normalized fractionally-lower order moment (FLOM) algorithm exhibits fast convergence but high excess mean squared error (MSE) when the order is less than 2. This paper proposes a method using variable order moments to adaptively changing the order during adaptation, thus achieving both fast initial convergence and low excess MSE in the steady state. The algorithm is applied to both Gaussian and heavy-tailed non-Gaussian clutter suppression in phased array applications. The results show better performances of the proposed algorithm over the normalized FLOM and normalized Least Mean Square (NLMS) algorithms. The proposed algorithm also performs well in other adaptive filtering applications such as system identification and noise/echo suppression.

I. INTRODUCTION

Space-time adaptive processing refers to combined spatial beamforming and temporal filtering of radar/sonar returns in phased array systems. It uses multiple antenna elements followed by tapped-delay-lines to coherently process multiple pulses, thus providing superior ability to suppress jammers and clutters while preserving desired signal target [1]. Since its introduction, STAP has been rigorously researched and has been shown to provide significant performance gains in interference suppression and target detection [2]. Many STAP algorithms deal with common scenarios where clutters and noises are complex Gaussian, which leads to mathematically tractable solutions [1]–[4]. However, recent studies and field measurements have found [6]–[8], [10] that heavy-tailed non-Gaussian clutters often occur in backscatters from mountain tops, dense forest canopy, rough sea surfaces, and manmade concrete objects, etc. These radar clutters are spiky, impulsive in nature and cause significant performance degradation in STAP and target detection. Many technical issues still remain unsolved for non-Gaussian environments.

To combat heavy-tailed non-Gaussian clutters, a fractionally lower-order moments (FLOM) adaptive algorithm has been proposed in [10] and an improved normalized FLOM is presented in [11] for adaptive array beamforming. The FLOM and NFLOM algorithm differs from the commonly used Minimum Variance Distortionless Response (MVDR) beamformers and/or the Normalized Least Mean Square (NLMS) algorithm, in that it minimizes the p -th order moment ($0 < p \leq 2$) of the output signal rather than its variance ($p = 2$). It has been shown [11] that the NFLOM algorithm with a smaller order p converges faster in both Gaussian and heavy-tailed clutter environments. In contrast, the steady-state Mean Square Error (MSE) is lower when the order p is larger.

In this paper, we propose a variable-order FLOM algorithm that uses a small order p at the beginning of the adaptation and gradually increases the order to a large p to achieve both fast convergence and low steady-state MSE. The proposed algorithm is evaluated in both Gaussian and compound K clutter scenarios. The results show that the variable-order FLOM algorithm outperforms the plain NFLOM and NLMS algorithms.

II. COMPOUND K DISTRIBUTIONS

Several statistical models have been used to describe the impulsive non-Gaussian clutter environment including the compound K and generalized Gaussian distributions [6]. The compound complex Gaussian model is a popular approach, where the clutter/noise process is the product of two random processes: $Z = \sqrt{G} \cdot X$, with G being the texture and X the speckle component. If X is complex Gaussian, then the envelop of X , denoted as R , follows the Rayleigh distribution with a random mode \sqrt{G} . The conditional probability density function

of R is given as

$$f_R(r|G) = \frac{r}{G} \exp\left(-\frac{r^2}{2G}\right) \quad (1)$$

Assume that G is gamma distributed with a shape parameter ν and a scale parameter β

$$f_G(g) = \frac{2}{\beta\Gamma(\nu)} \left(\frac{2g}{\beta}\right)^{\nu-1} \exp\left(-\frac{2g}{\beta}\right), \quad g \geq 0 \quad (2)$$

where $\Gamma(\cdot)$ is the Euler gamma function. The texture component G is often temporally correlated with an exponential autocorrelation function $\phi_{GG}(\tau) = E[g(t)g(t-\tau)] = C \exp(-\lambda\tau)$ [9], where C and λ are constants and $E[\cdot]$ denotes the mathematical expectation. The marginal pdf of R is the compound K distribution obtained by averaging $f_R(r|G)$ with respect to G

$$f_R(r) = \frac{4}{\sqrt{\beta}\Gamma(\nu)} \left(\frac{r}{\sqrt{\beta}}\right)^\nu K_{\nu-1}\left(\frac{2r}{\sqrt{\beta}}\right), \quad r \geq 0 \quad (3)$$

where K_ν is the modified Bessel function of the second kind and with order ν . The pdf of the compound K distribution is plotted in Fig. 1. The tails of the compound K pdfs are much higher than the Rayleigh distribution which is the envelope pdf of the complex Gaussian process. Note that the Rayleigh distribution is a special case of the compound K distribution when $\nu \rightarrow \infty$ and $E(R^2) = \beta\nu$ remains constant. The smaller the ν , the heavier the tail of the K distribution, the more impulsive the clutters.

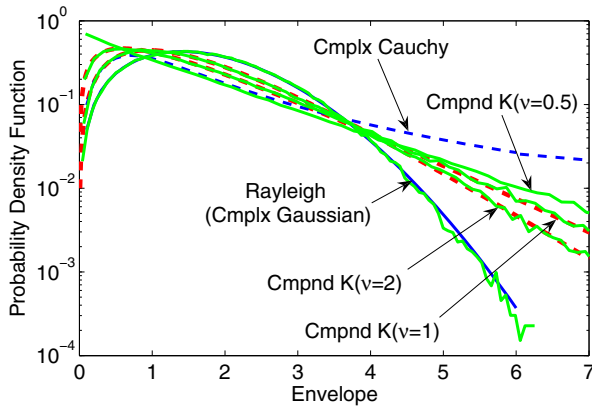


Fig. 1. The envelop Probability Density Function (pdf) of the compound K clutters, in comparison to complex Gaussian (Rayleigh) and complex Cauchy [5] clutters. The compound K distributions exhibit heavier tails than Rayleigh (special case with $\nu = \infty$), but has lighter tails than the envelop of complex Cauchy distributions.

III. STAP AND THE NORMALIZED LMS ALGORITHM

Consider an arbitrary radar array antenna consisting of M elements with the m -th element located at Θ_m in a spherical coordinate system. Coherent bursts of K pulses are transmitted at a constant pulse repetition frequency (PRF) $f_r = 1/T_r$, where T_r is the pulse repetition interval (PRI). Radar returns are collected over a coherent processing interval (CPI) of length KT_r . Within each PRI, there are L time (range) samples collected to cover the range interval. This multidimensional data set can be visualized as a $M \times K \times L$ cube of complex samples [1]. For STAP performed in the space-slow-time domain, denote the received samples at range bin l as $u_{k,m}(t)$ with slow-time index $k = 1, 2, \dots, K$, and array element index $m = 1, 2, \dots, M$, and the sampling time index t . Let $N = M \times K$, then the $N \times 1$ concatenated space-time sample vector is

$$\mathbf{U}(t) = [\mathbf{u}_1^H(t), \dots, \mathbf{u}_k^H(t), \dots, \mathbf{u}_K^H(t)]^H, \quad (4)$$

$$\mathbf{u}_k(t) = [u_{k,1}(t), u_{k,2}(t), \dots, u_{k,M}(t)]^H, \quad (5)$$

where superscript $()^H$ denotes conjugate transpose.

The radar return vector $\mathbf{U}(t)$ is a mixture of the target echo (\mathbf{U}_s) with the uncorrelated jammer (\mathbf{U}_J), uncorrelated clutters (\mathbf{U}_c), and background noise (\mathbf{U}_n):

$$\mathbf{U}(t) = \mathbf{U}_s(t) + \mathbf{U}_J(t) + \mathbf{U}_c(t) + \mathbf{U}_n(t), \quad (6)$$

where

$$\mathbf{U}_s(t) = S(t)\mathbf{b}(\omega_s) \otimes \mathbf{a}(\Theta_s),$$

$$\mathbf{U}_J(t) = \sum_{i=1}^{N_J} S_{J_i} \mathbf{g}_{J_i} \otimes \mathbf{a}(\Theta_{J_i})$$

$$\mathbf{U}_c(t) = \sum_{i=1}^{N_c} S_{c_i} \mathbf{b}(\omega_{c_i}) \otimes \mathbf{a}(\Theta_{c_i}).$$

where the point target $S(t)$ is located at Θ_s and with Doppler frequency f_0 . The operator \otimes denotes the Kronecker matrix product, the temporal steering vector $\mathbf{b}(\omega_s) = [1, \dots, e^{-jk\omega_s}, \dots, e^{-j(K-1)\omega_s}]^H$ with the normalized Doppler frequency $\omega_s = 2\pi f_s/f_r$, and the spatial steering vector $\mathbf{a}(\Theta_s) = [1, e^{-j\Omega(\tau_{2s}-\tau_{1s})}, \dots, e^{-j\Omega(\tau_{Ms}-\tau_{1s})}]^H$ for location Θ_s , where $\tau_{ms} = |\Theta_m - \Theta_s|/c$ is the propagation delay from the signal source to the m -th array element, c is the wave propagation speed, and Ω the operating frequency. The N_J jammers S_{J_i} are at locations Θ_{J_i}

with gains $\mathbf{g}_{J_i} = [g_{J_i}(1), \dots, g_{J_i}(k), \dots, g_{J_i}(K)]^T$. The N_c independent clutter patches are uniformly distributed in a circular ring/sphere around the radar platform [1] with the i -th patch at Θ_{ci} and having a Doppler frequency ω_{ci} proportional to its angular location. The receiver noise \mathbf{U}_n appears as a uniform noise floor throughout the angle-Doppler plane.

The STAP system consists of a tapped-delay-line attached to each array element. Let \mathbf{W} be the concatenated weight vector of the STAP processor, then the output of the STAP $y(t)$ can be expressed in a matrix form as $y(t) = \mathbf{W}^H \mathbf{U}(t)$. For the Gaussian clutter environment, the Minimum Variance Distortionless Response (MVDR) method is commonly used for adapting the weight vector \mathbf{W} .

$$\min_{\mathbf{W}} E \{ |y(t)|^2 \}, \quad \text{subject to} \quad \mathbf{C}^H \mathbf{W} = \mathbf{h}, \quad (7)$$

where $E\{\cdot\}$ is the expectation operator, $E \{ |y(t)|^2 \} = \mathbf{W}^H \mathbf{R}_{\mathbf{uu}} \mathbf{W}$, and $\mathbf{R}_{\mathbf{uu}}$ is the covariance matrix of the concatenated input vector \mathbf{U} . The matrix \mathbf{C} is a set of linear constraints and \mathbf{h} is the desired response vector. For example, a simple point constraint [11] may be chosen as $\mathbf{C} = \mathbf{b}(\omega_s) \otimes \mathbf{a}(\Theta_s)$ and $\mathbf{h} = 1$, which enforces a unit gain response at the target location Θ_s and the Doppler frequency f_s . The optimal solution to the constrained minimization problem (7) is well-known assuming that the covariance matrix $\mathbf{R}_{\mathbf{uu}}$ has a full rank [3]:

$$\mathbf{W}_{opt} = (\mathbf{C}^H \mathbf{R}_{\mathbf{uu}}^{-1} \mathbf{C})^{-1} \mathbf{R}_{\mathbf{uu}}^{-1} \mathbf{C} \mathbf{h} \quad (8)$$

Direct implementation of (8) requires the knowledge of the covariance matrix of the array input vector and the Sample Matrix Inversion (SMI) method is often employed in practice [2]. Alternatively, the Normalized Least Mean Square (NLMS) algorithm provides a low-complexity iterative solution. Define

$$\mathbf{W}_q = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{h}, \quad (9)$$

$$\mathbf{B} = \mathbf{I} - \mathbf{C}[\mathbf{C}^H \mathbf{C}]^{-1} \mathbf{C}^H, \quad (10)$$

The iterative algorithm is then

$$\begin{aligned} \mathbf{W}(0) &= \mathbf{W}_q = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{h}, \\ \mathbf{W}(t+1) &= \mathbf{B} \left[\mathbf{W}(t) + \mu \frac{y^*(t) \mathbf{U}(t)}{\delta + \mathbf{U}^H(t) \mathbf{U}(t)} \right] + \mathbf{W}_q. \end{aligned} \quad (11)$$

where μ is the step size, and δ is the regularization parameter which prevents the numerical instability when the inputs are small [3], [4].

IV. THE VARIABLE-ORDER FLOM ALGORITHMS

In the severe, impulsive clutter environment, the conventional STAP algorithm suffers from performance loss due to two reasons: one is the high probability of outliers in the received samples; another is the large eigenvalue spread of the sample covariance matrix. An approach to combat these problems is the normalized fractionally lower-order moment (FLOM) algorithm which minimizes the p -th order moment rather than the variance of the STAP output [10], [11]

$$\min_{\mathbf{W}} E \{ |y(t)|^p \}, \quad \text{subject to} \quad \mathbf{C}^H \mathbf{W} = \mathbf{h}, \quad (12)$$

There is no closed-form solution for the optimal coefficients that minimizes the cost function, but a gradient descent method is available. Similar to the normalized LMS algorithm, the Normalized Fractionally-Lower Order Moment (N-FLOM) algorithm is iteratively adaptive as

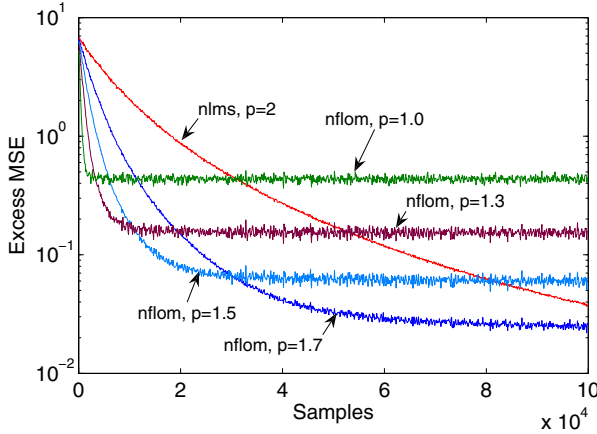
$$\begin{aligned} \mathbf{W}(0) &= \mathbf{W}_q = \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{h}, \\ \mathbf{W}(t+1) &= \mathbf{B} \left[\mathbf{W}(t) + \mu \frac{|y(t)|^{p-2} y^*(t) \mathbf{U}(t)}{\delta + \sum_i |u_i(t)|^p} \right] + \mathbf{W}_q \end{aligned} \quad (13)$$

where $u_i(t)$ are the elements of the input vector $\mathbf{U}(t)$. The parameters μ and δ are the same as those in the NLMS algorithm (11).

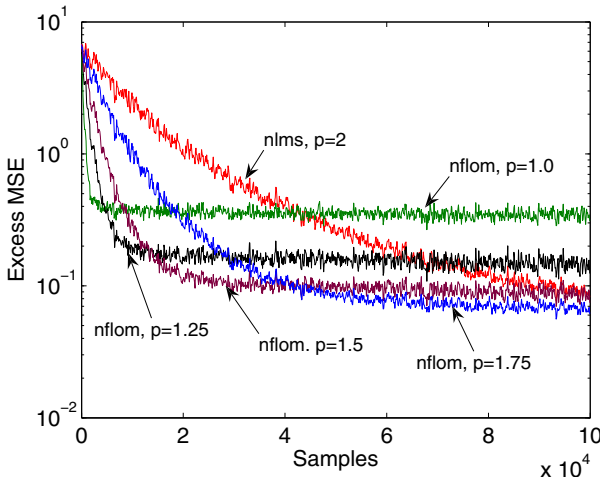
The N-FLOM algorithm reduces to the NLMS algorithm when $p = 2$ and to the median algorithm when $p = 1$ [5]. Numerical analysis has found [11] that, when the order p is smaller, the N-FLOM algorithm converges faster but exhibits larger steady-state mean square errors (MSE). This phenomenon is observed in both Gaussian and heavy-tailed clutters, as shown in Fig. 2, where the simulated excess MSE curves are the ensemble averages of 100 trials.

This observation motivates us to propose a variable-order FLOM algorithm to achieve both fast convergence and low steady-state MSE. Intuitively, the variable order p shall start with a small order and then gradually increases to $p = 2$. The proposed variable-order FLOM follows the procedures:

- 1) Choose $\mathbf{P} = \{P_l\} = [P_{min} : \Delta P : P_{max}]$. Set $l = 1$ and the initial order as $p = P_l$;
- 2) Select the estimation window size D and the threshold T_h ; Set the output energy of the previous window $\mathcal{E}_0 = DP_U$, where P_U is the total power of the input signal $\mathbf{U}(t)$;



(a) In Gaussian Clutters



(b) In Compound K Clutters ($\nu = 0.7$)

Fig. 2. The convergence curves of the N-FLOM algorithm in comparison to the NLMS and median algorithms. The array consists of $M = 10$ equi-spaced elements with $K = 7$ taps. Two wideband jammers and clutters have 30 dB power above background noises. The smaller the order p is, the faster the NFLOM converges, but with larger MSE in the steady state.

- 3) Adapt the filter coefficients $\mathbf{W}(t)$ based on (13) using the current order p . Estimate the output energy of the current window as $\mathcal{E}_1 = \sum_{i=1}^D |y(i)|^2$;
- 4) Compare \mathcal{E}_0 to \mathcal{E}_1 . If $\mathcal{E}_1 - \mathcal{E}_0 > DT_h^p$, then increment l and update the order p to P_l .
- 5) Set $\mathcal{E}_0 = \mathcal{E}_1$ and repeat Step 3 - 4 until $p = P_{max}$.

The parameter selection of the algorithm determines the convergence rate and the steady-state MSE. The threshold T_h can be set at the 1% to 10% of the signal-to-noise-ratio (SNR) or clutter-to-noise-ratio (CNR) level. The window size is normally chosen at several

hundred to several thousand samples. The selection of $\mathbf{P} = \{P_l\} = [P_{min} : \Delta P : P_{max}]$ is rather flexible with $P_{min} \geq 1$ and $P_{max} = 2$ for complex Gaussian clutters. For heavy-tailed clutters, slightly smaller P_{min} and P_{max} normally provide better results.

V. NUMERICAL EXAMPLES

A linear phased array is used to demonstrate the performances of the proposed variable-order FLOM algorithm. The array consists of $M = 10$ equally spaced elements at half wavelength of the operation frequency. The coherent pulse interval (CPI) is $K = 7$. The target signal has a power of 0 dB with respect to the background noise and is located at the angle of arrival (AoA) 20° with a normalized Doppler frequency of 0.25. The noises are independent among antenna elements and CPI taps with white Gaussian spectrum. Two wideband jammers present at AoA of -20° and $+50^\circ$, respectively. Each jammer has a full Doppler spectrum and with 15 dB power. In addition, many clutters impinge from different AoAs which are uniformly distributed between -180° and 180° . The Doppler frequencies of the clutters depend on their AoAs. The envelop of the clutters may be Gaussian or compound K with a total power of 30 dB.

The variable-order FLOM algorithm was evaluated for three clutter scenarios:

- *Example 1: A Complex Gaussian Clutter Scenario;*
- *Example 2: A Moderate Heavy-tail Scenario with $\nu = 2$ Compound K Clutter;*
- *Example 3: A Very Heavy-tail Scenario with $\nu = 0.7$ Compound K Clutter.*

The compound K clutters were simulated using the momentless nonlinear transformation method [9] and the autocorrelation of the gamma texture was $E[g(t)g(t - \tau)] = \exp(-\lambda\tau)$ with $\lambda = 100$ samples. For performance comparison, the design parameters for the variable-order FLOM were chosen commonly for all three scenarios: the window size $D = 1000$ samples, the threshold $T_h = 0.01$, the step size $\mu = 0.002$, and the regulation parameter $\delta = 20\text{CNR}$. The order p is variable as $[1.0:0.1:2.0]$ for all three examples. The convergence performance were evaluated by the excess MSE $J_{ex}(t)$ defined as [3]

$$J_{ex}(t) = E[|\mathbf{W}^H(t)\mathbf{U}(t)|^2] - J_{min} \quad (14)$$

and $J_{min} = E[|\mathbf{W}_{opt}^H \mathbf{U}(t)|^2]$. The MVDR optimal solutions were used here for all three examples although

the NFLOM algorithms are designed to minimize the lower order moments which yield better performance in terms of L_p norms.

Figures 3 and 4 plot the ensemble average of 100 trials of the convergence curve $J_{ex}(t)$. For all three clutter examples, the NLMS algorithm does not converge after 10^5 sample iterations, while the variable-order FLOM algorithm converges after 3×10^4 sample iterations, as shown in Fig. 3. For the Gaussian clutter environment, the excess MSE of the converged variable-order FLOM algorithm is much smaller than that achievable in heavy-tailed clutters. It has been verified that the variable-order FLOM converges to the same solution as the NLMS algorithm. Figure 4 shows that the variable-order FLOM algorithm take the initial fast convergence property of the median algorithm as $P_{min} = 1$. After it converges with $p = 1$ at a large excess MSE, it automatically switches to higher fractional orders and then continues to reduce the excess MSE. It achieves the lowest excess MSE with final order being $P_{max} = 2$. Consequently, it achieves both fast convergence and low excess MSE.

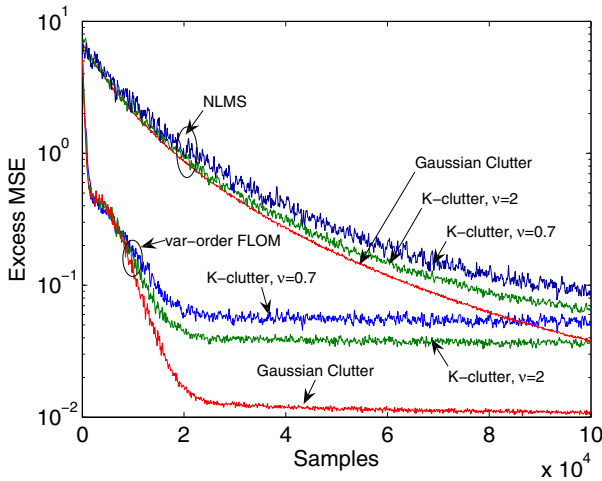


Fig. 3. Convergence curves of the variable-order FLOM algorithm in comparison to the NLMS algorithm. Three clutter environments: complex Gaussian ($\nu = \infty$), compound K clutter with shape parameter $\nu = 2.0$, and compound K-clutter with $\nu = 0.7$. The order parameters are $P_{min} = 1.0$, $\Delta P = 0.1$, $P_{max} = 2.0$, the threshold $T_h = 0.01$, and the window size $D = 1000$.

The beampattern of the STAP is also evaluated, which is defined as

$$\Psi(\Theta, f_d) = |W_{opt}^H \mathbf{b}(f_d) \otimes \mathbf{a}(\Theta)|^2 \quad (15)$$

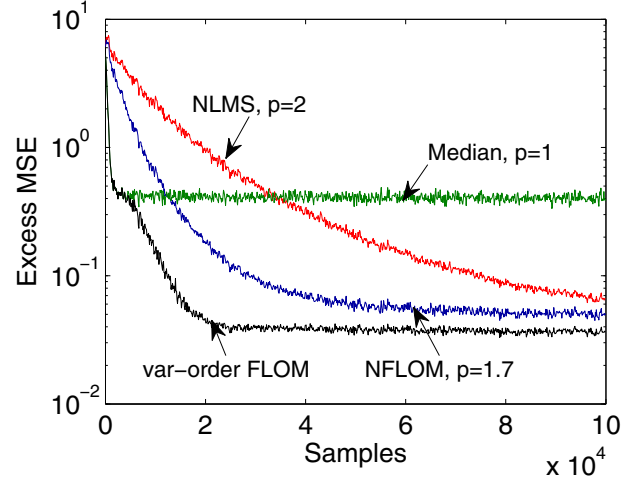
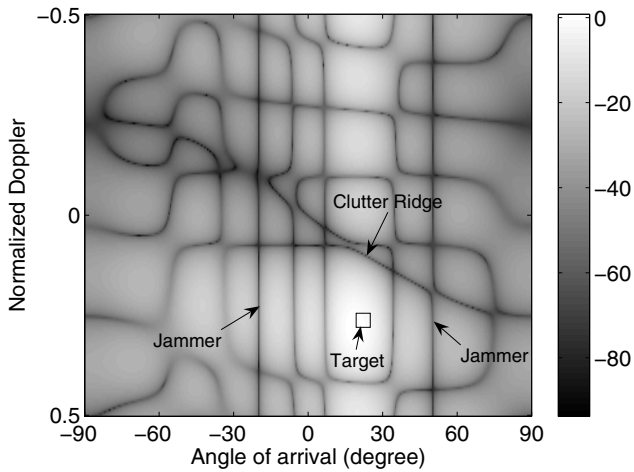


Fig. 4. Convergence curves of the variable-order FLOM algorithm in comparison to the NLMS, median, and NFLOM algorithms. The interference environment consists of compound K clutters with shape parameter $\nu = 2.0$ and two wideband jammers. The order parameters are the same as those in Fig. 3. The variable-order FLOM outperforms the conventional algorithms.

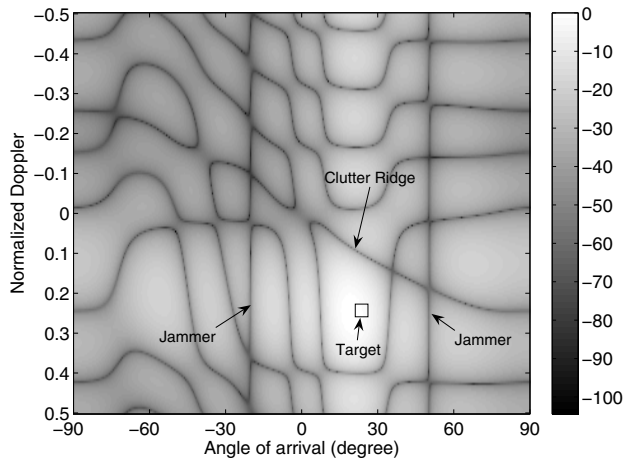
The beampatterns of the variable-order FLOM algorithm under Gaussian and compound K clutters are plotted in Fig. 5. The location of the target is indicated by the small square on the angle-Doppler plane at $f_d = 0.25$ and $AoA = 20^\circ$. When the noise and clutters are Gaussian, the proposed algorithm can effectively suppress the clutters and jammers by placing deep nulls at jammer locations and the clutter ridge, as shown in Fig. 5(a). Similar performance is achieved for heavy-tailed clutter with $\nu = 2$, as shown in Fig. 5(b).

VI. CONCLUSION

A variable-order FLOM adaptive algorithm has been proposed for space-time adaptive processing (STAP) of phased array radar systems. The algorithm improves upon the NFLOM algorithm and the commonly used NLMS algorithm in that the weight adaptation is proportional to a variable p -order moment of the error rather than a fixed order moment or the mean square error, where p is often a fractional order between 0 and 2. When the order P varies from a small order to a large order, the proposed algorithm achieves fast initial convergence associated with small order NFLOMs and low steady-state errors associated with the large order NFLOM (or NLMS). The excess mean squared error (MSE) curves have been evaluated for both Gaussian



(a) In complex Gaussian clutters



(b) In heavy-tailed clutters

Fig. 5. Beam patterns of the variable-order FLOM algorithm for STAP in Gaussian/non-Gaussian clutters. Deep nulls are formed at the jammer locations and the clutter ridge, while the target is passed with unit gain.

clutter and non-Gaussian, heavy-tailed clutter scenarios. The results show that the proposed variable-order FLOM converges much faster than the plain NFLOM and NLMS algorithms.

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