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A Statistical Simulation Model for Mobile Radio Fading Channels

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Abstract— Recently, a Clarke's model-based simulator was proposed for Rayleigh fading channels. However, that model, as shown in this paper, may encounter statistic deficiency. Therefore, an improved model is presented to remove the statistic deficiency. Furthermore, a new simulation model is proposed for Rician fading channels. This Rician fading simulator, which is using the improved Rayleigh fading simulator with finite number of sinusoids plus a *zero-mean* stochastic sinusoid as the specular (line-of-sight) component, is different from all the existing Rician fading simulators, which have *non-zero mean* deterministic specular component. The statistical properties of the proposed Rayleigh and Rician fading channel models are analyzed in detail, which shows that these statistics either exactly match or quickly converge to the theoretically desired ones. Additionally and importantly, the probability density function of the Rician fading phase is not only independent from time but also uniformly distributed, which is fundamentally different from that of all the existing Rician fading models. The statistical properties of the new simulators are evaluated by numerical results, finding good agreement in all cases.

I. Introduction

Mobile radio channel simulators are commonly used in the laboratory because they allow system tests and evaluations which are less expensive and more reproducible than field trials. In the past, there are many different approaches to the modeling and simulation of a mobile radio channel [2]-[25]. Among them, the well known mathematical reference model due to Clarke [2] and its simplified simulation model due to Jakes [5] have been widely used for Rayleigh fading channels for about three decades. However, Jakes' simulator is a deterministic model, and it has difficulty to create multiple uncorrelated fading waveforms for frequency selective fading channels and multiple-input multiple-output (MIMO) channels, therefore different modifications of Jakes' simulator have been reported in the literature [9], [15], [17], [18]. Despite the extensive acceptance and application of Jakes' simulator, some important limitations of the simulator were determined and discussed in detail recently [20]. It was shown in [20] that Jakes' simulator is wide-sense nonstationary. It was further pointed out in [23] that the second-order statistics of Jakes' models [5] and its various modifications [9], [15], [17], [18], [20] do not match the desired ones of Clarke's reference model. Moreover, even in the limit as the number of sinusoids approaches infinity, the autocorrelations and cross-correlations of the quadrature components, and the autocorrelation of the squared envelope of the modified simulators fail to match the desired correlation statistics. These statistic deficiencies are removed by those models proposed in [24], [25]. Recently, Pop and Beaulieu [22] proposed an-

other simulation model directly based on Clarke's reference model, and they made some interesting view points. In this paper, it will be shown that Pop and Beaulieu's model [22] may encounter statistic deficiency as well, and an improved model will be presented.

For Rician fading channel simulations, all the existing Rician channel models assume that the specular (line-of-sight) component is either non-zero *constant* [1], [13], or *deterministic time-varying* parameter [4], [15]. These assumptions were probably made in favor of mathematically convenient derivations, but they are not reflecting the physical specular components. For example, constant specular component implies Direct Current (DC) signals which is impossible to get from fading channels; deterministic time-varying is contradicting the random nature of specular component from time to time and from mobile to mobile. Moreover, all these Rician fading models are non-stationary in the wide sense according to [4], and the probability density function (PDF) of the fading phase is time-dependent [4], [15], [27]. In this paper, a statistical simulation model will be proposed for Rician fading channels. The specular component will be assumed a *zero-mean* random variable with pre-chosen angle of arrival and random initial phase. This assumption implies that different specular component in different channel may have different initial phase, which is true in the physical channel.

The rest of this paper is organized as follows. Section II briefly reviews Pop and Beaulieu's Rayleigh fading simulator, especially on its statistical properties, then an improved simulator is proposed. Section III presents a new statistical sum-of-sinusoids simulation model for Rician fading channels, statistical properties of this new model are analyzed in detail. Section IV presents the performance evaluation of the new Rayleigh and Rician simulators by extensive numerical results. Section V draws the conclusion.

II. A WSS Rayleigh Fading Channel Simulator

In this section, we present some key second-order statistics of a recently proposed wide-sense stationary sum-of-sinusoids Rayleigh fading channel simulator. It is shown this WSS channel simulator has statistic deficiency. Furthermore, we propose an improved model for Rayleigh fading channels.

A. Pop and Beaulieu's Simulator

Based on Clarke's mathematical reference model [2], [27], Pop and Beaulieu [22] developed a Rayleigh fading simula-

tor whose low-pass fading process is given by:

$$X(t) = X_c(t) + jX_s(t) \qquad (1a)$$

$$X_c(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos\left(w_d t \cos\frac{2\pi n}{N} + \phi_n\right) \qquad (1b)$$

$$X_s(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \sin\left(w_d t \cos\frac{2\pi n}{N} + \phi_n\right), \qquad (1c)$$

where w_d is the maximum angular Doppler frequency, ϕ_n are mutually independent and uniformly distributed over $[-\pi, \pi)$ for all n. Note that a normalization constant is used to make $X(t)$ have unit power. In [22], Pop and Beaulieu gave excellent and detailed discussion on the PDF of the fading envelope, and the autocorrelation of the complex envelope of this model. In order to further reveal the statistical properties of this model, we present some second-order statistics of this model as follows:

$$R_{X_c X_c}(\tau) = \frac{1}{2N} \sum_{n=1}^{N} \cos\left(w_d \tau \cos\frac{2\pi n}{N}\right) \qquad (2a)$$

$$R_{X_s X_s}(\tau) = \frac{1}{2N} \sum_{n=1}^{N} \cos\left(w_d \tau \cos\frac{2\pi n}{N}\right) \qquad (2b)$$

$$R_{X_c X_s}(\tau) = \frac{1}{2N} \sum_{n=1}^{N} \sin\left(w_d \tau \cos\frac{2\pi n}{N}\right) \qquad (2c)$$

$$R_{X_s X_c}(\tau) = -\frac{1}{2N} \sum_{n=1}^{N} \sin\left(w_d \tau \cos\frac{2\pi n}{N}\right) \qquad (2d)$$

$$R_{XX}(\tau) = 2R_{X_c X_c}(\tau) + j2R_{X_c X_s}(\tau) \qquad (2e)$$

$$R_{|X|^2|X|^2}(\tau) = 1 + 4R_{X_c X_c}^2(\tau) + 4R_{X_c X_s}^2(\tau) + \frac{1}{N}, \qquad (2f)$$

where $R_{X_c X_c}(\tau)$ and $R_{X_s X_s}(\tau)$ are the autocorrelations of the quadrature components, $R_{X_c X_s}(\tau)$ and $R_{X_s X_c}(\tau)$ are the cross-correlations of the quadrature components, and $R_{XX}(\tau)$ and $R_{|X|^2|X|^2}(\tau)$ are the autocorrelations of the complex envelope and the squared envelope, respectively. It is noted that $R_{|X|^2|X|^2}(\tau)$ contains fourth-order statistical information of the quadrature components of this model.

The proof of these statistics shown above is similar to the proof procedure of Theorem 1 of [24], details are omitted here for brevity. Figs. 1-3 show the autocorrelations of the complex envelope and squared envelope for this model with $N = 17$, $N = 18$ and $N = \infty$.

From eqns (2) and Figs. 1-3, we can observe and make the following remarks:

Remark 1: Although the statistics of this model with $N = \infty$ are the same as the desired ones of Clarke's mathematical reference model, when N is finite, the statistics of this model is different from the desired ones.

Remark 2: The statistics of this model are not asymptotically converging to the desired ones when N increases. This is agreeable to the discussion of the real part of $R_{XX}(\tau)$ in [22].

Remark 3: When N is finite and odd, the imaginary part of $R_{XX}(\tau)$ can be significantly different from zero,
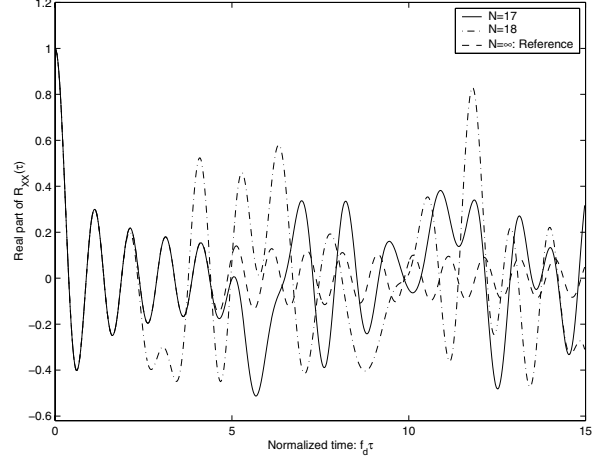

Fig. 1. The real part of the autocorrelation of the complex envelope of the model in [22] with $N = 17$, $N = 18$ and $N = \infty$ for reference.
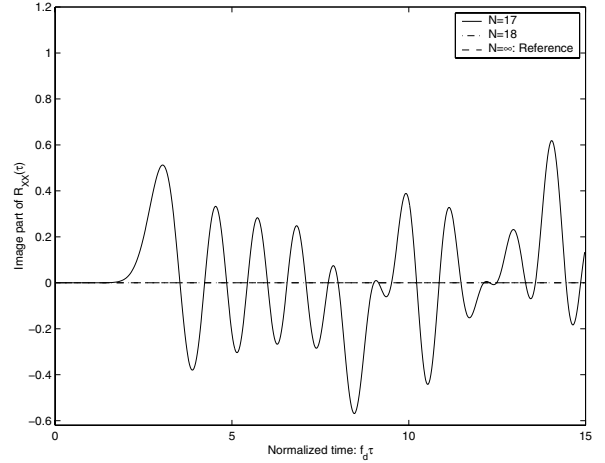

Fig. 2. The imaginary part of the autocorrelation of the complex envelope of the model in [22] with $N = 17$, $N = 18$ and $N = \infty$ for reference.
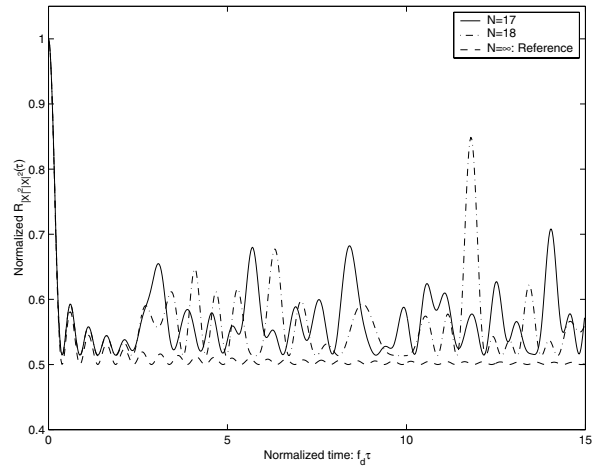

Fig. 3. The autocorrelation of the squared envelope of the model in [22] with $N = 17$, $N = 18$ and $N = \infty$ for reference.

the desired statistics for Clarke's mathematical reference model, which also implies that the quadrature components of this model is statistically correlated when N is odd. This was not realized in [22].

B. A New Rayleigh Fading Channel Simulator

To remove the shortcoming of Pop and Beaulieu's model, we propose an improved simulation model as follows:

Definition 1: The normalized low-pass fading process of a new statistical sum-of-sinusoids simulation model is defined by

$$Y(t) = Y_c(t) + jY_s(t) \tag{3a}$$

$$Y_c(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos(w_d t \cos\alpha_n + \phi_n) \tag{3b}$$

$$Y_s(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \sin(w_d t \cos\alpha_n + \phi_n) \tag{3c}$$

with

$$\alpha_n = \frac{2\pi n + \theta_n}{N}, \qquad n = 1, 2, \cdots, N, \tag{4}$$

where ϕ_n and θ_n are statistically independent and uniformly distributed over $[-\pi, \pi)$ for all n. It is noted that the difference between this improved model and Pop and Beaulieu's model is the introduction of random variables θ_n to the angle of arrival.

It can be shown that the first-order statistics of this improved model is the same as those of Pop and Beaulieu's model. However, the second-order statistics of this improved model are different, and they are presented as follows:

Theorem 1: The autocorrelation and cross-correlation functions of the quadrature components, and the autocorrelation functions of the complex envelope and the squared envelope of fading signal $Y(t)$ are given by

$$R_{Y_c Y_c}(\tau) = \frac{1}{2} J_0(w_d \tau) \tag{5a}$$

$$R_{Y_s Y_s}(\tau) = \frac{1}{2} J_0(w_d \tau) \tag{5b}$$

$$R_{Y_c Y_s}(\tau) = 0 \tag{5c}$$

$$R_{Y_s Y_c}(\tau) = 0 \tag{5d}$$

$$R_{YY}(\tau) = J_0(w_d \tau) \tag{5e}$$

$$R_{|Y|^2 |Y|^2}(\tau) = 1 + J_0^2(w_d \tau) + \frac{1}{N}. \tag{5f}$$

Proof: The proof is similar to those of Theorems 1 and 2 in [24], details are omitted for brevity. ∎

It should be emphasized here that the autocorrelation and cross-correlation functions given by (5a)-(5e) do not depend on the number of sinusoids N, they match the desired second-order statistics exactly irrespective to the value of N. Furthermore, the autocorrelation function of the squared envelope asymptotically approaches the desired one [27] as the number of sinusoids N approaches infinity, while good approximation has been observed when

N is as small as 8. These analytical statistics will be confirmed by numerical results in Section IV. Also, if we choose $\theta_n = \theta$ for all n, all the statistics of $Y(t)$ will be the same as shown above, but the convergence of the ensemble average in simulation is slower.

Before concluding this section, it is important to point out that the new simulation model can be directly used to generate uncorrelated faders for frequency selective Rayleigh channels, MIMO channels, and diversity combing techniques. Let $Y_k(t)$ be the kth Rayleigh fader given by

$$Y_k(t) = \sqrt{\frac{1}{N}} \left\{ \sum_{n=1}^{N} \cos\left[w_d t \cos\left(\frac{2\pi n + \theta_{n,k}}{N} \right) + \phi_{n,k} \right] \right.$$
$$\left. + j \sum_{n=1}^{N} \sin\left[w_d t \cos\left(\frac{2\pi n + \theta_{n,k}}{N} \right) + \phi_{n,k} \right] \right\}, \tag{6}$$

where $\theta_{n,k}$ and $\phi_{n,k}$ are mutually independent and uniformly distributed over $[-\pi, \pi)$ for all n and k. Then, $Y_k(t)$ retains all the statistical properties of $Y(t)$ which is defined by equations (3), furthermore, $Y_k(t)$ and $Y_l(t)$ are uncorrelated for all $k \neq l$, due to the mutual independence of $\theta_{n,k}$, $\phi_{n,k}$, $\theta_{n,l}$ and $\phi_{n,l}$ when $k \neq l$.

III. RICIAN FADING CHANNEL SIMULATOR

In this section, we present a statistical Rician fading simulation model and its statistical properties.

Definition 2: The normalized low-pass fading process of a new statistical simulation model for Rician fading is defined by

$$Z(t) = Z_c(t) + jZ_s(t) \tag{7a}$$
$$Z_c(t) = \left[Y_c(t) + \sqrt{K} \cos(w_d t \cos\theta_0 + \phi_0) \right] / \sqrt{1+K} \tag{7b}$$
$$Z_s(t) = \left[Y_s(t) + \sqrt{K} \sin(w_d t \cos\theta_0 + \phi_0) \right] / \sqrt{1+K} \tag{7c}$$

where K is the ratio of the specular power to scattered power, θ_0 and ϕ_0 are the angle of arrival and the initial phase, respectively, of the specular component, and ϕ_0 is a random variable uniformly distributed over $[-\pi, \pi)$.

We now present the correlation statistics of the fading $Z(t)$ in the following theorem.

Theorem 2: The autocorrelation and cross-correlation functions of the quadrature components, and the autocorrelation functions of the complex envelope and the squared envelope of fading signal $Z(t)$ are given by

$$R_{Z_c Z_c}(\tau) = [J_0(w_d \tau) + K \cos(w_d \tau \cos\theta_0)] / (2 + 2K) \tag{8a}$$
$$R_{Z_s Z_s}(\tau) = [J_0(w_d \tau) + K \cos(w_d \tau \cos\theta_0)] / (2 + 2K) \tag{8b}$$
$$R_{Z_c Z_s}(\tau) = K \sin(w_d \tau \cos\theta_0) / (2 + 2K) \tag{8c}$$
$$R_{Z_s Z_c}(\tau) = -K \sin(w_d \tau \cos\theta_0) / (2 + 2K) \tag{8d}$$
$$R_{ZZ}(\tau) = [J_0(w_d \tau) + K \exp(jw_d \tau \cos\theta_0)] / (1 + K) \tag{8e}$$
$$R_{|Z|^2 |Z|^2}(\tau) = \left\{ 1 + J_0^2(w_d \tau) + 2K \left[1 + J_0(w_d \tau) \cos(w_d \tau \cos\theta_0) \right] \right.$$
$$\left. + K^2 + \frac{1}{N} \right\} / (1 + K)^2. \tag{8f}$$

Proof: Based on the assumption that the initial phase of the specular component is random and uniformly distributed over $[-\pi, \pi)$, and it is independent from the initial phases of the scattered components, one can prove this theorem by using the results of Theorem 1. ∎

We now present the PDFs of the fading envelope $|Z|$ and phase $\Psi(t) = \arctan[Z_c(t), Z_s(t)]^1$.

Theorem 3: When N approaches infinity, the envelope $|Z|$ is Rician distributed and the phase $\Psi(t)$ is uniformly distributed over $[-\pi, \pi)$, and their PDFs are given by

$$
\begin{aligned}
f_{|Z|}(z) &= 2(1+K)z \cdot \exp\left[-K - (1+K)z^2\right] \\
&\quad \times I_0\left[2z\sqrt{K(1+K)}\right], \quad z \geq 0 \quad \text{(9a)}
\end{aligned}
$$

$$
f_\Psi(\psi) = \frac{1}{2\pi}, \qquad\qquad \psi \in [-\pi, \pi), \quad \text{(9b)}
$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind.

Proof: Since all the individual sinusoids in the sums of $Y_c(t)$ and $Y_s(t)$ are statistically independent and identically distributed, according to the central limit theorem [28], when the number of sinusoids N is large, $Y_c(t)$ and $Y_s(t)$ become to Gaussian random processes. Moreover, since $R_{Y_c Y_s}(\tau) = 0$ and $R_{Y_s Y_c}(\tau) = 0$, $Y_c(t)$ and $Y_s(t)$ are independent. Therefore, $Z_c(t)$ and $Z_s(t)$ defined by equations (7) are also independent.

When the initial phase ϕ_0 of the specular component is chosen, then the conditional joint PDF of $Z_c(t)$ and $Z_s(t)$ is given by

$$
f_{Z_c, Z_s}(z_c, z_s | \phi_0) = \frac{1}{\pi} \exp\left\{-[z_c - m_c(t)]^2 - [z_s - m_s(t)]^2\right\},
$$

where $m_c(t) = \sqrt{K}\cos(w_d t \cos\theta_0 + \phi_0)$ and $m_s(t) = \sqrt{K}\sin(w_d t \cos\theta_0 + \phi_0)$.

Since the initial phase ϕ_0 is uniformly distributed over $[-\pi, \pi)$, the joint PDF of $Z_c(t)$ and $Z_s(t)$ can be calculated by

$$
\begin{aligned}
f_{Z_c, Z_s}(z_c, z_s) &= \int_{-\pi}^{\pi} f_{Z_c, Z_s}(z_c, z_s | \phi_0) \cdot \frac{1}{2\pi} \cdot d\phi_0 \\
&= \frac{1}{\pi} \exp\left(-z_c^2 - z_s^2 - K\right) \cdot I_0\left[2\sqrt{K(z_c^2 + z_s^2)}\right].
\end{aligned}
$$

Applying the transformation of the Cartesian coordinates (z_c, z_s) to polar coordinates (z, ψ), we obtain the joint PDF of the envelope $|Z|$ and the phase $\Psi = \arctan(z_c, z_s)$ as follows:

$$
\begin{aligned}
f_{|Z|, \Psi}(z, \psi) &= \frac{(1+K)z}{\pi} \cdot \exp\left[-K - (1+K)z^2\right] \\
&\quad \times I_0\left[2z\sqrt{K(1+K)}\right], \quad z \geq 0, \ \psi \in [-\pi, \pi).
\end{aligned}
$$

Then, the PDFs of the envelope and the phase can be obtained by the following two equations

$$
f_{|Z|}(z) = \int_{-\pi}^{\pi} f_{|Z|, \Psi}(z, \psi) d\psi, \quad f_\Psi(\psi) = \int_0^\infty f_{|Z|, \Psi}(z, \psi) dz.
$$

This completes the proof. ∎

We now make the following three remarks based on Theorem 3.

Remark 4: Both the fading envelope and the phase are stationary because their PDFs are independent of time t.

---

^{1}The function $\arctan(x, y)$ maps the arguments (x, y) into a phase in the correct quadrant in $[-\pi, \pi)$.

This is very different from the previous Rician models [4], [15], where the PDF of the fading phase is a very complicated function depending on time t, and therefore the fading phase is not stationary as pointed out by Aulin in [4]. Here, the fading phase of our new model is not only stationary but also uniformly distributed over $[-\pi, \pi)$.

Remark 5: The fading envelope and phase of our new Rician model are independent from each other. The PDFs of the envelope and the phase of our Rician channel model cover the Rayleigh fading ($K = 0$) as a special case.

Remark 6: The PDF of the fading envelope of our Rician model can be derived by using two-dimensional random walk procedure. Details are omitted.

Another important second-order statistics associated with fading envelope are the *level crossing rate* (LCR). LCR is defined as the rate at which the envelope crosses a specified level in the positive slope. The following theorem provides the LCR result of our Rician model.

Theorem 4: When N approaches infinity and $\theta_0 = \pi/2$, then the level crossing rate $L_{|Z|}$ of the new simulator output is given by

$$
\begin{aligned}
L_{|Z|} &= \sqrt{2\pi(K+1)}\rho f_d \cdot \exp[-K - (K+1)\rho^2] \\
&\quad \times I_0\left[2\rho\sqrt{K(K+1)}\right] \quad \text{(10)}
\end{aligned}
$$

where ρ is the normalized fading envelope level given by $|Z|/|Z|_{rms}$ with $|Z|_{rms}$ being the root mean square envelope level.

Proof: When N approaches infinity, the fading envelope is Rician distributed as shown in Theorem 3. Using the same procedure provided in [27], one can prove eqn (10). ∎

It is noted here that if $K = 0$, then $Z(t) = Y(t)$ becomes to Rayleigh fading, and the LCR is simplified to be $L_{|Y|} = \sqrt{2\pi}\rho f_d \exp(-\rho^2)$; however, if $K \neq 0$ and $\theta_0 \neq \pi/2$, then the LCR has no closed form solution [27].

IV. Performance Evaluation

The performance evaluation of the proposed fading simulator is carried out by comparing the corresponding simulation results with those of the theoretical limit when N approaches infinity. Throughout the following discussions, the newly proposed statistical simulators have been implemented by choosing $N = 8$ unless otherwise specified, all the ensemble averages for simulation results are based on 500 random samples unless otherwise specified.

A. Evaluation of Correlation Statistics

The simulation results of the autocorrelations of the complex envelope and squared envelope of the simulator output are shown in Figs. 4-6, respectively.

As can be seen from Figs. 4-6, the simulation results show that the real part of the autocorrelation of the complex envelope, which contains the autocorrelation information of the quadrature components, and the imaginary part of the autocorrelation of the complex envelope, which contains the cross-correlation information of the quadrature components, match the theoretically desired ones very well even
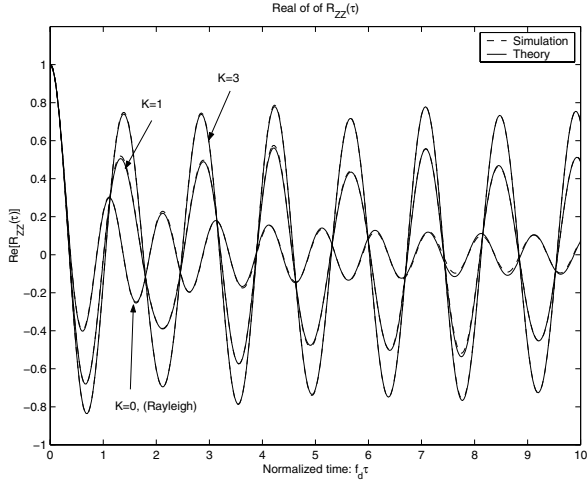
Fig. 4. The real part of the autocorrelation of the complex envelope $Z(t)$. $\theta_0 = \pi/4$ for $K = 1$ and $K = 3$ Rician cases.
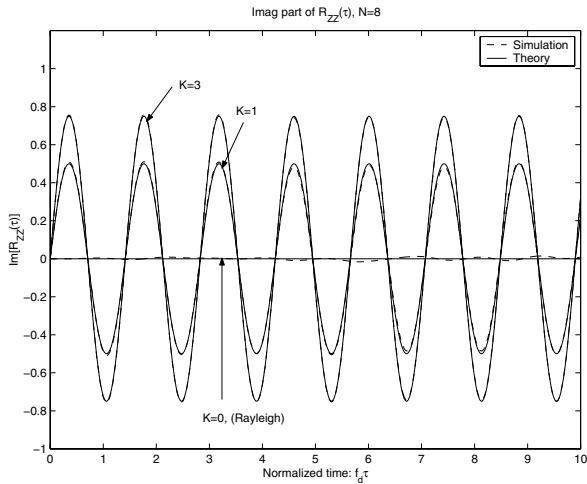

Fig. 5. The imaginary part of the autocorrelation of the complex envelope $Z(t)$. $\theta_0 = \pi/4$ for $K = 1$ and $K = 3$ Rician cases.
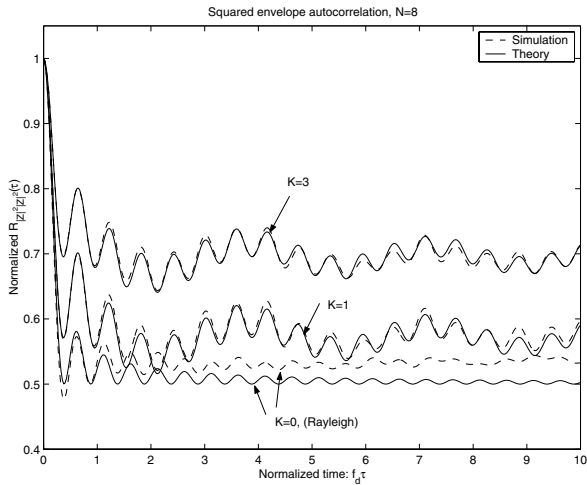

Fig. 6. The autocorrelation of the squared envelope $|Z(t)|^2$. $\theta_0 = \pi/4$ for $K = 1$ and $K = 3$ Rician cases.

though N is as small as 8. It is also shown in Fig. 6 that the autocorrelation of the squared envelope of the simulator is very close to the desired one, for both Rayleigh and Rician fading cases.

B. Evaluation of PDFs of the Envelope and Phase

Figs. 7 and 8 show that the PDFs of the fading envelope and phase of the simulator with $N = 8$ are in very good agreement with the theoretical ones. It is also noted that when $N > 8$, these PDFs will have even better agreement with the theoretically desired ones.
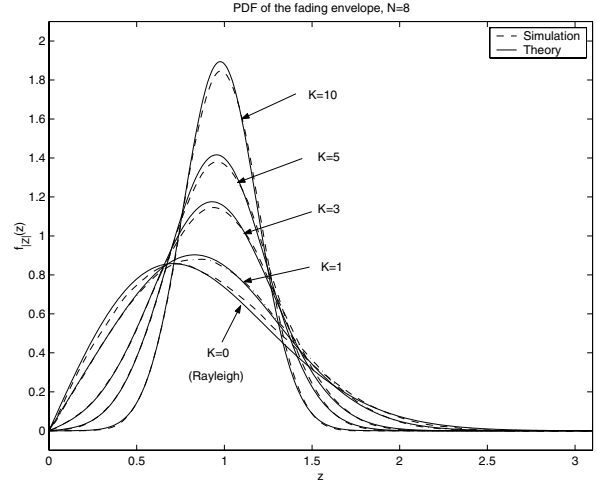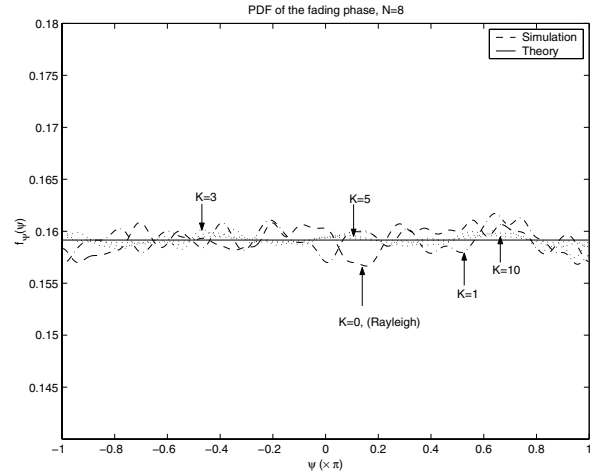

Fig. 7. The PDF of the fading envelope $|Z(t)|$.

Fig. 8. The PDF of the fading phase $\Psi(t)$.

C. Evaluation of LCR

The simulation results of the normalized level crossing rate (LCR), $\frac{L_{|X|}}{f_d}$, of the new simulator are shown in Fig. 9, where the theoretically calculated LCR are also included in the figure for convenient comparison, indicating good agreement in all cases.
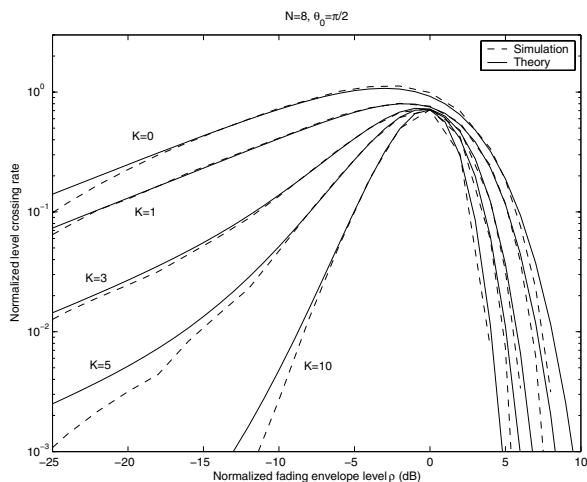
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Fig. 9. The normalized LCR of the fading envelope $|Z(t)|$.

V. CONCLUSION

In this paper, an improved sum-of-sinusoids statistical simulation model is proposed for Rayleigh fading channels. Based on this improved Rayleigh fading model, a new simulation model is presented for Rician fading channels. The specular (line-of-sight) component of this Rician fading model is a *zero-mean* stochastic sinusoid with a pre-chosen angle of arrival and a random initial phase. Compared to all the existing Rician fading models, which have *non-zero mean* deterministic specular component, the new model better reflects the random nature of specular component from time to time and from mobile to mobile, additionally, the PDF of the Rician fading phase is independent from time and uniformly distributed over $[-\pi, \pi)$, which is different from that of all the existing Rician fading models.

It has also been shown that the autocorrelation of the squared envelope, the PDFs of the fading envelope and phase, and the level crossing rate of the new simulator approach those of the theoretically desired ones as the number of sinusoids approaches infinity, while good convergence can be reached even when the number of sinusoids is small. All these statistical properties of the new simulator have been evaluated by extensive simulation results with excellent agreement in all cases. It has been pointed out that the new simulation model can be directly used to generate multiple uncorrelated faders for frequency selective channels, MIMO channels, and diversity combining techniques.

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