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Maciej Jan Zawodniok
Missouri University of Science and Technology, mjzx9c@mst.edu

Jagannathan Sarangapani
Missouri University of Science and Technology, sarangap@mst.edu

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A Distributed Power Control MAC Protocol for Wireless Ad Hoc Networks[1]

*Maciej Zawodniok and Sarangapani Jagannathan
Department of Electrical and Computer Engineering,
University of Missouri-Rolla
Rolla, Missouri 65401
mjzx9c@umr.edu, sarangap@umr.edu

Abstract—**A novel distributed power control (DPC)* scheme and a MAC protocol for wireless ad hoc networks in the presence of radio channel uncertainties such as path loss, Shadowing and Rayleigh fading is presented. The DPC quickly estimates the time-varying nature of the channel and uses the information to select a suitable transmitter power value in order to maintain a target Signal-to-Interference ratio (SIR) at the receiver. The standard assumption of a constant interference during a link's power update used in other works is relaxed. The performance of the proposed DPC is demonstrated analytically. The power used for all RTS-CTS-DATA-ACK frames is selected using the proposed DPC; hence, energy savings and spatial reuse are achieved. The hidden-terminal problem is overcome by periodically increasing the power. The NS simulator is used to compare the proposed scheme with 802.11. The proposed MAC protocol renders a significant increase in throughput in the presence of channel variations compared with 802.11 while consuming low energy per bit.**

Keywords-component; Distributed Power Control, Fading Channels, Wireless Adhoc Networks, Convergence

I. INTRODUCTION

The objectives of transmitter power control include minimizing power consumption while increasing the network capacity and prolonging the battery life of mobile units by managing mutual interference so that each mobile unit can meet its signal-to-interference ratio (SIR) and other quality of service (QoS) requirements. Rigorous work on distributed power control (DPC) was performed for cellular networks. A few DPC schemes [2][3][4][5] were developed for wireless ad hoc networks where the topology of an ad hoc network constantly changes due to node mobility and communication link failures.

In an ad hoc network, RTS and CTS messages are used to establish a connection for data transmission between a transmitter and a receiver. In [2][3], the authors propose using maximum transmitter powers for RTS-CTS followed by DATA-ACK transmission at a much lower power, which is calculated according to RTS-CTS reception conditions. On the other hand, work in [2] suggested that calculating the transmitter power using a DPC scheme for all or certain RTS-CTS-DATA-ACK frames in 802.11 results in a degradation of QoS. More collisions will occur, thus causing a huge number of retransmissions. Consequently, this yields higher power consumption, low throughput per node and low network utilization. Additionally, the DPC schemes reported for ad hoc networks [2][3][4][5] do not show performance guarantees analytically.

Unlike wired networks, radio channel uncertainties in a wireless network, for instance path loss, Shadowing and Rayleigh fading can attenuate the power of the transmitted signal and thus cause variations in the received SIR and degrading the performance of any DPC. Low SIR means high bit error rate (BER), which is unsatisfactory. Reported DPC schemes in [2][3][4][5] assume that: 1) only path loss is present, 2) no other channel uncertainty exists, and 3) the mutual interference among the users is held constant during power update of each user. Moreover, improvement of spatial reuse factor is not adequately addressed. The proposed work overcomes these limitations.

In this paper, a novel DPC scheme and a MAC protocol is presented for wireless ad hoc networks in the presence of radio channel uncertainties. The proposed protocol uses the novel DPC scheme, which estimates the time-varying channel, and uses this information to update the power so that a target SIR is maintained at the receiver provided the channel variation is slow compared to its estimation. In addition, the proposed DPC scheme shown to converge analytically to any target SIR value in the presence of channel uncertainties such as Shadowing and Rayleigh fading. Moreover, the proposed MAC protocol assigns power adaptively for RTS-CTS frames while overcoming the hidden-terminal problem. Consequently, a modest improvement in spatial reuse factor is observed. Also, a comparison with 802.11 is included.

II. DPC ALGORITHM

Radio channels involve many uncertain factors, so they are extremely random and do not offer easy analysis. We focus our effort on these main channel uncertainties, such as path loss, shadowing, and Rayleigh fading.

A. Radio channel with uncertainties

Path Loss: For path loss only, power attenuation is taken to follow the inverse 4[th] power law: $g_{ij} = \overline{g} / d_{ij}^{n}$, where $\overline{g}$ is a constant usually equal to 1, d_{ij} is the distance between the

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*** -** Contact author

transmitter of the j^{th} link and the receiver of the i^{th} link and n is the path loss exponent with value of n=4 used to model path loss in an urban environment.

Shadowing: High buildings, mountains and other objects block wireless signals. The term $10^{0.1\zeta}$ is often used to model the attenuation of the shadowing to the received power, where ζ is assumed to be a Gaussian random variable.

Rayleigh fading: In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. The Rayleigh distribution has a probability density function (pdf) given by:

$$p(x) = \begin{cases} \dfrac{x}{\sigma^2}\exp\left(-\dfrac{x^2}{2\sigma^2}\right) & (0 \le x \le \infty) \\ 0 & (x < 0) \end{cases} \tag{1}$$

where x is a random variable, and σ^2 is known as the fading envelope of the Rayleigh distribution.

Since the channel uncertainties can distort the transmitted signals, therefore, the effect of these uncertainties is represented via a channel loss (gain) factor that typically multiplies the transmitter power. The channel gain or loss, g, can be expressed as:

$$g = f(d,n,X,\zeta) = d^{-n} \cdot 10^{0.1\zeta} \cdot X^2 \tag{2}$$

where d^{-n} is the effect of path loss and $10^{0.1\zeta}$ corresponds to the effect of shadowing. For Rayleigh fading, it is typical to model the power attenuation as X^2, where X is a random variable with Rayleigh distribution. Typically the channel gain, g, is a function of time.

B. *Distributed Power Control (DPC)*

Suppose there are $N \in Z_+$ links in the network. Let g_{ij} be the power loss (gain) from the transmitter of the j^{th} link to the receiver of the i^{th} link. The power attenuation is considered to follow the relationship given in equation (2).

Calculation of SIR, $R_i(t)$, at the receiver of i^{th} link at the time instant t, is given by

$$R_i(t) = \frac{g_{ii}(t)P_i(t)}{I_i(t)} = \frac{g_{ii}(t)P_i(t)}{\sum_{j\neq i} g_{ij}(t)P_j(t) + \eta_i(t)} \tag{3}$$

where $i, j \in \{1,2,3,...,n\}$, $I_i(t)$ is the interference, $P_i(t)$ is the link's transmitter power, $P_j(t)$ are the transmitter powers of all other nodes, and $\eta_i(t)>0$ is the thermal noise at its receiver node. For each link *i* there is a lower SIR threshold γ_i and upper threshold γ^*. Therefore, we require

$$\gamma_i \le R_i(t) \le \gamma_i^* \tag{4}$$

for every *i* = 1,2,3,…n the lower threshold value for all links can be taken equal to γ for convenience, reflecting a certain QoS the link has to maintain in order to operate properly. An upper SIR limit is also set, in order to manage the interference.

C. *Proposed DPC algorithm*

In this paper, a novel DPC scheme is given where both $g_i(t)$ and the interference $I_i(t)$ are time-varying, and channel uncertainties are considered for all mobile users. In other words, in all existing works [1]-[5], both $g_i(t)$ and $P_j(t)$ are held constant, whereas in our work, this assumption is relaxed.

Considering SIR from (3) where the power attenuation $g_{ij}(t)$ is taken to follow the time-varying nature of the channel and differentiating (3) to get

$$R_i(t)' = \frac{(g_{ii}(t)P_i(t))' I(t) - (g_{ii}(t)P_i(t))I(t)'}{I_i^2(t)} \tag{5}$$

where $R_i(t)'$ is the derivative of $R_i(t)$ and $I_i(t)'$ is the derivative of $I_i(t)$.

To transform the differential equation into the discrete time domain, $x'(t)$ is expressed using Euler's formula as $\frac{x(l+1)-x(l)}{T}$, where T is the sampling interval. Equation (5) can be expressed in discrete time as

$$\begin{aligned} R_i(l)' &= \frac{(g_{ii}(l)P_i(l))' I(l) - (g_{ii}(l)P_i(l))I(l)'}{I_i^2(l)} \\ &= \frac{1}{I_i^2(l)}[(g_{ii}'(l)P_i(l))I(l) + (g_{ii}(l)P_i'(l))I_i(l) \\ &\quad - (g_{ii}(l)P_i(l))\left(\sum_{j\neq i} g_{ij}(l)P_j(l) + \eta_i(t)\right)'] \end{aligned} \tag{6}$$

In other words,

$$\begin{aligned} &R_i(l+1) = \\ &\left[\frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\left\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) + [P_j(l+1)-P_j(l)]g_{ij}(l)\right\}}{I_i(l)}\right]R_i(l) + \\ &+ g_{ii}(l)\frac{P_i(l+1)}{I_i(l)} \end{aligned} \tag{7}$$

Now, define

$$\begin{aligned} &\alpha_i(l) = \\ &\frac{g_{ii}(l+1)-g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\left\{[g_{ij}(l+1)-g_{ij}(l)]P_j(l) + [P_j(l+1)-P_j(l)]g_{ij}(l)\right\}}{I_i(l)} \\ &= \frac{\Delta g_{ii}(l)}{g_{ii}(l)} - \frac{\sum_{j\neq i}\Delta g_{ij}(l)P_j(l) + \Delta P_j(l)g_{ij}(l)}{I_i(l)} \end{aligned}$$

$\beta_i(l) = g_{ii}(l)$ and $v_i(l) = \frac{P_i(l+1)}{I_i(l)}$

Equation (7) can be expressed as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) \tag{8}$$

with the inclusion of channel noise, equation (7) is written as

$$R_i(l+1) = \alpha_i(l)R_i(l) + \beta_i(l)v_i(l) + r_i(l)\omega_i(l) \tag{9}$$

where $\omega(l)$ is the zero mean stationary stochastic channel noise with $r_i(l)$ is its coefficient.

The SIR of each link at time instant, l, is obtained using (9). Carefully observing (9), it is clear that the SIR at the time instant $l+1$ is a function of channel variation from time instant l to $l+1$. The channel variation represented as α is not known; it has to be estimated for DPC development.

Now define $y_i(k)=R_i(k)$ in (9). Also α_i, β_i and r_i are considered unknown. In this scenario, equation (9) can be expressed as

$$y_i(l+1) = \begin{bmatrix}\alpha_i(l) & r_i(l)\end{bmatrix}\begin{bmatrix} y_i(l) \\ \omega_i(l)\end{bmatrix} + \beta_i(l)v_i(l) \tag{10}$$
$$= \theta_i^T(l)\psi_i(l) + \beta_i(l)v_i(l)$$

where $\theta_i(l) = \begin{bmatrix}\alpha_i(l) & r_i(l)\end{bmatrix}$ is a vector of unknown parameters, and $\psi_i(l) = \begin{bmatrix} y_i(l) \\ \omega_i(l)\end{bmatrix}$ is the regression vector. Now selecting feedback for DPC as

$$v_i(l) = \beta_i^{-1}(l)\left[-\hat{\theta}_i(l)\psi_i(l) + \gamma + k_v e_i(l)\right] \tag{11}$$

where $\hat{\theta}_i(l)$ is the estimate of $\theta_i(l)$, then the SIR error system is expressed as

$$e_i(l+1) = k_v e_i(l) + \theta_i^T(l)\psi_i(l) - \hat{\theta}_i^T(l)\psi_i(l) \tag{12}$$
$$= k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)$$

where $\tilde{\theta}_i(l) = \theta_i(l) - \hat{\theta}_i(l)$ is the error in estimation. From (12), the SIR error system is driven by channel estimation error. If the channel conditions are properly estimated, then estimation error tends to zero. In this case, SIR error goes to zero with time. In the presence of estimation error, only an SIR error bound can be shown. Next, Assumption 1 is required.

Assumption 1: The channel changes slowly compared to the parameters' updates.

Remark: This assumption is valid due to the performance of embedded computer systems and the computationally simple estimation scheme.

Theorem 1: Given the DPC scheme above with channel uncertainties, if the feedback from the DPC scheme is selected as (11), then the mean channel estimation error along with the mean SIR error converges to zero asymptotically, if the parameter updates are taken as

$$\hat{\theta}_i(l+1) = \hat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) \tag{13}$$

provided

$$\sigma\|\psi_i(l)\|^2 < 1 \tag{14}$$

$$k_{v\max} < 1/\sqrt{\delta} \tag{15}$$

where $\delta = \dfrac{1}{1-\sigma\|\psi_i(l)\|^2}$ and σ is the adaptation gain.

Proof: See Appendix. ■

Consider now the closed-loop SIR error system with channel estimation error, $\varepsilon(l)$, as

$$e_i(l+1) = k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l) + \varepsilon(l). \tag{16}$$

Theorem 2: Assume the hypothesis as given in Theorem 1, with the channel uncertainty is now estimated by

$$\hat{\theta}_i(l+1) = \hat{\theta}_i(l) + \sigma\psi_i(l)e_i^T(l+1) - \left\|I - \psi_i^T(l)\psi_i(l)\right\|\hat{\theta}_i(l) \tag{17}$$

where $\varepsilon(l)$ is an estimation error which is considered bounded above $\|\varepsilon(l)\| \le \varepsilon_N$, with ε_N a known constant. Then the mean error of SIR and the estimated parameters are bounded, provided (14) and (15) hold. ■

III. POWER CONTROL MAC PROTOCOL

For DPC implementation, the original MAC protocol has to be modified. These modifications occur at different layers. Further, a novel idea based on [2] to overcome the hidden terminal problem is included. Then, the power selection for RTS-CTS-DATA-ACK is discussed.

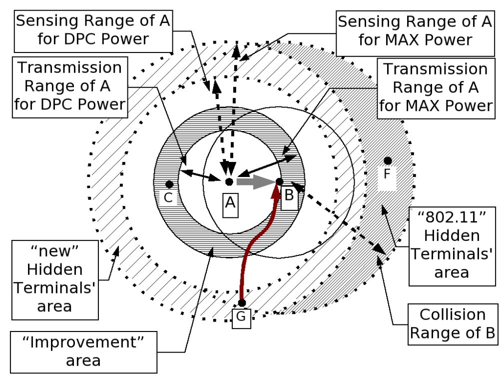

Figure 1. Hidden terminal problem for DPC schemes.

The hidden terminal problem occurs in a wireless network when a third node causes collision to an ongoing communication between any two nodes. The problem is illustrated in Figure 1, where node A transmits data to node B. Node F is outside the sensing range of transmitter A, and it will not detect transmission of the DATA frame that is sent from node A. Thus, F initiates transmission, which will collide with the DATA frame at the receiving node B. In Figure 1, all nodes located inside a marked region, called "Hidden Terminals' area", are potential sources for this problem. The proposed DPC scheme uses much lower power for the transmission of frames. Hence, it results in increased collisions. A solution similar to others in [2] is proposed, though an improvement in contention time is observed.

Generally when a lower transmission power is used, the transmission and sensing range of a node will decrease [2] as shown in Figure 1. Node G will fail to detect transmission from node A and it will initiate transmission by assuming that the channel is idle. If node G uses maximum power, then a collision will occur at receiver node B. Hence, the probability of hidden terminal problem occurring in the network will increase when a low transmitter power is used.

To overcome this problem, in our proposed methodology, a train of short frames (pulses) with increased transmitted power is used periodically during transmission. The RTS, CTS, DATA and ACK frames are transmitted by using the power dictated by the proposed DPC along with the train of pulses. The pulses use the maximum transmission power defined by the links of the network. This ensures that all the nodes in the sensing range of the transmitter will detect the pulses and update their NAV vectors accordingly. Thus, the nodes in the sensing range of the transmitter node will not cause collision. Figure 2 shows the difference in handling of NAV vector with and without pulse train.

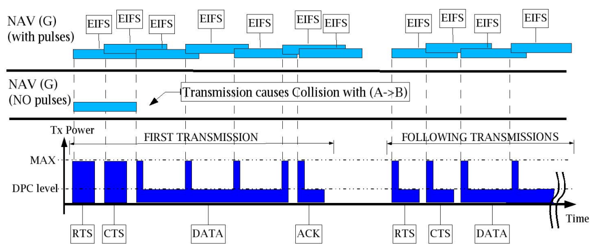

Figure 2. Periodic increase of power during frame transmission

A. Proposed MAC protocol

In the proposed protocol, only the initial RTS-CTS frames during link set-up have to be transmitted using maximum power defined by the link. Subsequently all frames, including RTS-CTS-DATA-ACK frames, will use transmission power calculated according to the proposed DPC scheme. The MAC header needs to be changed to allow power information to be sent between the communicating nodes.

In other words, the MAC frames have to embed the power information that is used for the current packet as well as for any subsequent response. This process repeats for all transmissions occurring between any two nodes A and B despite an increase in overhead, which causes a decrease in throughput. However, the observed increase in throughput, due to the smaller contention time, overcomes the penalty introduced by the overhead bits. Moreover, this overhead can be further reduced by using power levels. Consequently, the number of bits used for storing the power value in the header will decrease thus rendering low overhead. Additionally, these fields can be defined as optional in the frame header by using a one-bit flag.

Whenever the power level changes, this flag will be set and this indicates the receiving node to calculate its transmitting power for the subsequent transmission. Otherwise, the flag will be cleared and the node will continue to use its previous value.

Since a lower power is selected for transmission of the MAC frames, during severe fading, certain frames will be unsuccessfully decoded due to poor reception and they will be dropped. Consequently, retransmissions will occur with the same power value causing poor reception and further retransmissions. Hence, to prevent such a problem, the proposed protocol increases the transmission power by a given factor before each retransmission in order to reduce the packet drops.

B. Increase in Channel Utilization

Though hidden terminal problem increases with lower transmitter powers, however, an increase in channel utilization and throughput will be seen by using the proposed MAC protocol. Consider Figure 1 to understand the improvement in utilization that will occur when a lower transmitter power is used for subsequent RTS-CTS transmissions and when RTS-CTS handshake fails for any given nodes A and B. Then for instance node C will be able to start transmission earlier than in the case of using maximum power for RTS-CTS. As a result, the contention time for frames from certain nodes, such as C, decreases. Two scenarios are presented: first – maximum power used for RTS-CTS, where there is no improvement, and second, with RTS-CTS sent with power calculated by the DPC algorithm, where the improvement occurs. In both cases, node B will not respond since it is unable to receive the RTS message due to a collision. This will occur, for example, if node F will transmit at the same time as node A is trying to send the RTS frames. After predefined number of retransmissions, node A will cease to send the frame.

In the first scenario, node C will decode the RTS frame because it was sent with the maximum transmitter power defined by the links. Then node C will update its NAV vector using the RTS frame data. No transmission occurs; hence, the channel is idle. In the second scenario, where the RTS frame is sent at a power level calculated by the DPC, the node C will only detect the RTS frame and will set its NAV vector to the EIFS time. Hence, shortly after EIFS, node C is free to initiate communication. Due to the availability of channel to C, the throughput increases.

This second scenario applies to all nodes within the "improvement area" depicted in Figure 1. Given the high density of nodes in the case of wireless ad hoc networks, the probability of a node accessing the channel is quite high. Therefore, an increase in aggregated throughput is observed with the proposed protocol.

Due to better channel utilization, the *spatial reuse factor*, which is defined as the number of successful transmissions within a given time interval for a given area, will increase for the proposed DPC scheme. For 802.11, the NAV vector will be set for an entire expected duration of flow transmission; hence, there will be time intervals when no transmissions take place. As the result, there will be fewer transmissions for a given time interval in comparison with the theoretical capacity of the radio channel. In our scheme, these idle periods are detected and nodes can transmit earlier, and thus the total number of successful transmission within a given time increases. Consequently, the spatial reuse factor increases with the proposed DPC when compared to 802.11.

C. Contention time

The change in contention time for the proposed DPC scheme is due to two major factors: more retransmissions during fading channel conditions and improved channel utilization, which is explained in Section III.B. During fading channel conditions, retransmissions will increase with the proposed DPC due to a possibility of undesirable reception. As a result, the average contention time increases. Additionally, high utilization due to the proposed DPC will increase in throughput causing congestion. Under these conditions, certain frames will be delayed longer compared to the case when 802.11 is used. Therefore, the contention time increases with the proposed DPC.

D. Protocol Implementation

The NS-2 simulator has been used for evaluating the proposed scheme. The modifications made to incorporate the DPC algorithm and protocol mainly focus on two layers of 802.11:

- Physical layer – modified to collect necessary data, like Interferences
- Medium Access (MAC) layer – here the DPC algorithm and protocol are implemented.

Floating-point variables are used for communicating power values. Separate short frames simulate the train of pulses. It is ensured that short-frames do not interfere with normal communication.

IV. SIMULATIONS

The 802.11 and proposed MAC protocols are evaluated under similar channel conditions, identical node placements, node movement and data flows (type, rate, start time, source, destination, etc.), and SIR thresholds, with a deterministic propagation model. The effect of path loss, shadowing and Rayleigh fading were introduced into the propagation model. The path loss effect is calculated as in *Propagation/Shadowing* object from NS-2 simulator. The Shadowing and Rayleigh fading effects has been calculated in advance and stored in a sample file. This ensured that the channel uncertainties for compared simulations are the same. The calculation of power attenuation for shadowing and Rayleigh fading employ models from equation (3). Figure 3 shows sample attenuation observed for a random receiver node. The results were repeated for a number of different, randomly generated scenarios and they were averaged.

A. Simulator Parameters

The DSR routing protocol with 2Mbps radio channel rate was used. The topology consists of a 1000m x 1000m square area with 100 nodes placed and moving at random. The simulations are executed for 20 seconds. The CBR traffic is used for 50 flows, starting randomly during the first 2 seconds. Each data flow generates steady traffic via 512 byte-long packets. The results were averaged over simulation trials using different fading effects, node placement and movement. The simulations were executed by varying the per-flow rates. The radio channel with 2Mbps bandwidth was used.

The maximum power used for 802.11 and proposed DPC is equal to 0.2818 [W]. The proposed DPC should maintain a target SIR of 5, while the minimum SIR for error-free reception is equal to 4 (~6db). The channel condition can change between time when the power was calculated and the time when this power is used. Hence, to overcome the unpredictable fading, the power calculated by the algorithm is multiplied by a factor equal to 2. For the proposed DPC, the design parameters are selected as Kv=0.01, and σ = 0.01. The power increase factor in case of retransmissions is set to 1.5 for the proposed DPC scheme.

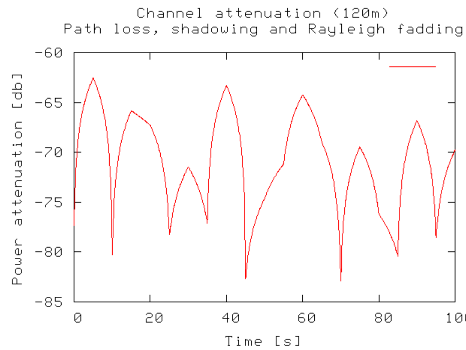

Figure 3. Shadowing, Path Loss and Rayleigh fading effects

In random topology scenario, each flow can use different number of hops (mini-flows) between the source and the destination pair. This can lead to different end-to-end throughput depending on number of hops used in particular scenario. Hence, the mini-flow transmissions were used instead of end-to-end transmissions.

B. Results

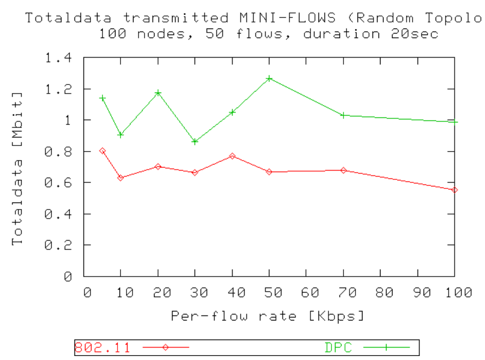

Figure 4. Total data transmitted (mini-flows)

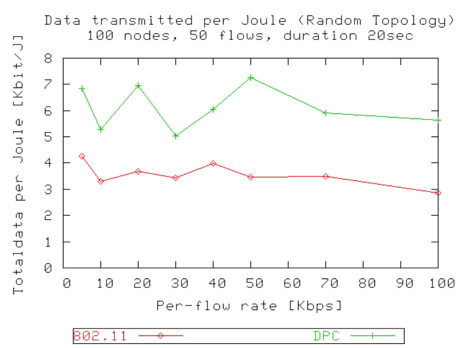

Figure 5. Total data per joule transmitted

The total data transmitted for all mini-flows is presented in Figure 4. Compared to 802.11, the proposed DPC transmits more data regardless of traffic flow rate. As the amount of data increases throughput does not increase suggesting that the channel is saturated. Figure 5 shows the energy efficiency characteristics of the protocols. Regardless of the traffic load, the proposed protocol allows transmission of more data per joule when compared to 802.11. The average contention time is presented in Figure 6. The proposed protocol yields slightly higher contention time since more number of retransmissions will occur with lower power values. When a channel condition severely deteriorates, frame decoding could fail due to reduced power at the receiver, retransmission will increase thus increasing the average contention time. Additionally, the higher channel utilization causes an increase in congestion, and in consequence increases the contention time.

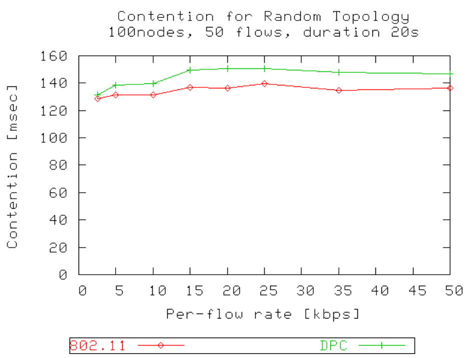

Figure 6. Contention time

V. CONCLUSIONS

A novel MAC power control protocol is presented for wireless ad hoc networks. It was seen that the proposed DPC scheme allows fully distributed power control and has rendered better performance in the presence of radio channel uncertainties. The proposed DPC scheme uses significantly less transmitter power per bit compared to 802.11; hence the energy is saved and life-time of wireless nodes extended. Additionally, the network capacity is maximized in terms of high aggregate throughput. In conclusion, the proposed power control MAC protocol offers a superior performance in terms of convergence and maximizes the network capacity compared to 802.11 standards. Results justify theoretical conclusions.

The pulse train is sent at maximum power to prevent the hidden terminal problem. Hence, the increase in throughput is due to increased channel utilization and not due to the spatial reuse factor. To further increase the throughput and spatial reuse factor, the transmission power has to be changed dynamically for all frames. This amounts to adaptive selection of the first RTS-CTS exchange as well as power used for the train of pulses.

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APPENDIX

Proof of Theorem 1: Define the Lyapunov function

$$J_i = e_i^T(l)e_i(l) + \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right] \tag{A.1}$$

whose first difference is

$$\begin{aligned}\Delta J &= \Delta J_1 + \Delta J_2 \\ &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &+ \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l+1)\tilde{\theta}_i(l+1) - \tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right]\end{aligned} \tag{A.2}$$

Consider J_1 from (A.2) and substituting (12) to get

$$\begin{aligned}\Delta J_1 &= e_i^T(l+1)e_i(l+1) - e_i^T(l)e_i(l) \\ &= \left(k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right)^T\left(k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right) - e_i^T(l)e_i(l)\end{aligned} \tag{A.3}$$

Taking the second term of the first difference from (A.2) and substituting (13) yields

$$\begin{aligned}\Delta J_2 &= \frac{1}{\sigma}\kappa\left[\tilde{\theta}_i^T(l+1)\tilde{\theta}_i(l+1) - \tilde{\theta}_i^T(l)\tilde{\theta}_i(l)\right] \\ &= -2[k_v e_i(l)]^T\tilde{\theta}_i^T(l)\psi_i(l) - 2\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] \\ &+ \sigma\psi_i^T(l)\psi_i(l)\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[k_v e_i(l) + \tilde{\theta}_i^T(l)\psi_i(l)\right]\end{aligned} \tag{A.4}$$

Combining (A.3) and (A.4) to get

$$\begin{aligned}\Delta J &= -e_i^T(l)\left[I - \left(1 + \sigma\psi_i^T(l)\psi_i(l)k_v^T k_v\right)\right]e_i(l) \\ &+ 2\sigma\psi_i^T(l)\psi_i(l)[k_v e_i(l)]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] \\ &- \left(1 - \sigma\psi_i^T(l)\psi_i(l)\right)\left[\tilde{\theta}_i^T(l)\psi_i(l)\right]^T\left[\tilde{\theta}_i^T(l)\psi_i(l)\right] \\ &\le -\left(1 - \delta k_{v\max}^2\right)\left\|e_i(l)\right\|^2 - \left(1 - \sigma\left\|\psi_i(l)\right\|^2\right) \\ &\left\|\tilde{\theta}_i^T(l)\psi_i(l) - \frac{\sigma\left\|\psi_i(l)\right\|^2}{1 - \sigma\left\|\psi_i(l)\right\|^2}k_v e_i(l)\right\|^2\end{aligned} \tag{A.5}$$

where δ is given after (15). Taking now expectations on both sides yields

$$\begin{aligned}E(\Delta J) &\le -E(\left(1 - \delta k_{v\max}^2\right)\left\|e_i(l)\right\|^2 - \left(1 - \sigma\left\|\psi_i(l)\right\|^2\right) \\ &\left\|\tilde{\theta}_i^T(l)\psi_i(l) + \frac{\sigma\left\|\psi_i(l)\right\|^2}{1 - \sigma\left\|\psi_i(l)\right\|^2}k_v e_i(l)\right\|^2)\end{aligned} \tag{A.6}$$

Since $E(J) > 0$ and $E(\Delta J) \le 0$, this shows the stability in the mean via sense of Lyapunov provided the conditions (14) and (15) hold, so $E[e_i(l)]$ and $E\left[\tilde{\theta}_i(l)\right]$ (and hence $E\ \hat{\theta}_i(l)$) are bounded in the mean if $E[e_i(l_0)]$ and $E\ \tilde{\theta}_i(l_0)$ are bounded in a mean. Sum both sides of (A.6) and take limits $\lim_{l\to\infty} E(\Delta J)$, the SIR error $E\left[\left\|e_i(l)\right\|\right] \to 0$. ■