01 Oct 2010

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Ganesh K. Venayagamoorthy
Missouri University of Science and Technology

E. C. Anene

Recommended Citation

G. K. Venayagamoorthy and E. C. Anene, "PSO Tuned Flatness Based Control of a Magnetic Levitation System," *Proceedings of the IEEE Industry Applications Society Annual Meeting, 2010. IAS '10*, Institute of Electrical and Electronics Engineers (IEEE), Oct 2010.
The definitive version is available at https://doi.org/10.1109/IAS.2010.5615717

PSO Tuned Flatness Based Control of a Magnetic Levitation System

Ejike Anene
Electrical Engineering Programme
Abubakar Tafawa Balewa University
PMB 0248, Bauchi, Nigeria
ejikeanene@yahoo.com

Ganesh K. Venayagamoorthy
Real-Time Power and Intelligent Systems Laboratory
Missouri University of Science and Technology
Rolla, Missouri, USA
gkumar@ieee.org

Abstract – **Investigation on the application of flatness-based feedback linearization to the magnetic levitation model of INTECO[Tm] Maglev system is presented in this paper. The MAGLEV system dynamics studied consists of a set of third order nonlinear differential equations. Using computational techniques proposed by Levine, it is verified that the ball position is the flat output. The derived flat output is applied in the construction of a nonlinear control law used to control the levitation to a set point as well as tracking a sine function trajectory. The controller gains are obtained and optimized using particle swarm optimization. The simulation results compared very well with the default PID control. Real-time and non real-time simulation using the MATLAB/ SIMULINK real workshop environment is presented.**

Index Terms-- **Feedback linearization, flatness, flat output, magnetic levitation, particle swarm optimization**

I. Introduction

Application of magnetic levitation systems (MLS) are increasingly getting into diverse areas including: trains, magnetic bearings, pumps, centrifuges, turbines etc. The control of magnetic levitation has evolved over the years from the linearized controls to nonlinear controls. The MLS is an impressive dynamic system and its synergetic system integrates sensors, drivers and controls making it a challenging control problem that can be used as an excellent project for use in control education [1]. Experimental models for teaching have been built and are being used in many departments of engineering colleges to teach the principles of magnetic positioning, sensors, control and so on [2]-[4]. Magnetic levitation phenomenon is based on the principles of electromagnetism. It causes ferromagnetic objects to be levitated by the magnetic force induced by electric current flowing through the coils around a solenoid. The system is inundated with electromagnetic fluctuations and is naturally unstable [5]. The simplest form of a magnetic bearing consists of a pair of opposing horseshoe electromagnets. The attractive force exerted on the levitated object by each electromagnet is proportional to the square of the current in each coil and is inversely dependent on the square of the gap. The coil is highly inductive and the rate of change of the current is limited [6]. The electromagnetic force is nonlinear giving rise to difficulties to get closed-loop stability [7].

If the electromagnet used to suspend the object were simply operated with a fixed amount of current, this would not be able to maintain any kind of control over the position of the object. If the object were too close to the electromagnet, it would be pulled right up to it. If it were too far away from the electromagnet, it would fall to the floor. There would be no way to adjust or compensate for the slight variations that take place in order to maintain the object at a fixed distance from the electromagnet [8].

The system is both inherently nonlinear and open-loop unstable. This has led to the use of feedback control to stabilize the system. Many authors have applied the analog lead compensator using classical frequency response design to control a one-dimensional magnetic levitator [5]. Methods for feedback control design typically use a linearized model of the system, but for bearing applications, it is highly nonlinear properties can limit the performance of the overall system. Reference [9] described a nonlinear control system for a magnetic bearing designed using a combination of feedback linearization and backstepping concepts implemented with a floating-point digital signal processor. The author in [10] designed negative feedback and phase-lead controllers to stabilize a levitation system. Several other control methods had been used to stabilize the MLS.

In this paper the flatness-based feedback linearization approach is applied to control the MLS through stabilization and tracking. Differential Flatness allows a feedback linearization strategy in which system states are defined as functions of the system flat output and its higher order derivatives. If the flat output or any variable linked to it is measureable then the states can be completely parameterized and subsequently used to implement the control law. But first the system has to be shown to possess a flat output or simply put flat [11]. In this concept, the feedback law is constructed as a function the flat output and its derivatives up to the order of the system control plus one on which the loop is closed. The gain structure of the closed loop law has characteristics that allows for the system performance to be optimized. The paper presents the investigations carried out by optimizing the system gains using a meta-heuristic approach.

The maglev levitation system model used is described in Section II. In Section III, the flat output is computed while the particle swarm optimization algorithm used to optimize the controller gains is presented in Section IV. Section V discusses the studies carried out in non-real time and real-time in the MATLAB/SIMULINK environment. Conclusions are given in Section VI.

This work was supported by the National Science Foundation, under CAREER Grant #0348221.

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II. THE MAGNETIC LEVITATOR MODEL

The INTECO maglev system is a complete laboratory tool for studying classical control techniques, real time control and signal analysis. It is a single degree of freedom levitation system. The system is configured to run real-time experiments executed in the MATLAB/Simulink environment using the real time workshop and real time workshop target toolboxes. It is also equipped with maglev hardware and a dedicated DSP card for real-time implementations. Since the purpose is to implement the flatness-based controller using this model, the parameters of the system dynamics is assumed the same. In the model development, INTECO used empirical analysis to model control of the current that goes to the electromagnet. The resulting linear relationship is found to be a straight line $i(u) = au + b$ with a dead zone. The constants a and b are determined from the experimental data. The system dynamics are described in (1) – (3).

$$\dot{x}_1 = x_2 \tag{1}$$

$$\dot{x}_2 = g - \frac{1}{m} x_3^2 \left(\frac{f\_p_1}{f\_p_2} \right) e^{\left(\frac{-x_1}{f\_p_2} \right)} \tag{2}$$

$$\dot{x}_3 = (k_i u + c_1 - x_3) \frac{1}{\left(\frac{p_1}{p_2} \right) e^{\left(\frac{-x_1}{p_2} \right)}} \tag{3}$$

Where g is gravitational force, m is mass of object, f_p_1, f_p_2, p_1, p_2, k_i, c_1 are system constants.

III. FLATNESS-BASED FEEDBACK CONTROLLER

The system

$$f(\dot{x}, x, u) = 0 \tag{4}$$

with $x \in R^n$ and $u \in R^m$ is differentially flat if one can find a set of variables called flat output;

$$y = h(x, u, \dot{u}, \ddot{u}, \ldots\ldots, u^{(r)}) \tag{5}$$

$y \in R^m$ and system variables,

$$x = \alpha(y, \dot{y}, \ddot{y}, \ldots\ldots, y^{(q)}) \tag{6}$$

and control,

$$u = \beta(y, \dot{y}, \ddot{y}, \ldots\ldots, y^{(q+1)}) \ . \tag{7}$$

A. Flat output

Given the dynamics (1)-(3), the flat output can be determined using Levine's method [12]. Applying the implicit function theory and eliminating the dynamics with control, the variational equation is given by:

$$d\ddot{x}_1 - a\, e^{(\frac{-x_1}{b})} x_3^2\, dx_1 - a\, e^{\left(\frac{-x_1}{b}\right)} x_3^2\, dx_3 \tag{8}$$

Where $a = \frac{1}{m}\left(\frac{f\_p_1}{f\_p_2} \right)$.

The polynomial matrix will therefore be

$$p(f) = \left[\frac{d^2}{dt^2} - a\, e^{\left(\frac{-x_1}{f\_p_2}\right)} x_3^2 \quad - a\, e^{\left(\frac{-x_1}{f\_p_2}\right)} x_3^2 \right] \begin{bmatrix} dx_1 \\ dx_3 \end{bmatrix}$$

(5)

or compactly

$$p(f) = \begin{bmatrix} A & -b \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_3 \end{bmatrix} \tag{9}$$

Where $A = \frac{d^2}{dt^2} - a\, e^{\left(\frac{-x_1}{f\_p_2}\right)} x_3^2$ - a polynomial and $b = -a\, e^{\left(\frac{-x_1}{f\_p_2}\right)} x_3^2$. Using Smith's algorithm for the manipulation of polynomial matrices, the following right Smith steps are performed.

$\begin{bmatrix} A & -b \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -\frac{1}{b} & \frac{1}{b} A \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$, therefore $\hat{U} = \begin{bmatrix} 1 \\ \frac{1}{b} A \end{bmatrix}$, such that $Q\hat{U} = \begin{bmatrix} 1 & 0 \\ \frac{1}{b} A & -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ as required [12].

Therefore,

$$Q\, dx = \begin{bmatrix} 1 & 0 \\ \frac{1}{b} A & -1 \end{bmatrix} \begin{bmatrix} dx_1 \\ dx_3 \end{bmatrix} \tag{10}$$

Such that the first line reads $dy = dx_1$ which gives $y = x_1$ the flat output, while the second line is identically equal to zero from (9) showing the flatness of the system dynamics.

B. Control law

From the computed flat output the control law follow from the following compensator

$$
\begin{aligned}
y &= x_1 \\
\dot{y} &= \dot{x}_1 = x_2 \\
\ddot{y} &= \ddot{x}_1 = \dot{x}_2 \\
\dddot{y} &= \dddot{x}_1 = \ddot{x}_2 = u_L
\end{aligned} \tag{11}
$$

Since $\dot{x}_2$ equals (2), then from $\ddot{x}_2$ we obtain

$$
x_3 = \left(\frac{m(g-\dot{x}_2)}{\dfrac{f\_p_1}{f\_p_2} e^{\frac{-x_1}{f\_P_2}}} \right)^{\frac{1}{2}} \tag{12}
$$

And from $\dot{x}_3$ the control law is computed as

$$
u = x_3 - c_1 + \frac{1}{2}\left(\frac{m\ddot{x}_2 + (m(g-\dot{x}_2))\dfrac{1}{f\_p_2}\dot{x}_1 M_p}{(m(g-\dot{x}_2))^{\frac{1}{2}}\left(\dfrac{f\_p_1}{f\_p_2} e^{\left(\frac{-x_1}{f\_P_2}\right)}\right)^{\frac{1}{2}}} \right)\frac{1}{k_i} \tag{13}
$$

where $M_p = \dfrac{p_1}{p_2} e^{\frac{-x_1}{p_2}}$

And the linear control is given by

$$
u_L = -k_1(\delta - \delta^*) - k_2(\dot{\delta} - \dot{\delta}^*) - k_3(\ddot{\delta} - \ddot{\delta}^*) \tag{14}
$$

The gains k_i are chosen such that the linear time invariant error dynamics

$$
e^{(3)} = -k_1 e - k_2 \dot{e} - k_3 \ddot{e} \tag{15}
$$

where $e^{(j)} = \delta^{(j)} - (\delta^*)^{(j)}$ are stable. To compute the gains, (14) can be rewritten as a Hurwitz polynomial by

$$
s^3 + k_3 s^2 + k_2 s + k_1 = 0 \tag{16}
$$

The closed loop characteristic polynomial of a third order equivalent system is given in terms of the natural frequency and damping ratio by

$$
(s^2 + 2\xi\omega_n s + \omega_n^2)(s+\beta) \tag{17}
$$

such that comparing (14) and (15) gives

$$
k_1 = \beta\omega_n,\ k_2 = 2\xi\omega_n\beta + \omega_n^2,\ k_3 = \beta + 2\xi\omega_n
$$

IV. Implementation of Particle Swarm Optimization Tuning of Flatness-based Excitation Controller (FEC)

The PSO uses a pseudorandom algorithm to search the solution space of an optimization problem. First proposed by Kennedy and Eberhart, it makes use of the inference that the social behavior of birds requires them to flock together and migrate from place to place. It therefore makes use of a collection of possible solutions called particles whose individual velocity and position are updated according to two basic expressions. The current position of each solution particle is constantly compared with the previous ones and the best is used along with the groups' best solution particle to determine the next direction of search, thereby narrowing the search space using the following relations [13].

$$
\begin{aligned}
v_i(t+1) = w v_i(t) &+ c_1\, rand * (x_{pi}(t) - x_i(t)) \\
&+ c_2\, rand * (x_{Gb} - x_i(t))
\end{aligned} \tag{18}
$$

$$
x_i(t+1) = x_i(t) + v_i(t+1) \tag{19}
$$

(18) and (19) are used to update the particles' velocity and position at each iteration. x_{pi}, x_{Gb} represent each particle's personal best solution and the populations' best solution respectively. w, c_1, c_2 are the inertia constant, and two positive numbers referred to as the cognitive and social acceleration constants respectively. These PSO parameters have to be chosen to ensure fast and accurate convergence of the PSO. *Rand* is a random number with uniform distribution in the interval [0, 1]. The fitness function is designed for optimal selection of feedback gains.

The fitness function which is used to update the particles' velocity and position is the square of the area under the curve of the object's position trajectory during stabilization and is given by:

$$
J = \int_{t_1}^{t_2} |e(\tau)|^2 d\tau < \varepsilon \tag{20}
$$

Where $e = (\theta_t - \theta_{ref})$ The controller gains are tuned using the PSO algorithm with 15 particles, each of three dimensions corresponding to the feedback gains k_1, k_2, k_3. Table I list the PSO parameters and computed gains after 300 iterations for n particles.

TABLE I

PSO PARAMETERS

n	w	c_1	c_2	$v_{\min}$	$v_{\max}$	$iteration$
15	0.6-0.8	2	2	-30	30	300

The optimized gains are given by

$$k_1 = 4306.4, \quad k_2 = 709.3136, \quad k_3 = 29.2527$$

V. RESULTS

Fig. 1 gives the PSO fitness after 300 iterations showing convergence at the 155 iteration to a set of gains.

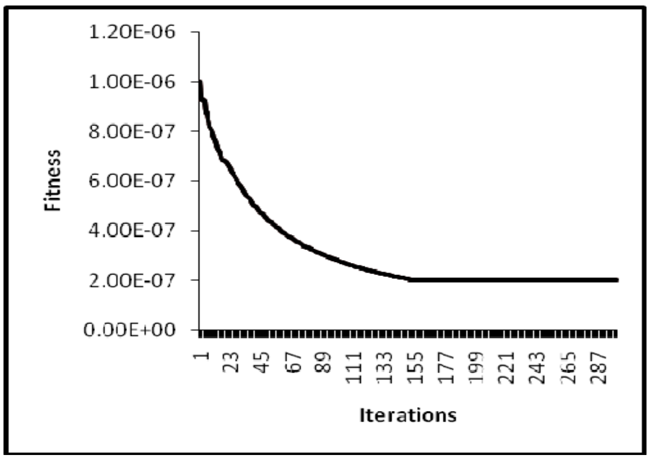

Fig. 1 Typical Fitness plot for 300 iterations

Figs. 2-5 show the results of a ten second simulation of the maglev system for a ball set point of 0.006 m.

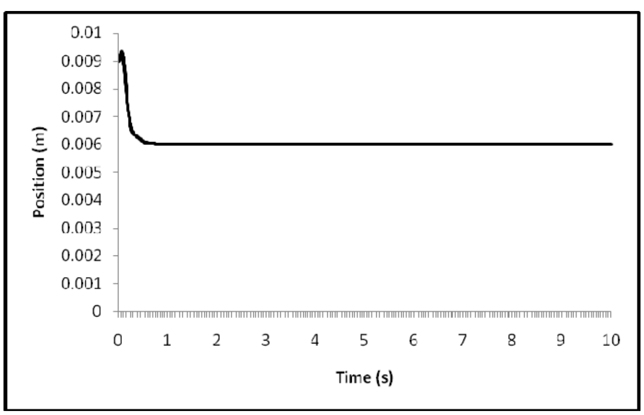

Fig. 2. Ball position for a ten second simulation

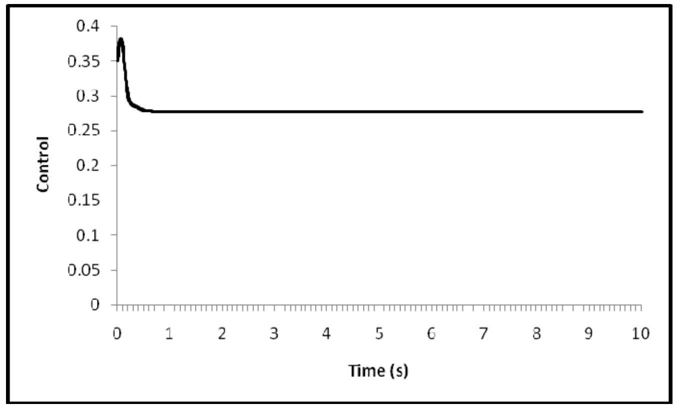

Fig. 3. Control to stabilize the ball position for a ten second simulation

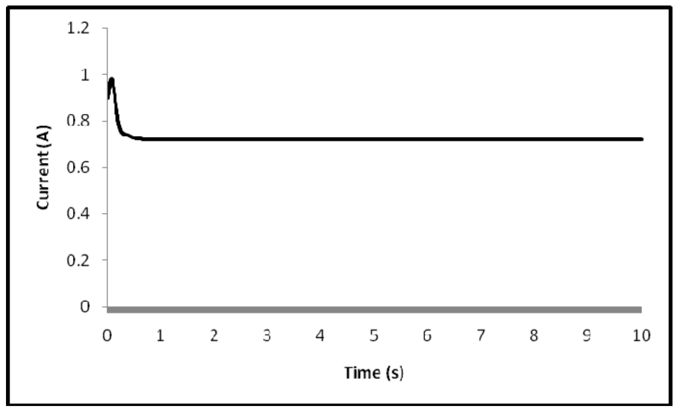

Fig. 4. Current to drive the electromagnet during levitation for a ten second simulation

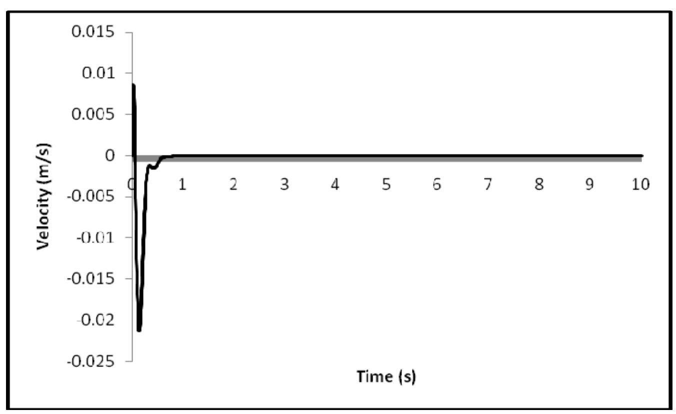

Fig. 5. Velocity showing the transient dip before stabilization of the ball position for a ten second simulation

Fig. 6 shows the results of a ten second real-time study of the maglev levitation system with a flatness based controller. The parameters of the flatness based controller are tuned with PSO. The input applied for tracking is $2e^{-3}\sin t$.

It can be observed from the non real-time and real-time studies and results that the flatness based controller is able to provide satisfactory control results.

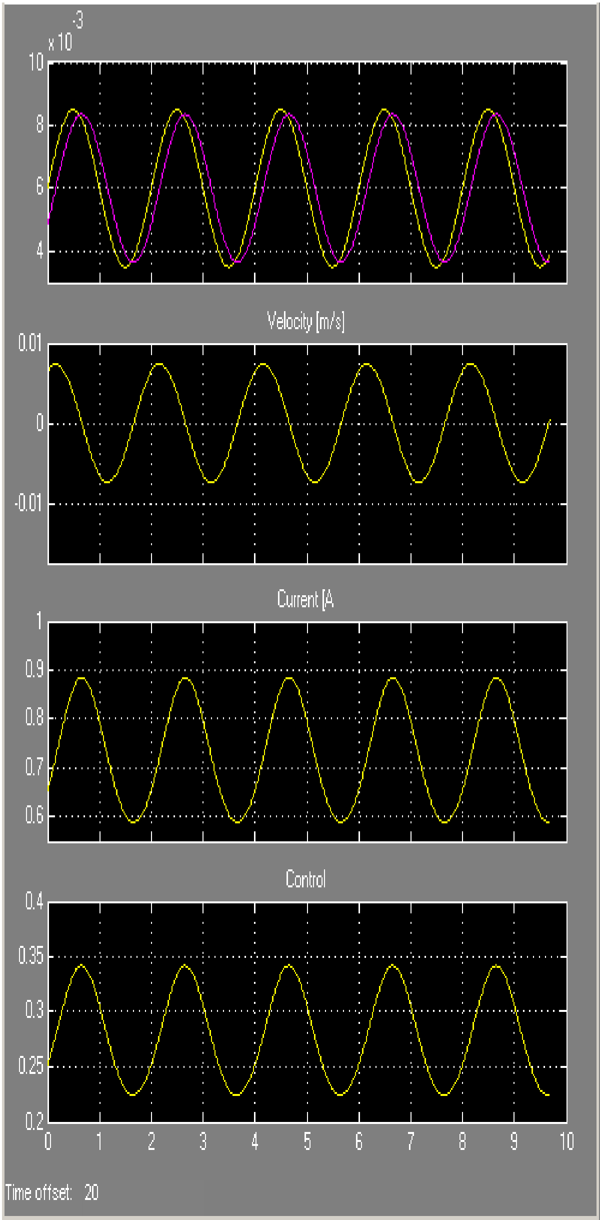

Fig. 6. Real time tracking performance simulation in MATLAB for an input $2e^{-3}\sin t$ using the flatness based controller tuned with PSO

VI. Conclusion

The dynamics of a magnetic levitation system considered in the paper possess a flat output on which the control law used to stabilize the system was constructed. The control law was designed and applied to the system to stabilize the displacement to a set point. The system with flat output based controller also performed satisfactorily in tracking in the real-time workshop based experiments. Future work will be focused in the hardware design and implementation of a simple MAGLEV plant using different control strategies as well as the flatness-based controller on the FPGA platform.

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