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Failure of a Twenty-Foot High Retaining Wall

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SYNOPSIS A cantilever retaining wall, designed in apparent accord with provisions in a civil engineering handbook, failed soon after construction. Analyses of the causes of the failure are presented.

INTRODUCTION

Cantilever retaining walls are typically designed using either the Rankine or Coulomb equations to estimate applied forces and considering failure modes involving sliding, overturning, toe failures, and overall failure. The standard design techniques are in civil engineering handbooks and have been in use for decades. Designers would seem to have the right to expect that a wall designed in accord with handbooks would perform adequately well.

A cantilever retaining wall with a maximum height exceeding twenty-six feet (eight metres) was designed by a registered professional engineer, in accord with a standard handbook. The designer was a generalist who performed design work in most areas of civil engineering and was thus not trained specially in geotechnical engineering. The designer used factors of safety in excess of two against the usual failure modes. The wall collapsed and subsequent litigation led to losses in excess of \$1 million for the designer.

The purpose of this paper is to present the case history, examine the original design calculations, present a more appropriate, but still simple, set of analyses, and draw relevant conclusions.

CASE HISTORIES OF RETAINING WALL FAILURES

Anecdotal evidence of unacceptable performance of retaining walls, can be heard but well-defined case histories seem to be few. Ireland (1964) made a survey of walls used to support railway cuts. He categorized the backfill and subsoil simply as "sand" (including gravel) and "clay". The distribution of data for walls that had not performed properly are shown in Table 1.

These cases confirm the general view that the major problems are associated with cohesive soils.

Table 1 Summary of Cases of Retaining Walls Engaging in Unacceptable Behavior (Ireland, 1964)

Subsoil	Backfill	% of Walls
clay	clay	43
clay	sand	17
clay	unknown	8
sand	clay	8
unknown	unknown	24
sand	sand	0

DEVELOPMENT OF SITE

A developer selected a site in Central Texas for a shopping center. The site fronted on a major street and was generally flat until near the back edge and left side where the surface began to slope downwards into a large tract of land that was left wild and used as a nature preserve. The ground in the preserve sloped down at about 15 to 20 degrees.

Economic considerations dictated a certain floor area for the shopping center, and city ordinances then required a certain area for parking spaces and driveways. It was discovered that it would be necessary to provide parking and driveways to a line near the edges of the property and thus that it would be necessary to use a 1300-foot (400-m) long retaining wall along the left side and the rear, to support fill. The maximum height of the actual wall, measured from the top of the stem to the bottom of the keyway, was 26 feet 9 inches (8.2 m).

City environmentalists objected to the presence of a high concrete wall at the upper edge of the nature preserve so a solution was worked out such that there would be a relatively low single wall over much of the length and a double wall (Fig. 1) over a 700-foot (210-m) section where the wall was higher than about fifteen feet (4.6 m). Vegetation below the lower wall would obscure that wall from view from below, and vegetation in the flat area

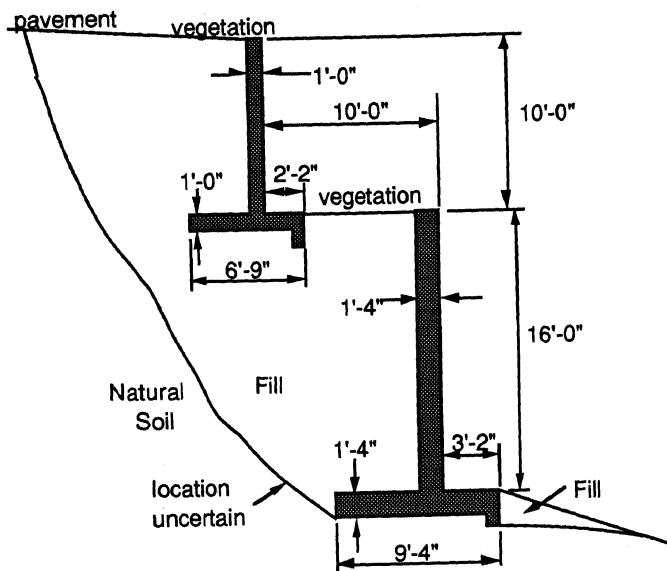


Fig. 1 Wall Configuration in the Area of the Failure

DESIGN OF THE WALL

In view of the fact that subsurface materials were generally limestone or stiff to hard clay shale, that there was an interest in getting the project in operation quickly, and the wall may have been a late addition to the design, the designer apparently saw little need to perform expensive soil tests. The designer attempted to follow recommendations in the Standard Handbook for Civil Engineers (Merritt, 1983), literally tracking through each successive calculation using the same symbols as in the handbook. The designer selected an apparently conservative active earth pressure coefficient of 0.4 ($\phi \sim 25$ degrees) and a total unit weight of 100 pcf for the backfill.

The factor of safety against overturning was calculated as the ratio of the potential resisting moment to the apparent overturning moment, about the toe. The designer analyzed the two wall sections separately and ignored the effect of the upper wall on the lower one. The calculated factor of safety of the lower wall was 2.4.

For the sliding mode of failure, a wall-surface friction coefficient of 0.62 was used ($\delta \sim 32$ deg.), and the factors of safety defined using forces, were inexplicably around 1.0.

The designer estimated the toe stress by taking moments about the heel. In consideration of the presence of limestone and clay shale, the designer set a limit on the calculated toe stress of 5000 psf (240 kPa) based on an undefined handbook and a description of the soil as "clay shale". The actual calculated toe stress was 4.3 ksf (206 kPa).

No slope stability type analyses were performed to examine the possibility of an overall failure.

The designer specified that 2-inch (50-mm) diameter PVC drainage tubes be cast into both the upper and lower walls, near the base of their stems, on 20-foot (6-m) centers. A one-cubic foot (0.03 cu.m.) bag of clean 3/4-inch (18-mm) gravel was to be placed on the soil side of each drainage hole to provide filtration.

No specifications were provided for the backfill but it was believed that local practice, and city specifications were followed. The city specifications set no limit on the maximum particle size but specified that "The percentage of fines shall be sufficient to fill all voids and insure a uniform and thoroughly compacted mass of proper density". Construction records show that several density tests were performed. The backfill apparently ranged from tar weathered, limestone gravel, to a brown gravelly clay. Field density tests indicated 95% to 100% of standard Proctor compaction.

between the walls and above the top wall would obscure the upper wall and shopping center. An irrigation system was planned at the top and at the mid-height.

SITE INVESTIGATION AT DESIGN TIME

About thirty soil borings were made at the site as part of several engineering studies. However, discussion in the engineering reports was restricted to the buildings and to pavements. The wall may not have been contemplated during the main design time. The boring closest to the wall was one-hundred feet (30 m) away. Over much of the site there was one to four feet (1 to 1.2 m) of rocky clay on top of limestone. Several borings penetrated the limestone and encountered a layer of clay shale at a depth of fifteen to twenty feet (4.5 to 6 m). The limestone layer was missing over part of the site, with the clay shale exposed. The clay shale was also exposed in the slope behind the structure and that slope was probably at the angle of repose. No water table was encountered in any soil boring.

The boring closest to the wall showed a soil profile of four feet (1.2 m) of clay over limestone with the limestone described as weathered near its surface but very hard at a depth of six feet (1.8 m).

In accord with usual practice, most of the soil tests were of the classification type. Atterberg limit tests indicated that most of the cohesive soil had liquid limits in the range of 30 to 50% and plasticity points plotted above the A line. Unconfined compressive strengths of shallow clays were generally 1 to 3 tsf (100-300 kPa), with strengths up to 7 tsf (700 kPa) at greater depths in fissured clay shale.

BEHAVIOR OF THE WALL

Construction History

Construction records were not entirely available but it is known that backfilling of the wall occurred during April and it is believed that the wall was completely backfilled in May.

Wall Failure

In late July, after a period of heavy rain, a section of the lower rear wall, with a length of about 135 feet (41 m), suddenly began to displace horizontally. My first site inspection was about seven hours after the failure apparently began and, at that point, the lower wall had displaced horizontally about fifteen feet (5 m) (Fig. 2). A substantial vertical scarp had formed in the fill because the upper wall had dropped into the void formed when the lower wall displaced. The lower wall continued to displace horizontally at a slow rate. Some months later, when the wall approached the property line, the damaged portion of the wall was removed.



Fig. 2 View of the Failed Section of Wall

Site Observations

The exposed fill was mostly limestone gravel but there were several layers of clay. Water was leaking out of the most pervious layers in the fill.

Vegetation in front of the lower wall was dry and dead but vegetation that had been between or above the walls was green. The soil directly in front of the wall, and the slumped fill, was soft. Broken irrigation pipes could be seen in the fill between and above the walls.

Based on the lack of stains, many of the weep holes had apparently not functioned. Attempts to drive rods through them often failed. It appeared that the fill side of some of the holes was blocked with concrete. Water was spurting out of the top part of one of the weep holes, in a stream

with a diameter about the size of a pencil. Apparently, the back side of the weep hole was blocked except for one small hole, and there must have been a substantial water head behind the wall.

Examination of intact portions of the wall showed the presence of rust stains along vertical joints, to a height of about six feet (two metres) above toe slab of the lower wall.

POST-FAILURE SOILS INVESTIGATION

Post failure site access was difficult because of the debris. We made two borings, using a light, portable rig, about thirty feet outside the original position of the lower wall and one in the fill just behind the scarp.

One boring in front of the wall encountered four feet (1.2 m) of medium to stiff black clay, grading rocky near the bottom, and met refusal at four feet (1.2 m), probably due to limestone. The other boring encountered four feet (1.2 m) of soft, wet, rocky clay (probably fill), over three feet (1.0 m) of tan clay (LL=54%, PI=33%, w=31%), and then refusal. The third boring was in the fill that had not yet failed. It encountered four feet (1.2 m) of pavement, base, and a rocky clay; two feet (0.6 m) of dry granular fill; 3.5 feet (1.1 m) of soft, wet, gravelly clay (LL=30%, PI=13%, w=22%); one foot (0.3 m) of very soft tan clay; five feet (1.5 m) of very soft, wet, gravelly clay (LL=28%, PI=11%, w=35%), two feet (0.6 m) of medium tan clay, and four feet (1.2 m) of dry, medium to stiff, black clay. This boring confirmed the heterogeneous nature of the fill.

The fact that an essentially vertical scarp, with a height of about six feet (2 m), stood for some days after the failure, seemed to indicate that the fill was reasonably well compacted.

Because of the heterogeneous nature of the fill, we did not perform any shearing tests on fill material. We performed four drained direct shear tests on samples of natural clay taken from a depth of four to six feet (1.2-2.0 m) in one of the borings in front of the wall (Fig. 3).

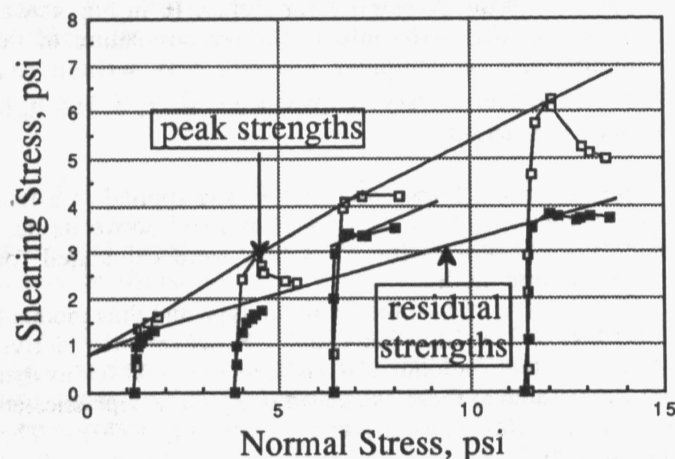


Fig. 3 Results of Direct Shear Tests on Natural Clay Shale

The "residual" friction angle was estimated simply by trimming a smooth failure surface in a direct shear sample and shearing the sample back and forth several times on that surface. Only the final stress path curves are shown in Fig. 3.

The estimated value of \bar{c} was 100 psf (4.8 kPa) for peak and residual, but $\bar{\phi}$ was about 25 deg. for peak and 16 deg. for residual conditions (Fig. 3). The measured shearing properties agreed well with a number of other shearing tests performed in our laboratory on samples from this formation, or from other similar formations in the area, except that the peak friction angle for the other tests was usually closer to 21 deg. and the residual friction angle generally was in the range of 12 to 15 degrees.

ANALYSIS OF STABILITY

The estimated wall configuration in the area of the failure is shown in Fig. 1. The location of the contact between fill and natural soil is uncertain. The actual location of the bottom of the lower wall is unknown because design drawings indicated that the elevation of that base slab would be decided in the field. It is probable that the slab in the middle part of the failed zone was on natural soil because that was the apparent intent of the specifications, but it may also have been on fill.

The uncertainties in wall geometries and soil conditions are such that a sophisticated analysis is not warranted. Instead, the analysis is of the type that a practicing engineer might have made during the design phase. The purpose of the analysis is to determine if a relatively routine approach, properly applied, would have predicted failure.

The analysis was performed in two phases. In the first, the upper wall and soil below it (Fig. 4) were taken out as a free body. Equations for force equilibrium in the vertical and horizontal direction make it possible to calculate the force P that this upper wall and associated soil applies to the vertical plane through the heel of the lower wall. The orientation of the potential shear surface (θ in Fig. 4) was obtained by trial. Because of the tedious nature of the computations, a computer program was written that allowed the user to vary the parameters (β , c , ϕ , H_w , q , δ , γ , and W) as required.

In the second phase, the force P was applied to a free body composed of the lower wall and soil above its heel slab (Fig. 5) and factors of safety were calculated for various failure modes.

The analysis was of the Coulomb type and thus moment equilibrium was not satisfied. However, for active conditions, the Coulomb analysis seems to yield forces that are comparable to those calculated using more sophisticated approaches (Morgenstern and Eisenstein, 1970). The location of P and its obliquity were unknown. Trial analyses were performed to estimate the importance of these uncertainties.

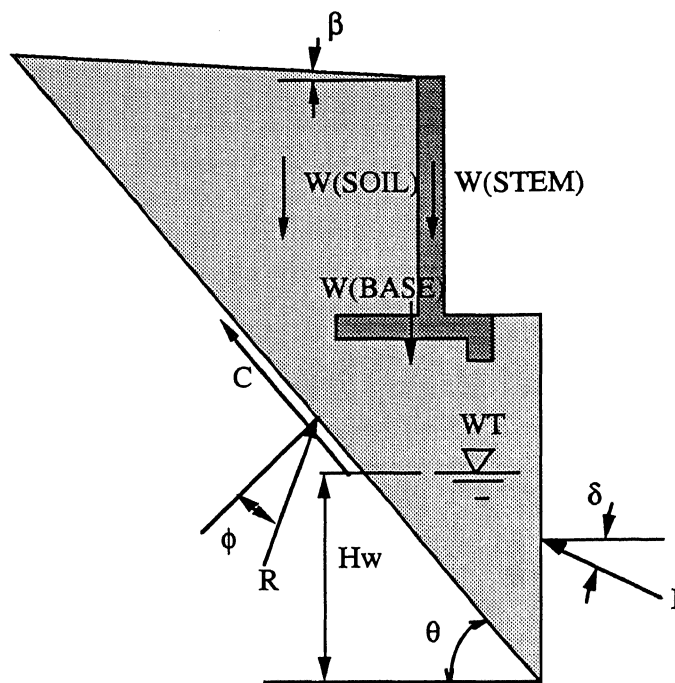


Fig. 4 Assumed Wedge of Soil Sliding Toward the Lower Wall

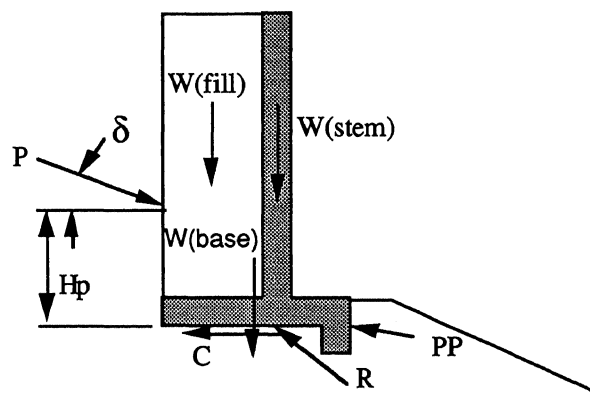


Fig. 5 Assumed Conditions for the Lower Wall Analyses to Obtain P

Assume the soil is saturated and undrained. Practicing engineers often assume that clays are saturated and undrained because then they can assume that $\phi=0$ and the analysis is greatly simplified. The presence of gravel layer in the backfill, with water flowing through them, makes it clear that this condition could not have existed throughout the backfill. However, the designer performed some computations based on this assumption so it will be used here as one extreme case. In any case, the wall did stand for some period of time and that stability needs to be explained.

Analyses using $\delta=0$ (Fig. 4) indicated that P dropped from 39 kips/foot when $c=100$ psf (4.8 kPa), to 0 when $c=720$ psf (34 kPa). Measured undrained strengths were in excess of 720 psf (34 kPa) for the natural soils and

estimated undrained strengths for the exposed clay layers in the backfill also exceeded 720 psf (34 kPa). The analysis confirms the field observation that a substantial height of the clayey part of the fill would stand unsupported as long as it was undrained. The assumption of no drainage could not apply to the fill prior to failure, because of the gravel layers, but it may be relevant for the cohesive parts of the backfill immediately after failure.

Assume the soil is fully drained. Because of the variation in fill material both vertically and horizontally, it seemed most reasonable to assume a homogeneous fill but do a sensitivity analysis for a reasonable range in properties. The cohesionless layers probably had friction angles of at least 35 degrees, and no cohesion. The clay layers probably had friction angles close to that of the natural soil, 25 degrees, with no cohesion. The results of part of the sensitivity study are shown in Table 2.

Table 2-Sensitivity Study for the Upper Wall

c psf	ϕ psf	H _w ft	δ deg.	P kips/foot
100	25	0	0	12
		10		14
100	25	0	0	12
			15	
0 100 200 300	25	0	0	15
				12
				8
				4
0	15	0	0	23
				19
				15
				12

The effect of depth of water (H_w) and obliquity of the resulting earth force (δ) are minor compared with the effects of mean c and ϕ . It would appear that a reasonably conservative design would use about 15 kips/foot for P.

Local Stability of the Lower Wall

Stability of the lower wall (Fig. 5) was analyzed for overturning, sliding, and bearing capacity. Equations for overturning and sliding are found in most books on foundation engineering and need not be repeated.

Some engineers, including the designer of this wall, do not calculate bearing capacity but rather calculate the tip stress assuming a linear variation in contact stress across the base. Such a computation makes little sense for a surface footing on a cohesionless subsoil because the tip bearing capacity depends on footing width and has the irrational value of zero for an imaginary strip footing of differential width at the toe. The more rational approach is to calculate the bearing capacity of the base slab. The bearing capacity of a strip footing subjected to an inclined and eccentric load, and located on a sloping ground surface, is

given by:

$$p_f = cN_c d_c i_c g_c + \gamma D_f N_q d_q i_q g_q + 0.5B' \gamma N_\gamma d_\gamma i_\gamma g_\gamma \quad (1)$$

where N is a dimensionless bearing capacity factor (depends on ϕ), d, i, and g are dimensionless factors to account for footing depth, inclined load, and effects of local ground slope, c is soil cohesion, γ is the soil unit weight above footing depth (q term) or below footing depth (γ term). The bearing capacity, p_f, is the vertical component of the applied force, divided by footing width. A variety of equations have been proposed for the various factors. Equations used here were as follows:

$$N_c = (N_q - 1) \cot(\phi) \quad (\text{Prandtl, 1920, 1921}) \quad (2)$$

$$N_q = \exp(\pi \tan \phi) \tan^2(45 + \frac{\phi}{2}) \quad (\text{Reissner, 1924}) \quad (3)$$

$$N_\gamma = (N_q - 1) \tan(1.4\phi) \quad (\text{Meyerhof, 1961}) \quad (4)$$

$$d_q = 1 + 0.1 \frac{D}{B} \quad (\text{Hansen, 1970}) \quad (5)$$

$$d_c = \frac{d_q s_q N_q - 1}{N_q - 1} \quad (\text{deBeer, 1970}) \quad (6)$$

$$d_\gamma = 1 \quad (7)$$

$$s_q = 1 + \frac{B}{L} (\tan \phi_{\text{triaxial}}) \quad (\text{deBeer, 1970}) \quad (8)$$

$$i_c = i_q = (1 - \frac{\alpha}{90})^2 \quad (\text{Meyerhof, 1953}) \quad (9)$$

where α is the inclination of the resultant force on the base relative to a line normal to the plane of the base,

$$i_\gamma = (1 - \frac{\alpha}{\phi})^2 \quad (\text{Meyerhof, 1953}) \quad (10)$$

$$g_c = (1 - \frac{\beta}{147}) \quad (\text{Hansen, 1970}) \quad (11)$$

where β is the slope of the ground surface measured positively downwards from the horizontal (degrees)

$$g_q = g_\gamma = [1 - 0.5 \tan(\beta)]^5 \quad (\text{Hansen, 1970}) \quad (12)$$

$$B' = B - 2e \quad (\text{Meyerhof, 1953}) \quad (13)$$

and e is the eccentricity of the resultant force applied to the subsoil by the base.

Analyses were performed to determine the factors of safety against failure of the lower wall. A sensitivity study was again performed because of the uncertainties in the various input variables. The results of a part of those computations are shown in Table 3. For all analyses, it was assumed that there was 1.3 feet (0.4 m) of soil producing passive resistance at the toe.

Table 3 Sensitivity Study for the Lower Wall

P kp f	H p ft	δp de g.	δ B de g.	αB ps f	ϕ de g.	c ps f	F _{sbc}	F _{so}	F _{ss}
15	5	0	25	0	25	10	0.08	1.13	0.47
13						0	0.19	1.31	0.55
11							0.31	1.55	0.65
15	3	0	25	0	25	10	0.49	1.89	0.47
	5					0	0.08	1.31	0.47
	6						0.00	---	---
15	5	0	25	0	25	10	0.08	1.13	0.47
		15				0	0.33	2.35	0.61
15	5	0	30	0	30	10	0.12	1.13	0.58
			25		25	0	0.08	1.13	0.47
15	5	0	25	0	25	10	0.08	1.13	0.47
						0	0.03	1.13	0.46
						0			
15	5	0	0	10	0	10	0.16	1.13	0.76
				00		00			

The first group of three analyses with variable force P shows that the lower wall was unstable in bearing capacity and sliding modes for any reasonable forces that would be predicted from the previous analyses.

The second set of analyses show that even a small increase in the assumed height of application of force would result in a bearing capacity failure because of the resulting eccentricity of resultant force on the base.

The third set of analyses shows that increasing the assumed obliquity of the applied force (positive for a downwards vertical component) results in an increase in the factors of safety, mostly because the horizontal component of the force is then reduced.

The fourth set of analyses shows that increasing the friction angles in the reasonable range had only a small numerical influence on the factors of safety.

Similarly, the sixth set of analyses showed that changing the cohesion, through a reasonable range, also had a small effect on the factors of safety.

The final analysis was for the undrained case and used the smallest measured strengths. In that case the wall was unstable for bearing capacity. The largest measured strengths were required to bring the bearing capacity factor of safety up to 1.0. These analyses utilized the value of P corresponding to a drained condition of the backfill. Previous analyses showed that under undrained condition there would be essentially no applied force and the wall would then clearly be stable.

Overall Stability

Overall stability analyses were performed using a slope stability program but the value of those analyses was

limited by the uncertainties in soil properties once the clay shale became relatively hard. If the clay shale was assumed to possess a large effective cohesion at shallow depth, then the slope stability analysis gave factors of safety comparable to those obtained for bearing capacity and sliding.

CAUSES OF THE INADEQUATE DESIGN

The designer's computations were examined as part of the discovery phase prior to trial. In general, the main source of the problem was that the designer based the design on a standard engineering handbook without understanding the limitations of such an approach. On a more technical level:

1. the designer followed the common, but irrational, practice of limiting the apparent tip stress rather than estimating the bearing capacity of the base slab. The bearing capacity equations for cases involving inclined and eccentric load, and for footings on a slope, have a substantial degree of uncertainty in them but at least they provide a rational form of analysis.
2. the designer apparently did not understand the difference between the strengths of the soil under drained and undrained conditions.
3. for reasons that were never explained, the designer ignored the effect of the upper wall on the loads applied to the lower wall.

WHY DID THE WALL STAND UP AT ALL?

Based on the low calculated factors of safety, one wonders why the wall stood up at all. Some of the reasons include:

1. The maximum height of wall existed over a length of only about one-hundred feet (30 m). Stability increases rapidly as the wall height decreases so only a small portion of the wall was in danger. The unstable section of the wall probably derived limited amounts of support from adjacent stable portions.
2. Subsoil conditions were uncertain. Some of the wall may have been supported by a thin layer of limestone that was above the clay shale.
3. The clay shale became considerably harder with depth (could not be penetrated with thin-walled steel tubes) and may have been cemented, thus causing the factor of safety for bearing capacity to be higher than predicted.
4. Negative pore water pressures in the backfill during much of its life would have reduced applied forces greatly, perhaps essentially to zero. The rains that immediately preceded the failure apparently raised the pore water pressures in the fill and caused increased earth forces.
5. The drainage system, though widely used, was ineffective and large water pressures apparently built up on the wall after the rain. Because of the irregular nature of the fill, the water pressure was probably not hydrostatic but, instead, developed in the more pervious layers of fill.

CONCLUSIONS (RE)LEARNED

This case history demonstrates lessons that are generally taught in college but seem often forgotten in practice, including:

1. Engineers should not practice outside of their areas of training and experience.
2. Engineers must resist pressure from clients who want designs turned out immediately because of economic considerations. A careless design, made in haste, may cost the engineer dearly.
3. Design recommendations made in standard civil engineering handbooks, for geotechnical problems, often over simplify the problems and do not provide the kind of technical advice that is required for successful design.
4. Designers should consider both short term (undrained) and long term (drained) conditions unless a technical understanding of the problem makes it clear that one or the other is the critical case.
5. Positive means of controlling water pressures on walls is critical.

LIST OF VARIABLES

a_B	adhesion of base slab on subsoil
c	cohesion
F_{sbc}	factor of safety against a bearing capacity failure
F_{so}	factor of safety against an overturning failure
F_{ss}	factor of safety against a sliding failure
H_p	vertical distance from the heel to the point of application of the earth force
H_w	height of ponded water
P	force applied to the vertical plane through the heel of the lower wall (force/length)
δ	obliquity of resultant force P (degrees)
δ_P	obliquity of P on the vertical plane through the heel (degrees)
δ_B	obliquity of resultant force applied to the subsoil by the base of the lower wall (degrees)

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