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Optimal Allocation of a STATCOM in a 45 Bus Section of the Brazilian Power System using Particle Swarm Optimization

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Abstract - This paper introduces the application of Particle Swarm Optimization (PSO) to solve the optimal allocation of a STATCOM in a 45 bus system which is part of the Brazilian power network. The criterion used in finding the optimal location is based on the voltage profile of the system, i.e. the voltage deviation at each bus, with respect to its optimum value, is minimized. In order to test the performance of the PSO algorithm in this particular application, different approaches for inertia weight are investigated; also different values of acceleration constants, number of iterations and maximum velocity are considered. A sensitivity analysis with respect to these parameters is carried out to determine the importance of these settings. Results show that the application of PSO is suitable for this type of problem. The STATCOM location is found with less computational effort compared with a exhaustive search and with a low degree of uncertainty.

I. INTRODUCTION

A typical power system mainly consists of generators, transformers, transmission lines, switches, active or passive compensators and loads. Such a network is nonlinear and non-stationary, and in practice it is prone to several faults and disturbances. Reinforcing a power system can be done by increasing the voltage level or adding transmission lines. However, these solutions require considerable investment which is difficult to recover. Flexible AC Transmission System (FACTS) devices can be a solution to these problems [1].

Heuristic approaches are traditionally applied for determining the location of FACTS devices in a power system. For instance, shunt FACTS devices are normally connected to the bus with the lowest voltage in the system, while the series devices are usually connected into the lines with the highest power flow through them. While applying general guidelines is feasible for placement of FACTS devices in a small power system, more scientific methods are required for placing these devices in a larger power network.

Traditional optimization methods such as mixed integer linear and non linear programming have been intensely investigated; however difficulties arise due to multiple local minima and the overwhelming computational effort [2], [3].

Recently, Evolutionary Computation Techniques have been employed to solve the optimal allocation of FACTS devices with promising results. Different algorithms such as Genetic Algorithms (GA) [2], [4], [5], [6], and Evolutionary Programming [7] have been tested for finding the optimal allocation as well as the types of devices and their ratings.

Particle Swarm Optimization (PSO) is another evolutionary computation technique that can be used to solve the FACTS allocation problem. It has been applied to other power engineering problems such as: economic dispatch [8], generation expansion problem [9], short term load forecasting [10], and others, giving better results than classical techniques and with less computational effort.

This paper introduces the application of PSO for the optimal allocation of a shunt FACTS device: Static Compensator (STATCOM), in a 45 bus system. The criterion used in finding the optimal STATCOM position is to optimize the voltage profile of the system, i.e. voltage deviations at each bus with respect to its desired value (1 p.u.) are minimized. It is not the purpose of this paper to compare the suitability of PSO in this application with other optimization methods.

The description of the power system used in this study is presented in section II. In section III an exhaustive search method is presented in order to investigate the main characteristics of the objective function, and to find the global minimum value. Sections IV and V describe the PSO algorithm and its implementation for this particular application: different approaches are considered for changing the inertia weight and different set of parameters (acceleration constants, maximum velocity, etc).

The best set of parameters is chosen for this particular type of application and a sensitivity analysis is carried out to determine the relevance of each of them. Results, presented in section VI, show that the application of PSO is suitable for this type of problem, the STATCOM location is found with less computational effort as compared with the exhaustive search, and with a low degree of uncertainty. Finally, conclusions and future work are presented (section VII).

II. MULTIMACHINE POWER SYSTEM

The study of electric power systems is concerned with generation, transmission, distribution and utilization of electric power. The generation of electric power involves the conversion of energy from a non-electrical form to electricity. In a power system the generation is done by the generators or electrical machines which transform mechanical energy into electrical energy. A power system with more than one generator is known as a multimachine power system.

The transmission part of the system involves the high voltage transmission lines, which transport the electric energy from the generation area to the load area, i.e. cities and industries. Transmission lines generally involve different voltages levels depending in the amount of power to transport. As the power requirement increases the transmission voltage is higher.

Computations for a power system having more than one voltage level can become very tedious, therefore a set of values known as base values are defined for each voltage level and each variable is expressed as a fraction of is respective base. In this case, each value is represented by a per-unit (p.u.) quantity. Using the per-unit system, the voltage deviations at each bus are expressed as a deviation from the nominal value of 1 p.u..

The multimachine power system used for this study is presented in Fig. 1. It corresponds to a part of the Brazilian Power Network and consists of the following [11]:

- 45 Buses.
- 10 Generators.
- 17 Transformers.
- 14 transmission lines at 525 kV.
- 41 transmission lines at 230 kV.
- 24 load buses.
- 7 buses with shunt compensation.
- Generation level at 13.8 kV.
- Total installed capacity of 8,940 MVA.

The distribution and utilization part of the system are represented by equivalent loads at the buses where they are connected.

In such a network, it is desirable to keep the voltage deviations between ±5% to avoid voltage collapses during faulty conditions. In general, if the load requirements increase, the voltages at the corresponding buses may drop below 0.95 p.u. and consequently an additional voltage support is needed at that particular bus.

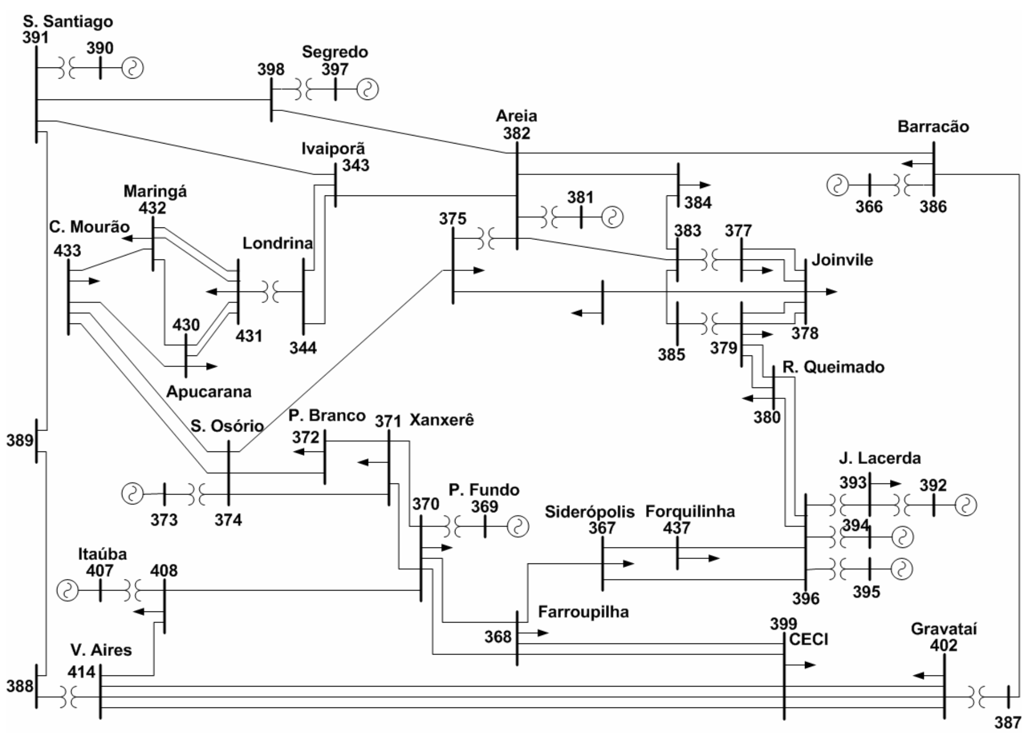

Fig. 1. One line diagram of the 45 bus 10 machine section of the Brazilian power system.

A Static Compensator (STATCOM) is a type of FACTS device that is generally used to mitigate against voltage variations, voltage depressions, and voltage collapses. It is connected to the network in parallel and can control the voltage at the point of connection by injecting reactive power to the system.

The amount of reactive power compensation that the STATCOM can provide depends on its control settings and rating (i.e. the maximum amount of reactive power that the device can inject). In this study a 350 MVA STATCOM is considered, its control settings are adjusted to regulate the voltage at the point of connection to 1 p.u.

The voltages at each bus of the system are determined by running a power flow calculation in PSAT software [12].

III. EXHAUSTIVE SEARCH

Since the multimachine power system has 10 generators, and the voltage at each generator bus is regulated by the generator itself, the corresponding generator buses are omitted from the searching process, thus leaving 35 possible locations for the STATCOM.

For this study, the bus numbers for the power system without the STATCOM are ordered from 1 to 35, with 1 representing the bus with the lowest voltage and 35 the bus with the highest voltage.

For each possible location of the STATCOM, a power flow is calculated in order to determine the voltage deviations at each bus with respect to the value of 1 p.u.; then the objective function J is calculated as the square root of the sum of all voltage deviations squared as follows.

$$J = \left(\sum_{i=1}^{45} (V_i - 1)^2 \right)^{1/2} \tag{1}$$

where:
V_i : is the value of the voltage at bus i in p.u.

Each power flow solution yields the steady state solution of voltages and power flows in Fig 1. The value of J for each of the 35 possible locations of the STATCOM (where case number 1 represents the STATCOM placed at location number 1) are graphically represented in Fig. 2.

The minimum objective function value (0.22182) corresponds to case number 4, with the STATCOM placed at bus 379, Blumenau 2.

Additionally, Fig. 2 shows the nature of the objective function: step type, non-differentiable function with two local minima (cases 1 and 6) and a global minimum (case 4).

IV. PARTICLE SWARM OPTIMIZATION

PSO is an evolutionary computation technique developed by Eberhart and Kennedy in 1995, and was inspired by the social behavior of bird flocking and fish schooling [13], [14], [15]. PSO has its roots in artificial life and social psychology as well as in engineering and computer science. It utilizes a population of individuals, called particles, which fly through the problem hyperspace with some given initial velocities.

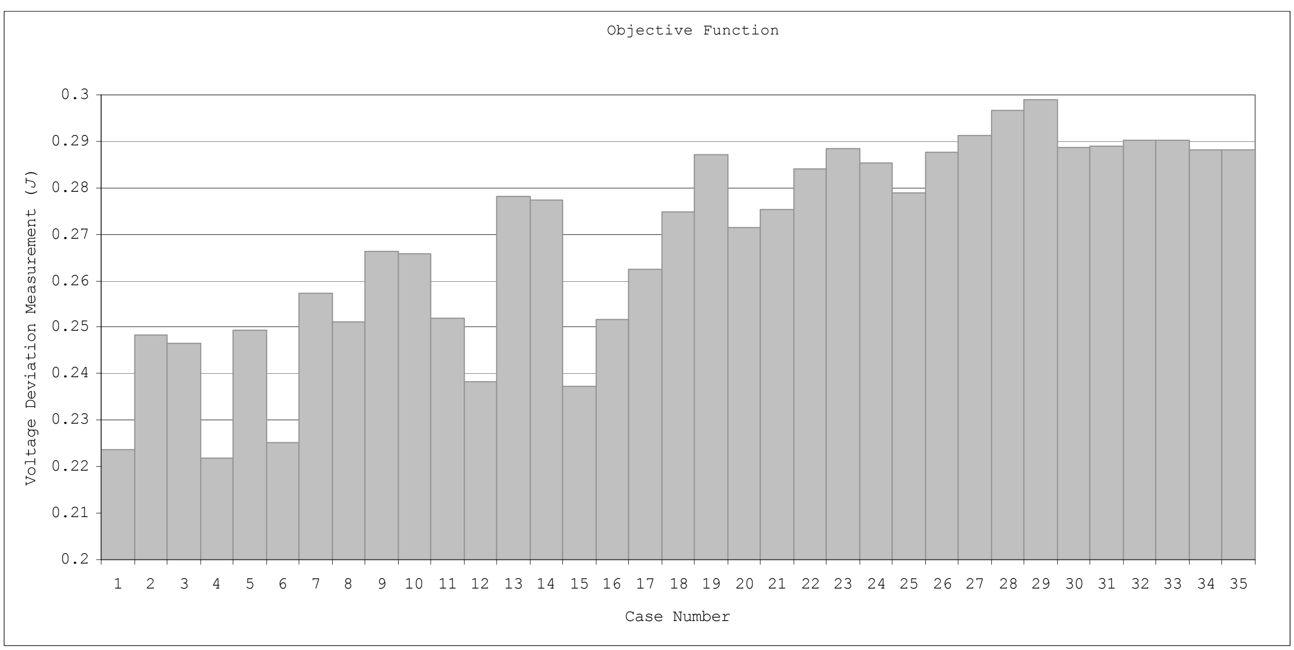

Fig. 2. Objective function value of the 45 bus 10 machine section of the Brazilian power system.

In each iteration, the velocities of the particles are stochastically adjusted considering the historical best position of the particles and their neighborhood best position; where these positions are determined according to some predefined fitness function [14], [16]. Then, the movement of each particle naturally evolves to an optimal or near-optimal solution. The name of "swarm" comes from the irregular movements of the particles in the problem space, more similar to a swarm of mosquitoes rather than flock of birds or school of fish [16].

In a real-number space, the position of each particle is given by the vector $\vec{x}_i \in \Re^n$. At iteration t, the particle position vector $\vec{x}_i(t)$, given in (2), is determined by the previous position vector $\vec{x}_i(t-1)$ and its movement given by the velocity applied to the particle $\vec{v}_i(t)$ [17].

$$\vec{x}_i(t) = \vec{x}_i(t-1) + \vec{v}_i(t) \tag{2}$$

At each iteration, the velocity of a particle is determined by both the individual's and group's experience:

$$\begin{aligned} \vec{v}_i(t) = w_i \cdot \vec{v}_i(t-1) + c_1 \cdot rand_1 \cdot (\vec{p}_i - \vec{x}_i(t-1)) + \ldots \\ c_2 \cdot rand_2 \cdot (\vec{p}_g - \vec{x}_i(t-1)) \end{aligned} \tag{3}$$

where:

w_i	is a positive number between 0 and 1.
c_1, c_2	are two positive numbers called the cognitive and social acceleration constants.
$rand_1$, $rand_2$	are two random numbers with uniform distribution in the range of [0, 1].

The velocity update equation (3) has three different components [18]:

i. The first component is sometimes referred to as "inertia", "momentum" or "habit". It models the tendency of the particle to continue in the same direction it has been traveling.

ii. The second component is a linear attraction towards the best position ever found by the given particle (*pbest)*. This component is variously referred to as "memory", "self-knowledge", "nostalgia" or "remembrance".

iii. The third component of the velocity update equation is a linear attraction towards the best position found by any particle (*gbest).* This component is variously referred to as "cooperation", "social knowledge", "group knowledge" or "shared information".

The maximum allowable velocity for the particles is controlled by the parameter V_{max}. If V_{max} is too high, then particles tend to move beyond a good solution; on the other hand, if V_{max} is small, then particles will be trapped in local minima.

V. IMPLEMENTATION OF PSO ALGORITHM

A. Fitness function

The PSO fitness function used to evaluate the performance of each particle corresponds to the objective function of equation (1).

B. PSO Parameters

For the selection of the parameters for the PSO, the following strategies are applied to the STATCOM allocation problem:

<u>Number of particles equals 4 for all cases</u>:
There is a trade-off between the number of particles and the number of iterations of the swarm. For this particular problem the exhaustive search of section III involves the computation of 35 power flows, therefore, it is desirable for the PSO to find the global minimum with a reduced number of computations. In that sense, each particle' fitness value has to be evaluated using a power flow solution at each iteration, thus the number of particles should not be large. A swarm of 4 particles is chosen as an appropriate population size as it will limit the number of power flow evaluations to meet the goal.

<u>Different types of inertia weight:</u>
Three approaches are considered for the inertia constant:

i. Fixed inertia weight: as in standard PSO definition.

ii. Linearly decreased inertia weight: the purpose is to improve the convergence of the swarm by reducing the inertia weight from 0.9 to 0.1 in even steps over the maximum number of iterations.

iii. Randomly decreased inertia weight: introduces a random factor in the previous approach to avoid the swarm to get trapped in a local minimum (equation 4)

$$w_i = k \cdot \left(0.9 - 0.8 \cdot \frac{iter - 1}{\max\_iter - 1}\right) \tag{4}$$

Where:

k	is a random number between 0 and 1.
$iter$	is the iteration number.
max_iter	is the maximum number of iterations.

<u>Different values for acceleration constants:</u>
The main purpose is to evaluate the effect of giving more importance to the individual's best or the swarm's best in solving the STATCOM allocation problem. A set of three

values for the individual acceleration constant are evaluated: c_1 = {1.5, 2, 2.5}. The value for the social acceleration constant is defined as: $c_2 = 4 - c_1$.

Different number of iterations:
Different numbers of iterations (from 5 up to 8) are carried out to evaluate the number of times in which the global minimum is found.

Different values for maximum velocity:
Three different values for maximum velocity are considered: 5 (smooth movement), 7 (normal velocity) and 9 (rapid changes allowed).

Table I presents a summary of the values tested for each parameter.

TABLE I: PSO PARAMETERS.

Parameter	Tested values
Number of particles	4
Number of iterations	{5, 6, 7, 8}
Inertia weight	Fixed inertia weight: {0.5, 0.7, 0.9} Linearly decreased inertia weight Randomly decreased inertia weight
Acceleration constant (c_1)	{1.5, 2, 2.5}
Maximum velocity	{5, 7, 9}

D. Integer PSO

For this particular application, the position of the particle is determined by an integer number (bus location); therefore the particle's movement as given by equation (2), is approximated to the nearest integer number. Additionally, the location number must belong to the interval [1-35] (feasibility of bus location). If the results of equation (2) are not between 1 and 35, then the particle's position is re-initialized to a random feasible position.

The application of PSO for the STATCOM allocation problem is illustrated in the flow chart shown in Fig. 3.

VI. SIMULATION RESULTS

A. Power Flow Results.

Power flow results when the STATCOM is located in its best position at bus 379 are shown in Table II. The values outside the range limits of ±5% are shown in bold for the cases with and without the STATCOM.

The system without the STATCOM has nine buses with voltages below 0.95 p.u., these buses correspond to two load centers (buses 430-433 and 377-380). Once the STATCOM is connected to bus 379 the voltage deviations improve in the closest load area (buses 377-380), giving an improvement of 19.5% in the objective function value.

With the STATCOM connected to bus 379, it is providing 279 MVA to the system. There are no buses with voltages over 1.05, thus the addition of a STATCOM is not producing overcompensation; however the reactive power injection is not enough to compensate for the entire system (buses 430, 432, 433 are below 0.95 p.u.) which suggests the need of a second compensation device or an increase in the STATCOM rating.

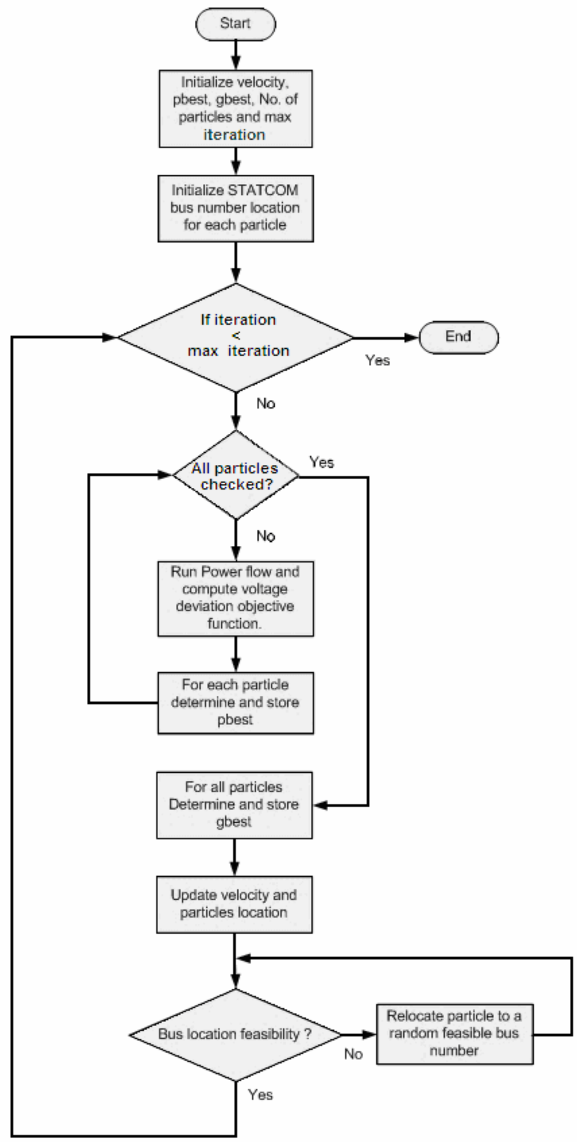

Fig. 3. Flow chart of the implemented PSO.

B. Results for Different PSO Parameters.

In order to find the best set of parameters for the PSO among all the alternatives presented above, a performance index called "Success Rate" (SR) is defined as the number of cases, over 100 trials, in which the minimum value for the objective function is found by any particle of the swarm. Ideally this value should be 100, but as the PSO is a stochastic optimization technique, this ideal value can not be accomplished. In practice, high values of SR are desirable.

The success rate can be understood also as the probability of the PSO to find the correct solution or degree of certainty.

TABLE II: POWER FLOW RESULTS.

Bus number	Voltage p.u. w/o STATCOM	Voltage p.u. with STATCOM
378	**0.9067**	0.9560
433	**0.9182**	**0.9181**
432	**0.9193**	**0.9200**
379	**0.9296**	1.0000
430	**0.9302**	**0.9310**
385	**0.9370**	0.9942
437	**0.9385**	0.9503
367	**0.9392**	0.9508
380	**0.9392**	0.9778
376	0.9516	0.9664
431	0.9520	0.9531
371	0.9542	0.9536
372	0.9549	0.9538
377	0.9580	0.9864
383	0.9640	0.9915
344	0.9711	0.9723
384	0.9713	0.9934
368	0.9752	0.9776
396	0.9766	0.9899
408	0.9805	0.9806
370	0.9816	0.9816
374	0.9819	0.9803
393	0.9830	0.9909
343	0.9922	0.9936
375	1.0000	0.9761
407	1.0000	1.0000
382	1.0028	1.0053
386	1.0117	1.0128
398	1.0118	1.0129
391	1.0120	1.0127
390	1.0180	1.0180
366	1.0200	1.0200
373	1.0200	1.0200
397	1.0200	1.0200
399	1.0206	1.0219
381	1.0220	1.0220
387	1.0224	1.0237
389	1.0284	1.0288
392	1.0300	1.0300
394	1.0300	1.0300
395	1.0300	1.0300
402	1.0320	1.0333
388	1.0384	1.0389
414	1.0391	1.0398
369	1.0400	1.0400

Table III presents the success rates for the three different approaches of inertia weight. For each type of inertia weight, all other parameters (acceleration constants, number or iterations and maximum velocity) are varied according to Table I. As mentioned before a higher success rate represents a larger probability for the PSO to find the correct solution and therefore a smaller degree of uncertainty.

From Table III it is observed that the performance using a randomly decreased inertia weight is worse compared with the other two approaches. Also, the maximum value for the success rate is found for a linearly decreased inertia weight algorithm, which indicates that this approach is more suitable for this type of application (step-type objective function, with $\vec{x}_i \in Z$).

TABLE III: SUCCESS RATE FOR DIFFERENT INERTIA WEIGHTS.

	Fixed Inertia weight	Linearly decreased Inertia weight	Randomly decreased Inertia weight
Max SR	71	77	65
Min SR	33	30	39
Average SR	52.0	53.5	52.0

For each of the three cases in Table III, the difference between the minimum and the maximum success rate is large, showing the importance of setting the parameters of the PSO algorithm correctly. Note also that for all three cases the average success rate is similar, which means that there is no major advantage in the type of algorithm unless the parameter setting is taken into account.

The parameters for the best case (Success Rate equal to 77%) are shown in Table IV.

TABLE IV: OPTIMAL PARAMETERS FOR PSO.

Parameter	Value
Number of Particles	4
Individual Acceleration Constant	2.5
Social Acceleration Constant	1.5
Maximum Velocity	9
Number of Iterations	8

According to Table IV, the best case occurs when the number of iterations is equal to 8, which implies that 32 power flows are computed by the PSO algorithm (8 iterations times 4 particles). Compared with the 35 power flows required in the exhaustive search, the PSO finds the optimal location with 8.5% less computational effort.

C. Sensitivities to Different PSO Parameters.

Tables V, VI and VII show how the Success Rate varies when one particular parameter of the PSO is modified keeping all the rest fixed, in other words they illustrate the sensitivity of the obtained best result to the parameters of the algorithm.

TABLE V: SUCCESS RATE FOR DIFFERENT NUMBER OF ITERATIONS.

Iterations	V_{max}	c_1	Success Rate
4	9	2.5	32
5	9	2.5	45
6	9	2.5	61
7	9	2.5	63
8	**9**	**2.5**	**77**

Table V shows that, as the number of iterations is increased, the probability of the swarm to find the global optimum also increases. In this particular example, since only a medium size power system is used, the number of iterations is limited by the computational effort (it is desirable to have less effort compared to an exhaustive search), thus the maximum success rate can not significantly improve from 77%; however in large power systems where the number of iterations will not represent a constraint to the PSO, it is expected that better success rates can be accomplished.

TABLE VI: SUCCESS RATE FOR DIFFERENT VALUES OF MAXIMUM VELOCITY.

Iterations	V_{max}	c_1	Success Rate
8	5	2.5	58
8	7	2.5	59
8	**9**	**2.5**	**77**

TABLE VII: SUCCESS RATE FOR DIFFERENT VALUES OF INDIVIDUAL ACCELERATION CONSTANT.

Iterations	V_{max}	c_1	Success Rate
8	9	1.5	61
8	9	2.0	70
8	**9**	**2.5**	**77**

Tables VI and VII show that the PSO algorithm is more sensitive to the maximum velocity rather than the individual and social acceleration constants. For this type of application, the results indicate that, to allow rapid changes, is the best strategy to find the optimal position of a STATCOM.

Additionally, giving priority to the self knowledge of the particles (individual acceleration constant equal to 2.5) and limiting the social interaction channel, also helps to increase the probability of finding the optimum placement.

VII. CONCLUSIONS AND FUTURE WORK

The paper has demonstrated the feasibility of the application of the PSO technique to the optimal allocation of a STATCOM in a 45 Bus section of the Brazilian power system. The technique is able to find the best location for the STATCOM in order to optimize the system voltage profile, with a low degree of uncertainty.

The best set of parameters for the PSO is determined using the success rate (SR) as the indicator for evaluation. In this case, the proposed technique is able to identify the optimal location of the STATCOM with a degree of certainty of 77% and less computational time as compared with the exhaustive search.

The obtained success rate results are very promising for this medium size power network. In large and very large power systems, where this kind of problem needs to be solved and the computational effort is an issue, the PSO algorithm can have a significant advantage with respect to exhaustive searches, allowing better success rates as the number of iterations of the swarm is increased.

For this particular type of application, a linearly decreased inertia weight, large maximum velocity and greater individual acceleration constant have proven to be more efficient.

Regarding the power flow solution, it is observed that after the STATCOM is applied, some of the bus voltages are still less than 0.95 p.u. This suggests that another compensating device or a larger MVA rating is required in order to keep all the bus voltages within the ± 5 % limits.

Future work can be done in two different directions. On the one hand, the allocation of more than one STATCOM, other types of FACTS devices and combinations of them can be investigated. On the other hand, different optimization criteria can be considered, e.g. a multi-objective problem in which more than one system feature is optimize, such as losses minimization and voltage profile improvement. Here stability issues could also be included.

VIII. REFERENCES

[1] N.G. Hingorani, L. Gyugyi, "Understanding FACTS; Concepts and Technology of Flexible AC Transmission Systems", IEEE Press, New York, 2000, ISBN 0-7803-3455-8.

[2] Mori, H., Goto, Y., "A parallel tabu search based method for determining optimal allocation of FACTS in power systems", Proceedings of the International Conference on Power System Technology, 2000.PowerCon 2000. Volume 2, 4-7 Dec. 2000 Page(s):1077 – 1082.

[3] N. Yorino, E.E. El-Araby, H. Sasaki, S. Harada, "A new formulation for FACTS allocation for security enhancement against voltage collapse", IEEE Transactions on Power Systems, Volume 18, Issue 1, Feb. 2003 Page(s):3 – 10.

[4] L.J. Cai, I. Erlich, G. Stamtsis, "Optimal choice and allocation of FACTS devices in deregulated electricity market using genetic algorithms", IEEE PES Power Systems Conference and Exposition, 2004. 10-13 Oct. 2004. Vol.1, Page(s):201 – 207.

[5] S. Gerbex, R. Cherkaoui, A.J. Germond, "Optimal location of multi-type FACTS devices in a power system by means of genetic algorithms", IEEE Transactions on Power Systems, Volume 16, Issue 3, Aug. 2001 Page(s): 537 – 544.

[6] S. Gerbex, R. Cherkaoui, A.J. Germond, "Optimal location of FACTS devices to enhance power system security", Power Tech Conference Proceedings, 2003 IEEE Bologna Volume 3, 23-26 June 2003 Page(s): 7-13.

[7] W. Ongsakul, P. Jirapong, "Optimal allocation of FACTS devices to enhance total transfer capability using evolutionary programming", IEEE International Symposium on Circuits and Systems, 2005. ISCAS 2005. 23-26 May 2005, Vol. 5 Page(s):4175 – 4178.

[8] Jong-Bae Park, Ki-Song Lee, Joong-Rin Shin, and Kwang Y. Lee, "A Particle Swarm Optimization for Economic Dispatch With Nonsmooth Cost Functions", IEEE Transactions on Power Systems, Vol. 20, No. 1, February 2005, pp. 34-42.

[9] S. Kannan, S. Mary Raja Slochanal, and Naraya Prasad Padhy, "Application and Comparison of Metaheuristic Techniques to Generation Expansion Planning Problem", IEEE Transactions on Power Systems, Vol. 20, No. 1, February 2005, pp. 466-475.

[10] Chao-Ming Huang, Chi-Jen Huang, and Ming-Li Wang, “A Particle Swarm Optimization to Identifying the ARMAX Model for Short-Term Load Forecasting”, IEEE Transactions on Power Systems, Vol. 20, No. 2, May 2005, pp. 1126-1133.

[11] Venayagamoorthy GK, del Valle Y, Qiao W, Mohagheghi S, Ray S, Harley RG, "Effects of a STATCOM, a SSSC and a UPFC on the Dynamic Behavior of a 45 Bus Brazilian Power System", IEEE PES Inaugural 2005 Conference and Exposition in Africa, Durban, South Africa, July 11 - 14, 2005, pp. 305 - 312.

[12] F. Milano, “An Open Source Power System Analysis Toolbox” IEEE Transactions on Power Systems, Volume: 20, No. 3, pp. 1199 - 1206, August 2005.

[13] J. Kennedy and R. Eberhart, “Particle swarm optimization,” in Proc. IEEE Int. Conf. Neural Networks, vol. 4, pp. 1942–1948, 1995.

[14] R. Eberhart and J. Kennedy, “A new optimizer using particle swarm theory,” in Proc. 6th Int. Symp. Micro Machine and Human Science (MHS ’95), pp. 39–43, 1995.

[15] Shuyuan Yang; Min Wang; Licheng jiao; “A quantum particle swarm optimization” Congress on Evolutionary Computation, 2004. CEC2004. Volume: 1, Pages: 320 - 324 Vol.1, 19-23 June 2004.

[16] J. Kennedy and R. C. Eberhart, “Swarm Intelligence”. San Francisco, CA: Morgan Kaufmann, 2001.

[17] J. Kennedy, “The particle swarm: Social adaptation of knowledge,” in Proc. IEEE Int. Conf. Evolutionary Computation, pp. 303–308, 1997.

[18] D.W. Boeringer, D.H.Werner, “Particle swarm optimization versus genetic algorithms for phased array synthesis”, IEEE Transactions on Antennas and Propagation, Volume: 52, Issue: 3, pp. 771 – 779, March 2004.