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SYNOPSIS The ground motion is caused by vibration propagation due to moving vehicles. Ground vibration recorded at a distance from a busy roadway or railway is analyzed assuming it to be a random and statistically stacionary function of time. The soil media are characterized as the visco-elastic halfspace. The analytical method for predicting ground vibration has been described. The spectral density of ground vibration for a particular case was calculated and the prediction is validated by experimental measurement.

INTRODUCTION

An analytical expression for the spectral density of ground vibration as a functions of distance from both roadways and railways respectively is formulated in terms of road roughness or irregularities on the rail head, vehicle characteristics and the frequency response function for the ground. The use of the random process theory to predict the level of ground vibration in the vicinity of busy roads (railways) via calculation of the spectrum of vibration at point is possible by the two principal ways: (i) using a computer implementation of the theoretical expression for the road roughness spectrum, the vehicle mass distribution spectrum and a model of vehicle dynamics and the frequency response function (FRF) of the ground by a method involving integral transform. (ii) using average response force spectrum derived from experimental data for authorized roadway category with corresponding road profile (e. g. Draft Standard ISO/TC 108/WG9) and the FRF of the ground and calculate spectrum vibration at point by the same way as mentioned in (i). The random process theory at the dynamic ground properties investigation can be utilized as well. The input signal (due to traffic) measurement into the ground and the output signal measurement passing through the ground, frequency response function, elastic and attenuation characteristics of the ground can be obtained, Bendat and Piersol (1971), Crandall and Mark (1963), Benčat (1986).

RESPONSE SPECTRUM DUE TO RANDOM LINE EXCITATION

Perhaps the most descriptive representation of the traffic influence on a halfspace is provided by a response spectrum. We consider response at a point due to random line excitation. On the surface of a linear visco-elastic halfspace, the displacement response spectrum $S_{ww}(\omega)$ can be expressed in terms of the spectrum of applied force, Newland (1984)

$$S_{ww}(\omega) = |H(r,\omega)|^2 S_{FF}(\omega) \tag{1}$$

where $H(r,\omega)$ is the frequency response function for halfspace and w is the displacement of the surface measured at a distance r from an applied surface froce F (fig. 1). For n independent point loads $F_1(t)$, $F_2(t)$, ... $F_n(t)$ acting at points x_1, x_2, ... x_2 ... x_n respectively on the surface of the halfspace, we can write the corresponding autospectral density functions as $S_{FF}(x_1,x_1,\omega)$, $S_{FF}(x_2,x_2,\omega)$ ... and likewise the n(n-1) cross spectral density functions $S_{FF}(x_1,x_2,\omega)$, $S_{FF}(x_1, x_3,\omega)$ ... Newland (1984) describes a method for analysing continuous systems subject to multiple, discrete random loads where it is shown that the spectrum of surface displacement at a point "A" (fig. 1) can be expressed as the summation

$$S_{ww}(\omega) = \sum_{j=1}^{n}\sum_{k=1}^{n} H^*(r_j,\omega)H(r_k,\omega)S_{FF}(x_j,x_k,\omega) \tag{2}$$

where $H^*(r,\omega)$ is the complex conjugate of $H(r,\omega)$ and r_j, r_k are the distances of the loads $F_j(t)$, $F_k(t)$ from the point "A".

Equation (2) would take the form

$$S_{ww}(a,\omega) = \sum_{j=1}^{n}\sum_{k=1}^{n} H^*(a,x_j,\omega)H(a,x_k,\omega)S_{FF}(x_j,x_k,\omega) \tag{3}$$

where $r_j^2 = a^2 + x_j^2$. For the case of continuously distributed line of random loads p(x,t) per unit length, where $f_j(t) = p(x_j,t)\Delta x$, an integral expression for the response spectral density can be written as

$$S_{ww}(a,\omega) = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 H^*(a,x_1,\omega)H(a,x_2,\omega)S_{pp}(x_1,x_2,\omega) \tag{4}$$

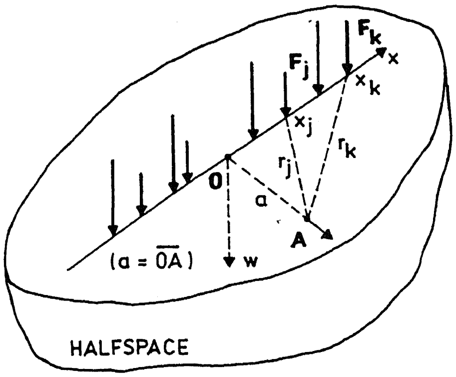


Fig. 1 Random line excitation - Response at a point "A"

where x_1 and x_2 are dummy variables. We assume $p(x,t)$ to be a stationary random process in space and time so that $S_{pp}(x_1,x_2,\omega)$ depends only on the spacing between loads $\varepsilon = x_2 - x_1$. Then by defining

$$G(a,\gamma,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}H(a,x,\omega)e^{-i\gamma x}\,dx \tag{5.1}$$

$$S_{pp}(\gamma,\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{pp}(\varepsilon,\omega)^{-i\gamma\varepsilon}\,d\varepsilon \tag{5.2}$$

and taking note of symmetry, eq. (4) can be expressed as, Newland (1984)

$$S_{ww}(a,\omega)=(2\pi)^2\int_{-\infty}^{\infty}|G(a,\gamma,\omega)|^2 S_{pp}(\gamma,\omega)\,d\gamma \tag{6}$$

where $\gamma = \omega/v$, and v is velocity of-all moving forces (for practical application it is assumed that all the vehicles move at the same speed v). This result is used to compute the spectrum of vibration at the distance from the road (railway) provided that the force spectrum $S_{pp}(\gamma,\omega)$ can be obtainded.

SPECTRUM OF APPLIED FORCE

Vehicles moving along the road are each considered as simple damped two degree of freedom oscilators (fig. 2). Based on an examination of typical heavy vehicles in ČSFR, the bounce frequency ω_1 and the "wheel hop" frequency ω_2 are assumed not to change significantly with vehicles. Their ratio $\alpha = \omega_2/\omega_1$ is assumed to be constant and the ratio of masses $\mu = m_2/m_1$ and damping $\beta = \xi_1/\xi_2$ are also assumed constant, as well. Such assumptions enable to evaluate $S_{pp}(\gamma,\omega)$ in terms of the distribution of vehicles on road, the vehicle dynamics and the road roughness. The derivation of the algorithm for the calculation of the predicted spectrum of surface ground vibration (acceleration) is given e. g. by Hunt (1987). The vehicles are assumed to be moving at a steady speed v along a rough surface whose Gaussian spectrum is given by (see also ISO/TC 109/WG9)

$$S_{YY}(\gamma)=S_{Y_0}(\gamma/\gamma_0)^{-n} \tag{7}$$

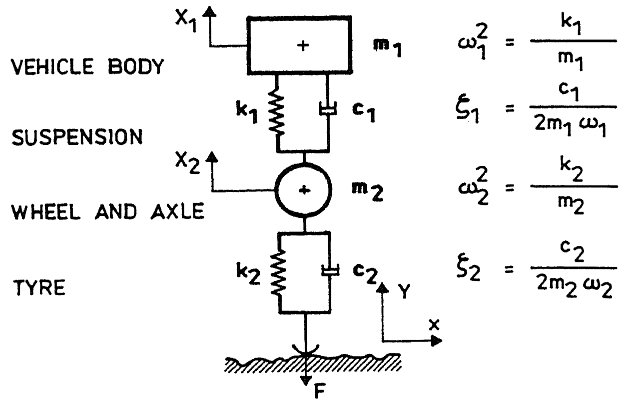


Fig. 2 Dynamic vehicle model

so that the spectrum of weighted acceleration can be written, Newland (1984), as

$$S_{\ddot{z}\ddot{z}}(\gamma)=v\,|H_{\ddot{z}Y}(\omega=v\gamma)|^2 S_{YY}(\gamma) \tag{8}$$

where $H_{\ddot{z}y}(\omega)$ is the frequency response function for the weighted average acceleration of the two masses $\ddot{z}=(\ddot{x}_1+\mu\ddot{x}_2)$ as output due to an input road displacement Y (fig. 2). It remains to describe the random distribution of vehicle masses m_1 along the road in terms of the spectral density $S_{m_1m_1}(\gamma)$ (with the units of m_1 being mass per unit length of road). The mean and standard deviation of the vehicle mass distribution (assumed Gaussian) are taken as $\bar{m}_1$ and σm_1 respectively (fig. 3). The tyre contact patch length is w_0 and u_0 is the mean vehicle spacing. Assuming the vehicle loads to be uncorrelated in space, the spectrum of mass distribution is given by Hunt (1987) as

$$S_{m_1m_1}(\gamma)=\frac{\bar{m}_1{}^2+\sigma^2 m_1}{2\pi u_0}\left(\frac{\sin\gamma w_0/2}{\gamma w_0/2}\right)^2 \tag{9}$$

The force applied to the road can now be written $F(t)=m_1\ddot{z}(t)$. Subject to the assumption that all

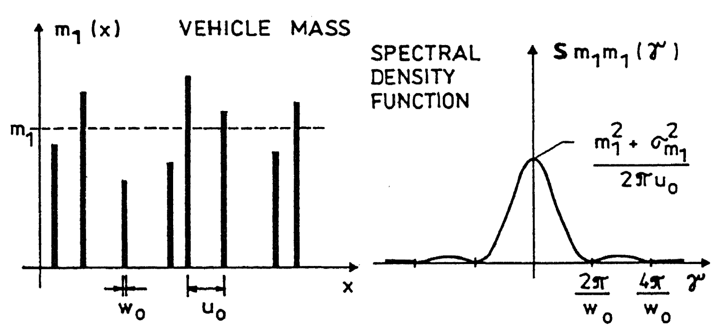


Fig. 3 Assumed vehicle mass distribution and its corresponding spectral density function

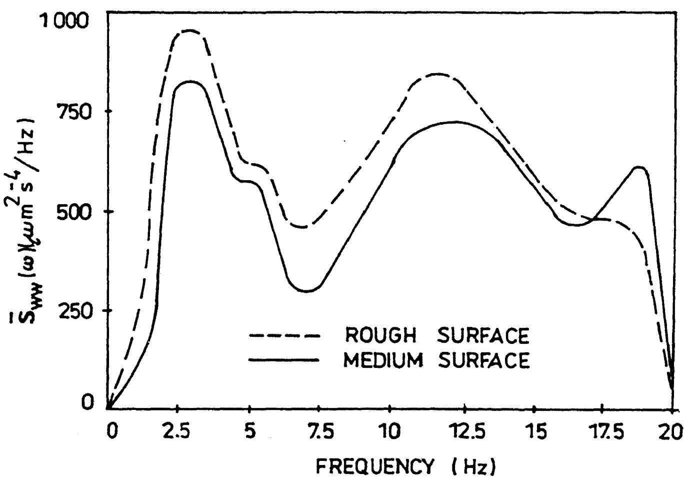
vehicles move at the same speed v and that the mass distribution translates at speed v but is otherwise unchanged, it can be shown that

$$S_{PP}(\gamma,\omega)=\frac{1}{v}\,S_{m_1m_1}(\gamma)S_{\ddot{z}\ddot{z}}(\gamma+\frac{\omega}{v}) \qquad (10)$$

which is the two dimensional form of the road force spectrum required by eq. (6). In the second case (ii) we can use pavement average acceleration response spectrum (PAARS) $\bar{S}_{ww}(\omega)$, derived from experimental data for the relevant category of roadway with the corresponding road profile, for calculation $S_{ww}(\omega)$ at a distance. Then the $S_{ww}(\omega)$ is calculated as

$$S_{ww}(\omega)=|H(r,\omega)|^2\,\bar{S}_{ww}(\omega) \qquad (11)$$

In Fig. 4 is shown smoothed PAARS for medium and rough pavement surface according to experimental data, Benčat (1992).



ig. 4 Pavement average acceleration response spectrums

XPERIMENTAL PROCEDURES

n experimental study of ground vibration ransmission from a busy road was carried out djacent to the SC.Nr.1/11 trunk road near Žilina ČSFR). The road is straight and well situated n level ground (sandy-clay and sandy-gravel, epth of over 20 m). This permits the ground to e modelled as a damped, viscoelastic halfspace. he viscoelastic model of simulation using omplex modulus $E=E(1+i\delta_E$ and $G=G(1+i\delta_G)$ espectively offers a very good approach to the ctual soil behaviour (E, G and $\delta_E\approx\delta_G$ are real nd imaginary components of complex modulus). ne basic equations used to describe the iscoelastic halfspace by Martinček (1968) as ell as the analysis of wave propagation through round with modulus in complex form cannot be ılly described here. The Rayleigh´s and shear aves propagating by halfspace are analysed in nis form in Benčat (1982, 1984). The experimenal test for the purpose of the evaluation of lastic and attenuation soil characteristics ere performed on the test site. The object of perimental measurements of stationary signals due to moving vehicle) was to find:

- the Rayleigh´s and shear wave velocities v_R and v_S by cross correlation function $R_{xy}(\tau)$ then derive the initial shear modulus $G_0= =v_R^2\rho F_{RE}$, where F_{RE} is a real component of complex roots of frequency equation, Martinček (1968) and ρ is mass density of soil
- the attenuation coefficient α (m^{-1}) obtained by standard deviations $\sigma(o)$, $\sigma(y)$ of amplitude vibration at the distance l_0, l_y from source of excitation using the displacement power spectral densities $G_{ww}^{(0)}$ and $G_{ww}^{(0)}$. The coefficient of attenuation is defined by

$$\alpha=(l_x-l_n)^{-1}\ln(k\sigma_{(0)}/\sigma(y)) \;;\quad k=(l_0/l_x)^{1/2} \qquad (12)$$

- the soil frequency characteristics expressed by the frequency response function $H(i\omega)$.

The results of the performing tests are as follows: $v_R=180.38\ ms^{-1}$, $G_0=68.90$ MPa, $\delta_G= =0.166$, the calculation includes the data $\lambda_R= 10.8$ m, $\rho=1982\ kgm^{-3}$, $F_{RE}=1.0695012$ and $\nu=0.38$. An approximate relationship between a damping parameter $\delta \doteq \delta_G \doteq \delta_E$ and wave length λ_R of the propagating Rayleigh´s wave is $\delta \doteq |\alpha|\lambda_R/\pi$. The analysis of the Rayleigh surface wave response of an elastic halfspace, Ewing, Jardetzky and Press (1966), can be extended to include the effect of light damping. The frequency response function for viscoelastic halfspace is expressed approximately as, Hunt (1987)

$$H(r,\omega)=\exp(-D\omega^2 r/2v_R)\,\frac{-i\omega K'}{2\rho v_R^3}\,H_0^{(2)}\left(\frac{\omega r}{v_R}\right) \qquad (13)$$

where K is a dimensionless material constant, a function only of Poisson ratio ν and in range $0.1\le K\le 0.22$, D is the damping coefficient derived for the complex modulus in term $E= =E(1+i\omega D)$, and $H_0^{(2)}$ is Bessel function expressed in standard notation. To obtain the parameter D and validation of parameters calculated from measuring of stationary signals, the impulse test was performed, as well. The object of experimental measurements of transient signal was to find:

- the Rayleigh´s and compressive wave fronts by using standard equipments of impulse-seismic method (ISM) which is similar to cross-hole method
- the same characteristics as mentioned in stationary signals investigation via the spectral and correlation theory means.

The impulse test results are as follows: $v_R=185.19\ ms^{-1}$, $\delta=0.182$, $D=0.00146$, $K'=0.191$.

TRAFFIC VIBRATION MEASSUREMENT

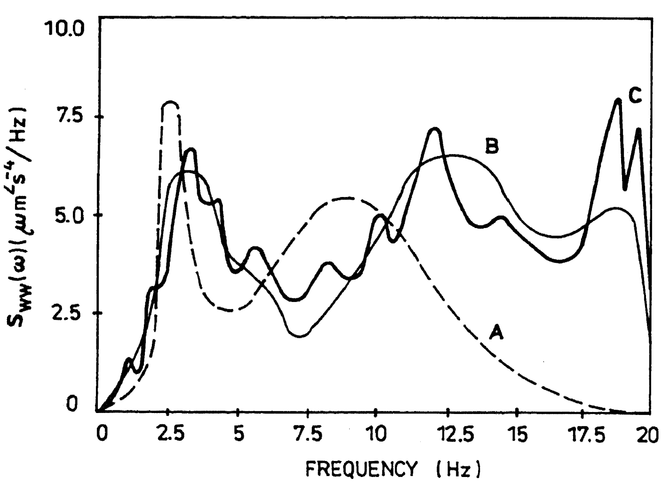
Ground vibration levels were measured at a distance of 18 m and 118 m respectively by accelerometers BK-8306 (Brüel-Kjaer) and recorded through amplifiers on a tape recorder BK-7005. The signals were then replayed in the laboratory and the output fed to the narrow-band real time analyser BK-2131 in conjuction with PC-AT calculator.

Assuming an average of three axles per vehicle (the vibration due to passenger cars is negligible), the parameters for Eq. 9 were deduced by considering each axle loading independently. Thus taking an average vehicle mass 24 000 kg and the observed average three--axle vehicle spacing of 150 m: m_1=8.000 kg, σ_{m_1}=2000 kg, u_0=50 m, v=30 ms^{-1}, w_0=200 mm.

The vehicle dynamics parameters for calculation $S_{\ddot{z}\ddot{z}}(\gamma)$, (Eq.8) are based on data averaged from several sources Cebon (1986), Plachý (1987) μ=0.15, ω_1=13.4 rad.s^{-1} (2.13 Hz), $\omega_2=3\omega_1$ (6.39 Hz), ξ_1=0.086, ξ_2=0.024.

For road roughness, the parameters for Eq.7 are those corresponding to a good profile (medium surface), Cebon (1986): $S_{Y_0}=0.318\times10^{-6}m^3.rad^{-1}$, γ =1.0 rad.m^{-1} and n=2.

By substituting the above parameters into Eqs.7--10 and Eq. 13 and by using Eq. 6, the predicted spectrum of surface ground vibration (acceleration PSD) is calculated and shown in fig. 5 (Curve-A). Also shown is the spectrum of vibration calculated by using Eq.11 via pavement acceleration response spectrum $S_{ww}(\omega)$ (Curve-B) and spectrum (smoothed) of vibration actually measured under the above conditions (Curve-C).



'ig. 5 Spectrums of ground vibration at 118 m. Curve (A), prediction spectrum (theory). Curve (B), prediction spectrum via experimental PAARS. Curve (C), experimental spectra for particular case

CLUSIONS

: predicted ground vibration spectrum is ıstantially borne out by experiment focused modeling accurately the transmission of und borne vibration from busy roads. In the st case (i) the sharp spectral peak dicted at 2.1 Hz reflects the assumption at all vehicles have the same "bounce" equency (fig. 5 - curve A). In practice, here appears to be sufficient variation in "bounce" frequency between vehicles to broaden the peak as observed without affecting the mean square of the spectrum. Substantial pitching modes in range 2÷5 Hz for typical heavy vehicles with 5 and more axles may also contribute to this effect (e.q.it is corresponding to the mode vibration: tractor nose diving, trailer antiphase pitch, tractor pitch and bounce, trailer pitch about 5th wheel, e.t.c.) A similar peak broadening is observed at the "wheel hop" frequency, but this is less prominent since the predicted peak at 8 Hz is itself very broadened but shifted about 3 Hz in comparison with the experimental ground vibration spectrum (fig. 5 - curve C). In the second case (ii) the predicted spectrum of ground vibration (fig. 5 - curve B) is closely associated with the experimental ground vibration spectrum for a particular case (fig. 5 - curve C). The vibration level measured at frequencies above 18 Hz are not associated with large scale vehicle dynamics discussed above, but reflects the level of ambient background vibration. The paper indicates that the analysis of random ground vibration due to both railway and road traffic respectively provides a useful and required information on the frequency composition of ground vibration, the surface wave velocity propagation, the soil attenuation and damping properties and viscoelastic properties of soils as well.

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