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# Critical behavior of a quantum spherical model in a random field

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We consider the influence of a quenched random field on the critical behavior of a quantum spherical model at the zero-temperature quantum phase transition. We find a complete solution of the model for arbitrary translationally invariant pair interactions. It turns out that the critical behavior for zero as well as finite temperatures is dominated by static random field fluctuations rather than by quantum or thermal fluctuations. Therefore the critical behavior close to the zero-temperature quantum phase transition is identical to that close to a finite-temperature transition. The system does not show a crossover from quantum to classical behavior.

While classical finite-temperature phase transitions are comparatively well understood by now, much less is known about zero-temperature phase transitions which are driven by quantum fluctuations rather than thermal fluctuations. These quantum phase transitions were introduced by Hertz[1] in the context of itinerant ferromagnets. Newer investigations include, e.g., metal-insulator transitions[2] and the superconductor-insulator transition,[3] as well as order-disorder transitions in quantum antiferromagnets[4] and spin glasses.[5] The study of quantum phase transitions in systems with quenched disorder is particularly interesting since the disorder which fluctuates in space but not in time destroys the symmetry between space and time dimensions. Therefore the critical behavior can be expected to be different from that of a classical phase transition in higher spatial dimensions.

Recently, these investigations of quantum phase transitions have stimulated renewed interest in the spherical model introduced as a toy model of a ferromagnet by Berlin and Kac.[6] The classical spherical model is one of the few models in statistical physics that can be solved exactly and shows nontrivial critical behavior. Thus it has been used to investigate a variety of problems since the first studies of phase transitions in ferromagnetic systems.[6,7] Later it was also used to investigate the influence of disorder on the critical behavior close to classical finite-temperature phase transitions in spin glasses,[8] random field systems,[9] and disordered electronic systems with localized states.[10] In order to study the critical properties close to quantum phase transitions, the classical spherical model has to be quantized. The quantization scheme is, however, not unique. Therefore different quantum versions of the spherical model have been proposed within the last years.[11,12] While the different models fall into the same universality class at a finite-temperature classical phase transition, the critical behavior at a zero-temperature quantum phase transition differs from model to model. Usually, it also differs from the finite-temperature critical behavior. In this case, for low but finite temperatures the quantum spherical models display a crossover from quantum to classical behavior.

In this paper we study the influence of a quenched random field on the zero-temperature phase transition of a quantum spherical model. Quenched random fields coupling linearly to the order parameter have drastic effects on the critical behavior. At a classical phase transition, they lead to an increase of the lower critical dimension $D_c^-$ by 1 (Ref. 13) (for the Ising universality class) or 2 (Ref. 11) (for the spherical model). The influence of a random field on quantum critical behavior was first studied by a perturbational renormalization group.[14] It was found that random field fluctuations rather than the quantum fluctuations dominate the critical behavior. However, in some cases perturbational results for the critical behavior of disordered systems have been proven to be wrong[15] since they neglect the very complicated structure of the phase space in disordered systems. Therefore, in this report, we investigate the problem by means of an exact solution of a random field quantum spherical model obtained by a canonical quantization scheme.

In the following we briefly summarize the canonical quantization of the spherical model.[12] In order to define the quantum spherical model we first consider a classical spherical model[6] of $N=L^D$ ($D$ is the spatial dimensionality) real variables $S_i$ ranging from $-\infty$ to $\infty$ that interact via a pair potential $U_{ij}$ which we assume to be translationally invariant [i.e., $U_{ij}=U(r_i-r_j)$] for simplicity. The Hamiltonian of the model is given by

$$H_{\rm cl}=\frac{1}{2}\sum_{i,j} U_{ij}S_iS_j+\sum_i (h+\varphi_i)S_i\,, \tag{1}$$

where $h$ represents an external "magnetic" field. The random field values $\varphi_i$ are independent random quantities characterized by the first two moments of the probability distribution

$$[\varphi_i]_\varphi=0, \quad [\varphi_i\varphi_j]_\varphi=\Phi^2\delta_{ij}\,, \tag{2}$$

where $[\cdots]_\varphi$ denotes the average with respect to the random field. The values of the spin variables $S_i$ are subject to the mean spherical constraint

$$\sum_i \langle S_i^2\rangle = N/4, \tag{3}$$

where $\langle\cdots\rangle$ denotes the thermodynamic average for a particular realization of the random field. In some of the previous studies of the spherical model this constraint was imposed not on the averages, but on the values of the variables themselves (strict spherical constraint). Usually, both versions lead to the same results for thermodynamic quantities. For a detailed discussion of the relation between the mean and strict spherical constraints see, e.g., Ref. 7. Here we have chosen the mean spherical constraint since it is easier to generalize to the quantum case than the strict constraint.[12]

We now reinterpret the variables $S_i$ as operators and define canonically conjugate "momentum" operators $P_i$ so that the following commutation relations are obeyed (with $\hbar$ set equal to 1):

$$[S_i,S_j]=0, \quad [P_i,P_j]=0, \quad [S_i,P_j]=i\delta_{ij}. \tag{4}$$

The quantum spherical model is then obtained from (1) by adding a kinetic energy term. As already mentioned, the choice of this term is by no means unique, and depending on the form of the kinetic energy the model displays different dynamical behavior (for comparison, see, e.g., Refs. 11 and 12). Here we choose the simplest possible kinetic energy, a sum over the squares of the momentum operators. Thus the Hamiltonian is given by

$$H=H_{\rm kin}+H_{\rm cl}=\frac{g}{2}\sum_i P_i^2+\frac{1}{2}\sum_{i,j}U_{ij}S_iS_j+\sum_i(h+\varphi_i)S_i +\mu\left(\sum_i S_i^2-\frac{N}{4}\right), \tag{5}$$

where the coupling constant $g$ determines the importance of quantum fluctuations; $g\to 0$ corresponds to the classical limit. The mean spherical constraint (3) is taken care of by a Lagrange multiplier $\mu$. We want to emphasize that the commutation relations (4) together with the quadratic kinetic term in the Hamiltonian (5) do not describe quantum Heisenberg-Dirac spins but quantum rotors. The quantum rotors can be seen as a generalization of Ising spins in a transverse field.[5] They also describe the low-temperature behavior of quantum antiferromagnets.[4,16]

The Hamiltonian of the quantum spherical model (5) is equivalent to that of a system of coupled displaced harmonic oscillators and can therefore be solved easily. A Fourier transformation yields

$$H=\sum_k\left[\frac{g}{2}P(k)P(-k)+\frac{1}{2g}\omega^2(k)S(k)S(-k) -\frac{\varphi(k)\varphi(-k)}{4\mu+2U(k)}\right]-\frac{Nh^2+2\sqrt{N}\varphi(0)h}{4\mu}-\frac{\mu N}{4}, \tag{6}$$

where $P(k)$, $S(k)$, and $\varphi(k)$ are the Fourier transforms of the operators and random field, respectively. The frequencies $\omega(k)$ are given by

$$\omega^2(k)=g[2\mu+U(k)]. \tag{7}$$

Here $U(k)$ is the Fourier transform of the interaction matrix $U_{ij}$, and we have fixed our energy scale by assuming that the Fourier component $U(0)$ for $k=0$ is equal to zero. Now the partition function for a fixed realization of the random field values can be written down easily since it factorizes with respect to $k$. We obtain

$$Z[\varphi]=\prod_k\left(2\sinh\frac{\beta\omega(k)}{2}\right)^{-1}\exp\left(\frac{\beta\mu N}{4}+\frac{\beta Nh^2+2\sqrt{N}\varphi_0 h}{4\mu} +\sum_k\frac{\beta\varphi(k)\varphi(-k)}{4\mu+2U(k)}\right), \tag{8}$$

where $\beta$ is the inverse temperature $\beta=1/k_BT$. Thus the free energy per site is given by

$$f[\varphi]=-\frac{\ln Z[\varphi]}{\beta N}=-\frac{\mu}{4}-\frac{h^2+2N^{-1/2}\varphi_0 h}{4\mu} -\frac{1}{N}\sum_k\frac{\varphi(k)\varphi(-k)}{4\mu+2U(k)}+\frac{1}{\beta N}\sum_k\ln\left(2\sinh\frac{\beta\omega(k)}{2}\right). \tag{9}$$

The Lagrange multiplier $\mu$ is determined by the equation for the spherical constraint (3):

$$0=\frac{\partial f[\varphi]}{\partial\mu}=-\frac{1}{4}+\frac{h^2+2N^{-1/2}\varphi_0 h}{4\mu^2}+\frac{1}{N}\sum_k\frac{\varphi(k)\varphi(-k)}{[2\mu+U(k)]^2} +\frac{1}{N}\sum_k\frac{g}{2\omega(k)}\coth\frac{\beta\omega(k)}{2}. \tag{10}$$

In principle, the value of $\mu$ could depend on the particular realization of the random field $\varphi_i$. A detailed analysis shows, however, that in the thermodynamic limit $N\to\infty$ $\mu$ is independent of the realization of the random field.[17] Thus the free energy $f$ and the Lagrange multiplier $\mu$ are self-averaging quantities. Therefore we can separately average (9) and (10) with respect to the random field to obtain the average free energy,

$$f=-\frac{\mu}{4}-\frac{h^2}{4\mu}-\frac{1}{N}\sum_k\frac{\Phi^2}{4\mu+2U(k)} +\frac{1}{\beta N}\sum_k\ln\left(2\sinh\frac{\beta\omega(k)}{2}\right), \tag{11}$$

with

$$0=-\frac{1}{4}+\frac{h^2}{4\mu^2}+\frac{1}{N}\sum_k\frac{\Phi^2}{[2\mu+U(k)]^2} +\frac{1}{N}\sum_k\frac{g}{2\omega(k)}\coth\frac{\beta\omega(k)}{2}. \tag{12}$$

In both equations the last term represents quantum and thermal fluctuations, whereas the third term, containing $\Phi^2$, represents the static random field fluctuations which have exactly the same form as in the classical spherical model.[9,10] In particular, the random field terms do not depend on $g$ or $\beta$.

As usual in the spherical model, the critical behavior is determined by the solution of Eq. (12) for the spherical constraint close to the critical point. At any finite temperature we can expand the coth terms in (11) and (12) in the long-wavelength limit $|k|\to 0$ and for small $\mu$. From this it follows that the leading terms are the same as in the classical random field spherical model.[9,10] Consequently, the critical behavior of the quantum spherical model close to a finite-temperature phase transition is identical to that of a classical spherical model as is expected from general renormalization group arguments.[1]

We now turn to the properties of the quantum phase transition at $T=0$ ($\beta=\infty$). Again, the critical behavior is determined by the solution of the equation for the spherical constraint close to the critical point $g=g_c$. $g_c$ is given by the value of $g$ which separates the regimes where a regular solution of (12) exists (large $g$, disordered phase) or does not exist (small $g$, ordered phase). Thus $g_c$ is the value at which (12) is fulfilled for $\mu=0$ and $h=0$. Since $T=0$ the hyperbel cotangent in (12) is equal to 1 and we obtain

$$0=-\frac{1}{4}+\frac{1}{N}\sum_k \frac{\Phi^2}{U^2(k)}+\frac{1}{N}\sum_k \frac{g_c}{2\sqrt{g_c U(k)}}, \qquad (13)$$

provided that both of the $k$ sums converge. Otherwise the system does not display a phase transition. Since the random field term in (13) contains the stronger singularity for $k\to 0$ than the term containing the quantum fluctuations, the properties of the random field term are responsible for the existence of a phase transition. Consequently, we find the same behavior as in the classical random field spherical model;[9] namely, a transition occurs for $D>D_c^-=2x$, where $x$ describes the behavior of $U(k)$, $U(k)\sim k^x$, close to $k=0$ (short-range interactions yield $x=2$). To determine the equation of state, we expand (12) for small $g-g_c$ and small $\mu$. Again, the leading contribution stems from the random field term. Thus, asymptotically close to the transition, we find

$$-t_g\sim\left(\frac{h}{\mu}\right)^2+\Phi^2\begin{cases} C\mu^{(D-2x)/x} & (D<3x), \\ C\mu\ln|\mu| & (D=3x), \\ C\mu & (D>3x), \end{cases} \qquad (14)$$

where $t_g=(g-g_c)/g_c$ is the distance from the critical point and the prefactor $C$ is a smooth function of $g$. If we define a "magnetization" $m=\partial f/\partial h=-h/2\mu$ [see Eq. (11)], we obtain the equation of state

$$-t_g\sim m^2+\Phi^2\begin{cases} C(h/m)^{(D-2x)/x} & (D<3x), \\ C(h/m)|\ln(h/m)| & (D=3x), \\ C(h/m) & (D>3x). \end{cases} \qquad (15)$$

This equation is identical to the equation of state of a classical random field spherical model close to its finite-temperature phase transition.[9,10] Consequently, we also find the same critical exponents $\alpha=(D-3x)/(D-2x)$, $\beta=\frac{1}{2}$, $\gamma=x/(D-2x)$, $\delta=D/(D-2x)$, $\nu=1/(D-2x)$, and $\eta=2-x$ below the upper critical dimension $D_c^+=3x$. Above $D_c^+$ we get the usual mean-field exponents $\alpha=0$, $\beta=\frac{1}{2}$, $\gamma=1$, $\delta=3$, $\nu=1/x$, and $\eta=2-x$.

In conclusion, we have investigated the influence of a quenched random field on the critical behavior of a quantum spherical model at zero temperature. In agreement with perturbational results,[14] we have found that static random field fluctuations rather than the quantum fluctuations dominate the critical behavior. Therefore the critical behavior at zero temperature is identical to that at finite temperature, and the system does not show a crossover from quantum to classical behavior at low but finite temperatures. We note that these results do not depend on the nature of the particular quantization scheme used to define the quantum spherical model. The reason is that in general random field fluctuations are stronger than thermal fluctuations in the classical spherical model. In addition, at any finite temperature thermal fluctuations are stronger than quantum fluctuations for any model. This corresponds to $T$ being a relevant variable at a quantum phase transition. From this it follows that in general random field fluctuations are stronger than quantum fluctuations in a quantum spherical model and thus they dominate the critical behavior.

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