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Critical correlations and susceptibilities in the random-field spherical model

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We investigate the behavior of the correlation function and the susceptibility of the random-field spherical model at the critical point. In particular we calculate the critical exponents η and $\bar{\eta}$ describing the divergences of the susceptibility and its disconnected part, respectively. In the case of short-range interactions we obtain $\eta=\bar{\eta}=0$. For power-law interactions $U_{ij}\sim r_{ij}^{-\sigma}$ we find $\eta=\frac{1}{2}\bar{\eta}=D+2-\sigma$ (for $D<\sigma<D+2$), where D is the spatial dimension. The Schwartz-Soffer exponent inequality $\bar{\eta}\leq 2\eta$ is satisfied and becomes an equality independent of the functional form of the interaction.

The influence of a quenched random field on the critical behavior of ferromagnetic systems has been extensively studied since the first work on this subject by Imry and Ma[1] (for recent reviews, see Nattermann and Villain[2] and Belanger and Young[3]). While there is a general agreement about upper and lower critical dimensions of the different models, the critical behavior of random-field systems is still poorly understood, and in particular the critical exponents and their relations are now known. In 1985 Schwartz and Soffer[4] obtained an inequality relating the critical behavior of susceptibilities and correlation functions in a general random-field system. While the so-called disconnected part of the susceptibility [see Eq. (5) below] vanishes in systems without a field, it does not vanish in random-field systems. Precisely at the phase transition the disconnected part is expected to diverge even more strongly than the susceptibility itself. A corresponding exponent inequality $\bar{\eta}\leq 2\eta$ relating the critical exponents η and $\bar{\eta}$ of the susceptibility and its disconnected part, respectively, was also derived by Schwartz and Soffer.[4] It has further been suggested[5] that both inequalities should actually occur as equalities. This is supported by numerical calculations for the random-field Ising model.[6-8] Rigorous results, however, are not known up to now.

In this paper we investigate the validity of the Schwartz-Soffer inequality and the corresponding exponent inequality on the basis of the random-field spherical model, which is easier to deal with than the corresponding Ising model. The spherical model was first introduced by Berlin and Kac[9] as an approximation for the nearest-neighbor Ising model. Later, it was also used to investigate systems with power-law interactions.[10] The effect of a random field on the properties of the spherical model was first studied by Schwartz.[11] Hornreich and Schuster[12] calculated the thermodynamic properties of a random-field spherical model with short-range interactions, and Vojta[13] extended this study to the case of long-range power-law interactions.

The random-field spherical model consists of $N=L^D$ dynamic variables S_i on the sites of a regular D-dimensional hypercubic lattice. The model has ferromagnetic pair interactions U_{ij} [which we take as translationally invariant, i.e., $U_{ij}=U(|\mathbf{r}_i-\mathbf{r}_j|)$, for convenience] between the variables S_i. The quenched random field φ_i couples linearly to the dynamic variables. The Hamiltonian of the model is given by

$$H=\sum_i \varphi_i S_i+\frac{1}{2}\sum_{i\neq j} U_{ij}S_iS_j \ . \tag{1}$$

The values φ_i of the random potential are independent random variables with a Gaussian probability distribution

$$W(\varphi_i)=\frac{1}{\sqrt{2\pi}\Phi}\exp\left[-\frac{\varphi_i^2}{2\Phi^2}\right] . \tag{2}$$

The dynamic variables S_i are continuous real variables ranging from $-\infty$ to ∞. To make the model well defined and to avoid states with diverging energy, a constraint on the values of the variables (the spherical constraint) is added:

$$\sum_i S_i^2=N/4 \ . \tag{3}$$

Our choice of the numerical value $N/4$ of this constant is influenced by the analogy of the model to the Coulomb glass model[14] and the respective analogy of the dynamic variables S_i and the occupation numbers $n_i=\pm\frac{1}{2}$.

The quantities we are interested in are the $\mathbf{q}$-dependent susceptibility

$$\chi(\mathbf{q})=\langle S_\mathbf{q}^2\rangle-\langle S_\mathbf{q}\rangle^2 \tag{4}$$

and its disconnected part

$$\chi^{\mathrm{dis}}(\mathbf{q})=\langle S_\mathbf{q}\rangle^2 \ , \tag{5}$$

where $\mathbf{q}$ is the wave vector and $S_\mathbf{q}$ is the Fourier transform of S_i. We note that we use a real version of the Fourier transformation which is appropriate for the spherical model.[9] The symbol $\langle\cdot\rangle$ denotes the thermodynamic average. Precisely at the ferromagnetic transition the configurational averages of these quantities are expected to diverge as

$$\langle\chi(\mathbf{q})\rangle_\varphi\sim|\mathbf{q}|^{-2+\eta} \tag{6}$$

and

$$\langle\chi^{\mathrm{dis}}(\mathbf{q})\rangle_\varphi\sim|\mathbf{q}|^{-4+\bar{\eta}} \ , \tag{7}$$

which defines the critical exponents η and $\bar{\eta}$.[3] Here $\langle\cdot\rangle_\varphi$ denotes the average over the random fields.

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The calculation of averaged quantities in random systems is a difficult task, and the usual way to overcome this difficulty is the replica trick.[15] However, in the case of the random-field spherical model, one can avoid the replica trick as was first noted by Schwartz[11] for a special case of the model.

The partition function for the random-field spherical model for a fixed configuration of the random potential is given by

$$Z=\prod_i \left[\int dS_i\right]\delta\left[\frac{N}{4}-\sum_i S_i^2\right] \times\exp\left[-\frac{\beta}{2}\sum_{i,j}U_{ij}S_iS_j-\beta\sum_i\varphi_i S_i\right], \tag{8}$$

where β is the inverse temperature $\beta=(kT)^{-1}$. Using the Fourier representation of the δ function, the Gaussian integrals over the variables S_i may be carried out. A single integral over the variable z that stems from the Fourier representation of the δ function remains,

$$Z=\frac{\pi^{N/2}}{2\pi i}\int dz\, e^{N\Psi(z)}, \tag{9}$$

where Ψ is given by

$$\Psi(z)=\frac{z}{4}-\frac{1}{2N}\mathrm{Tr}\ln\underline{V}+\frac{\beta^2}{4N}\varphi\underline{V}^{-1}\varphi . \tag{10}$$

The matrix $\underline{V}$ comprises the matrix elements $V_{ij}=z\delta_{ij}+\beta U_{ij}/2$. In the thermodynamic limit $N\to\infty$, the integral (9) can be calculated by means of the saddle-point method yielding

$$Z=\pi^{N/2}(2\pi N|\Psi''(z_0)|)^{-1/2}e^{N\Psi(z_0)}, \tag{11}$$

where Ψ'' denotes the second derivative of Ψ with respect to z. The saddle-point equation

$$0=\frac{1}{4}-\frac{1}{2N}\mathrm{Tr}\underline{V}^{-1}-\frac{\beta^2}{4N}\varphi\underline{V}^{-2}\varphi \tag{12}$$

determines z_0. In principle, the saddle-point value z_0 could depend on the special realization of the random potentials. However, a detailed analysis[16] shows that z_0 is *independent* of the realization of the random potentials in the thermodynamic limit $N\to\infty$.

To determine the susceptibility (4) and its disconnected part (5), we have to calculate the thermodynamic averages $\langle S_{\mathbf{q}}\rangle$ and $\langle S_{\mathbf{q}}^2\rangle$, which are given by

$$\langle S_{\mathbf{q}}^m\rangle=\frac{1}{Z}\prod_i\left[\int dS_i\right]\delta\left[\frac{N}{4}-\sum_i S_i^2\right]S_{\mathbf{q}}^m \times\exp\left[-\frac{\beta}{2}\sum_{i,j}U_{ij}S_iS_j-\beta\sum_i\varphi_iS_i\right]. \tag{13}$$

The calculation of $\langle S_{\mathbf{q}}\rangle$ and $\langle S_{\mathbf{q}}^2\rangle$ is completely analogous to the calculation of the partition function. After having carried out the saddle-point integration over z, almost all terms in the numerator and denominator cancel, and we obtain

$$\langle S_{\mathbf{q}}\rangle=-\frac{\beta\varphi_{\mathbf{q}}}{2\lambda_{\mathbf{q}}} \tag{14}$$

and

$$\langle S_{\mathbf{q}}^2\rangle=\frac{1}{2\lambda_{\mathbf{q}}}+\frac{\beta^2\varphi_{\mathbf{q}}^2}{4\lambda_{\mathbf{q}}^2}, \tag{15}$$

with

$$\lambda_{\mathbf{q}}=z_0+\beta U_{\mathbf{q}}/2 , \tag{16}$$

where $U_{\mathbf{q}}$ and $\varphi_{\mathbf{q}}$ are the Fourier transforms of the interaction and the particular realization of the random field, respectively. Equation (14) establishes a linear relation between the thermal averages of the variables $S_{\mathbf{q}}$ and the random field. We note that this relation was *assumed to be valid* in leading order in $\mathbf{q}$ by Schwartz and Soffer in their study[5] of critical correlations and susceptibilities. In the spherical model, this relation is a *rigorous result.* From Eqs. (14) and (15), we obtain

$$\langle\chi(\mathbf{q})\rangle_\varphi=\frac{1}{2\lambda_{\mathbf{q}}} \tag{17}$$

and

$$\langle\chi^{\mathrm{dis}}(\mathbf{q})\rangle_\varphi=\frac{\beta^2\Phi^2}{4\lambda_{\mathbf{q}}^2} . \tag{18}$$

The Schwartz-Soffer inequality[5] for the susceptibilities (4) and (5)

$$\beta^2\Phi^2\langle\chi(\mathbf{q})\rangle_\phi^2\le\chi^{\mathrm{dis}}(\mathbf{q}) \tag{19}$$

is obviously satisfied as an equality independent of the special choice of the interaction U_{ij}.

In the following we calculate the critical exponents η and $\bar\eta$. To do this we study the behavior of $\lambda_{\mathbf{q}}$ for small $|\mathbf{q}|$ precisely at the transition. In the case of short-range interactions we obtain $\lambda_{\mathbf{q}}\sim|\mathbf{q}|^2$, which can be easily seen by expanding $U_{\mathbf{q}}$ in powers of $\mathbf{q}$. We note that the term proportional to $\mathbf{q}^0$ vanishes at the transition, because $z_0=-\beta U_{\mathbf{q}=0}/2$ as shown by Berlin and Kac.[9] Inserting this into Eq. (17) yields $\langle\chi(\mathbf{q})\rangle_\varphi\sim|\mathbf{q}|^{-2}$, and therefore the critical exponent η in Eq. (6) is equal to zero. Analogously, we obtain $\langle\chi^{\mathrm{dis}}(\mathbf{q})\rangle_\varphi\sim|\mathbf{q}|^{-4}$, and the critical exponent $\bar\eta$ in Eq. (7) is also equal to zero. If the model contains long-range power-law interactions $U_{ij}\sim r_{ij}^{-\sigma}$ (with $\sigma>D$ to ensure the convergence of the average energy per site), the asymptotic behavior of $\lambda_{\mathbf{q}}$ precisely at the transition is given[13] by $\lambda_{\mathbf{q}}\sim|\mathbf{q}|^{\sigma-D}$ for $D<\sigma<D+2$. Inserting this into Eqs. (17) and (18), we obtain $\eta=\frac{1}{2}\bar\eta=D+2-\sigma$. For $\sigma>D+2$ the susceptibilities show the same behavior as for short-range interactions, i.e., $\eta=\bar\eta=0$.

In conclusion, we have studied the behavior of the susceptibility $\chi(\mathbf{q})$ and its disconnected part $\chi^{\mathrm{dis}}(\mathbf{q})$ in the random-field spherical model. The Schwartz-Soffer inequality relating these two quantities is satisfied as an equality independent of the choice of the interaction U_{ij}. Consequently, the exponent inequality $\bar\eta\le2\eta$ is also satisfied as an equality. We have calculated η and $\bar\eta$ for the random-field spherical model with short-range interactions as well as long-range power-law interactions. We have obtained $\eta=\bar{\eta}=0$ and $\eta=\frac{1}{2}\bar{\eta}=D+2-\sigma$ for $D<\sigma<D+2$, respectively. Since the spherical model is equivalent to the $n\rightarrow\infty$ limit of the n-vector model,[17] it seems to be interesting to calculate corrections to these results by means of an $1/n$ expansion starting from the spherical model.[18] This, however, remains a task of the future.

Note added in proof. In a recent paper[19] Gofman *et al.* investigated the validity of the Schwartz-Soffer inequality in the random-field Ising model (RFIM) by means of extensive high-temperature series expansions. They provided evidence that the inequality is valid for the short-range RFIM in arbitrary dimension.

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