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# Iterative Equalization using Improved Block DFE for Synchronous CDMA Systems

Sang-Yick Leong, Kah-Ping Lee, and Yahong Rosa Zheng

**Abstract**—Iterative equalization using optimal multiuser detector and trellis-based channel decoder in coded CDMA systems improves the bit error rate (BER) performance dramatically. However, given large number of users employed in the system over multipath channels causing significant multiple-access interference (MAI) and intersymbol interference (ISI), the optimal multiuser detector is thus prohibitively complex. Therefore, the sub-optimal detectors such as low-complexity linear and non-linear equalizers have to be considered. In this paper, a novel low-complexity block decision feedback equalizer (DFE) is proposed for the synchronous CDMA system. Based on the conventional block DFE, the new method is developed by computing the reliable *extrinsic* log-likelihood ratio (LLR) using two consecutive received samples rather than one received sample in the literature. At each iteration, the estimated symbols by the equalizer is then saved as *a priori* information for next iteration. Simulation results demonstrate that the proposed low-complexity block DFE algorithm offers very good performance gain over the conventional block DFE.

Keywords: DFE, Iterative Equalization, Turbo Equalization.

## I. INTRODUCTION

Iterative equalization which employs maximum *a posteriori* (MAP) algorithm is a powerful tool to combat frequency selective fading [1]. This so-called *Turbo-principle* [2] can be achieved easily due to the serial concatenation of two steps that formed by the channel encoder at the transmitter and the discrete-time equivalent multipath channels. The performance of a wireless system can be enhanced in the fashion of exchanging the *extrinsic* information iteratively among the soft-input/soft-output equalizer and channel decoder until convergence is achieved. Unfortunately, these optimum algorithms are not usually applicable to the practical communication systems in use today due to their high computational complexity. As a consequence, the study of low complexity suboptimal algorithms in turbo equalization are motivated.

In recent literature, the Turbo principle has been successfully applied to the code division multiple access (CDMA) systems to mitigate the multiple-access interference and intersymbol interference. Specifically, Moher derived an optimal iterative multiuser detector for synchronous coded CDMA system based on the cross-entropy minimization [3]. In [4], the near single user performance of the iterative multiuser detection is proposed by Reed, *et al.* Later, Wang and Poor developed a low-complexity

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iterative receiver structure to decode the multiuser information data. The minimum mean square error linear equalizer (LE) implemented in turbo equalization cancels the inter-symbol interference and multi-access interference successfully.

In this paper, we focus on the nonlinear multiuser block decision feedback equalizer in the synchronous CDMA systems. We address the drawback of the conventional block DFE algorithm in turbo equalization, which has error propagation. The effects of error propagation can be observed from the simulation results [6], [7]. Hence we propose a new approach to mitigate the error propagation in the DFE algorithm when used in iterative equalization and retaining low computational complexity. It estimates the data using the *a priori* information gleaned from the channel decoder and also the *a priori* detected data from last iteration to minimize error propagation. From simulation results, we show that the bit error rate performance of the improved block DFE algorithm has very good improvement when compared with the conventional block DFE algorithm.

## II. SYSTEM MODEL

Fig. 1 shows a convolutional coded CDMA system with  $K$  users in total and one receiver antenna. Prior to transmission, a frame of binary data symbol  $b_k(i) \in \{0, 1\}$  of user  $k$ ,  $k = \{1, 2, \dots, K\}$ , with length  $K_b$  is first encoded through a convolutional encoder with constraint length  $M$  and rate  $r$ . The output encoded data symbols of length  $K_c$  are next interleaved into different ordering by a random permutation function to reduce the influence of error bursts. Thereby, yielding a block of data symbols  $c_k(n) \in \{+1, -1\}$ , where  $n = \{1, 2, \dots, K_c\}$  denotes the symbol index of the interleaved symbols. The interleaver operation is denoted as  $c_k(n) = \Pi(c_k(i))$  and its reverse operator (de-interleaver) is denoted as  $\Pi^{-1}(\cdot)$ . At the final stage, each coded data symbol  $c_k(n)$  is then modulated by a spreading waveform  $s_k(t)$ . The modulated sequence due to the  $k$ th user can be written as

$$x_k(t) = A_k \sum_{n=0}^{K_c-1} c_k(n) s_k(t - nT), \quad 0 \leq t \leq (K_c - 1)T; \quad (1)$$

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} w_k(q) \psi(t - qT_c). \quad (2)$$

where  $A_k$  and  $s_k(t)$  are, respectively, the amplitude and  $k$ th user's normalized signature waveform that is supported only on the interval  $[0, T]$ .  $N$  is the processing gain and  $w_k(q)$ ,  $q \in \{0, 1, \dots, N-1\}$  is the signature code of user  $k$ ,  $T_c$  is the chip period,  $T = NT_c$  is the symbol period,  $\psi(t)$  is the chip waveform that is nonzero only on  $[0, T_c]$ .

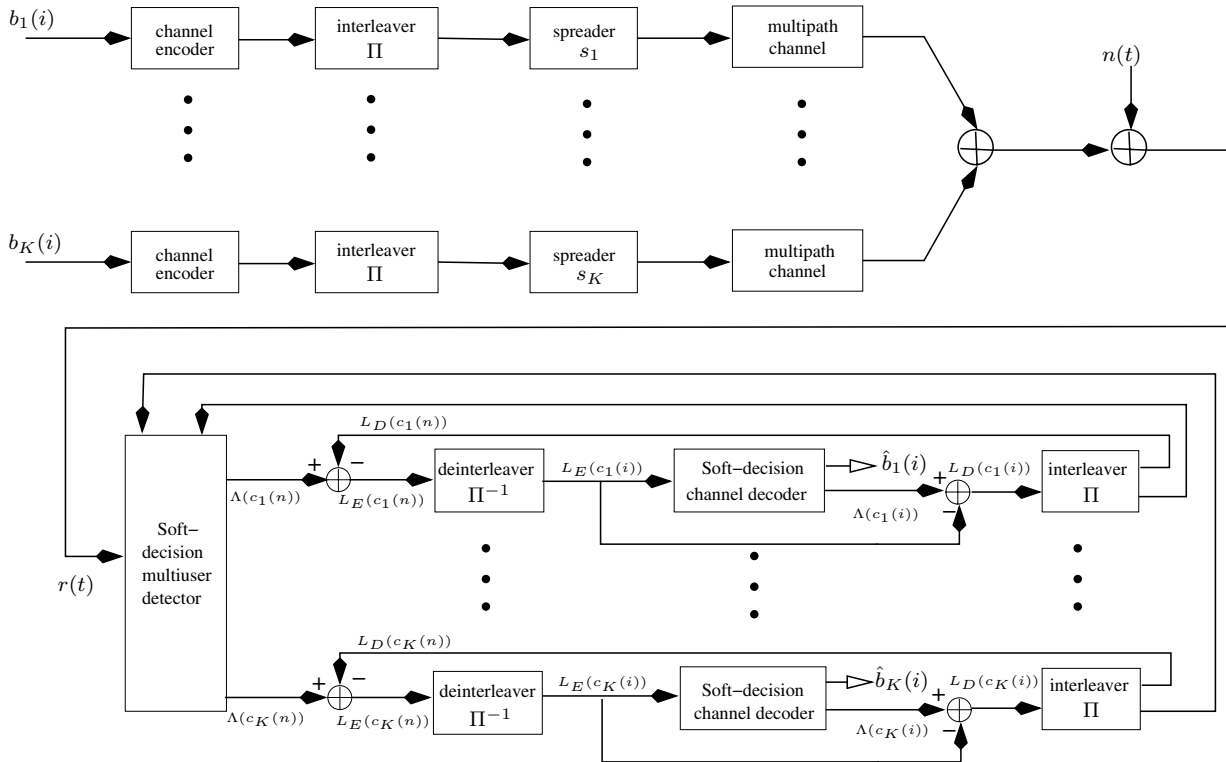


Fig. 1. Turbo Equalization system model consists of soft-decision equalizer and channel decoder

Assume the data sequence of  $k$ th user are transmitted in burst mode to the receiver and distorted by the multipath channel and additive white Gaussian noise (AWGN). The impulse response of the multipath channel propagated by the  $k$ th user is given by

$$h_k(t) = \sum_{l=0}^{L_k-1} h_{kl} \delta(t - \tau_{kl}) \quad (3)$$

where  $L_k$  is the total number of paths in the  $k$ th user channel and  $h_{kl}$  and  $\tau_{kl}$  are, respectively, the complex gain and delay of the  $l$ th path signal of  $k$ th user channel. In this paper, we implement a special case of CDMA channel model, where the transmitted signals arrive at the receiver synchronously and the multipath channel impulse response is given by  $h_k(t) = [h_{k1} \delta(t) h_{k2} \delta(t - T) \cdots h_{kL_k} \delta(t - L_k T)]$  and  $L_1 = L_2 = \cdots = L_K = L$ . Therefore, the baseband representation due to the  $k$ th user at the receiver is then given by

$$y_k(t) = x_k(t) \otimes h_k(t) \quad (4)$$

$$= A_k \sum_{n=0}^{K_c-1} c_k(n) \sum_{l=0}^{L-1} s_k(t - (n+l)T) h_{kl} \quad (5)$$

where  $\otimes$  is the convolution operator. As shown in Fig. 1, the received signal  $r(t)$  at the multiuser receiver is the combination of the  $K$  users' signal and the additive white Gaussian noise and written as follows,

$$r(t) = \sum_{k=1}^K y_k(t) + v(t) \quad (6)$$

where  $v(t) \sim \mathbb{N}(0, \sigma^2)$  is the zero mean complex Gaussian noise process with variance  $\sigma^2$ .

### III. MULTIUSER DETECTION FOR SYNCHRONOUS CDMA

In this section, we present the multiuser detection technique of the synchronous CDMA channel model described above. The key techniques developed in this paper later can be generalized to the general multipath CDMA channel such as asynchronous CDMA systems. For this synchronous case, it is sufficient to demodulate the  $k$ th user signal by choosing the output of a filter matched to  $s_k$  in the  $n$ th symbol interval, is given by

$$y_k(n) = \int_{nT}^{(n+1)T} s_k(t - nT) r(t) dt \quad (7)$$

$$= A_k \sum_{l=0}^{L-1} c_k(n-l) h_{kl} + \sum_{\forall j, j \neq k} A_j \rho_{kj} \cdot \sum_{l=0}^{L-1} c_j(n-l) h_{jl} + z(n) \quad (8)$$

$$\rho_{kj} = \int_0^T s_k(t) s_j(t) dt \quad (9)$$

where  $z(n)$  is a Gaussian noise and  $\rho_{kj}$  is the cross-correlation of the signal set  $s_1, \dots, s_K$ .

Denote  $\mathbf{Y}(n) = [y_1(n) \cdots y_k(n) \cdots y_K(n)]^T$  is the K-vector whose  $k$ th component is the filter output given in (7) and

$[\cdot]^T$  is the transpose operator. Based on equation (8), the vector  $\mathbf{Y}(n)$  can be arranged into the matrix form as follows

$$\mathbf{Y}(n) = \mathbf{R}\mathbf{A}\mathbf{H}\mathbf{C}(n) + \mathbf{Z}(n) \quad (10)$$

where  $\mathbf{R}$  is the normalized cross-correlation matrix:  $[\mathbf{R}]_{k,j} = \rho_{k,j}$  and  $\mathbf{A} = \text{diag}(A_1, \dots, A_K)$ . We further define other quantities into matrix form as follows

$$\mathbf{h}_k = [h_{k0} \ h_{k1} \ \dots \ h_{k(L-1)}] \in \mathbb{C}^{1 \times L} \quad (11)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_2 & \mathbf{0} & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{h}_K \end{bmatrix} \in \mathbb{C}^{K \times KL} \quad (12)$$

$$\mathbf{C}_k(n) = [c_k(n) \ c_k(n-1) \ \dots \ c_k(n-L+1)] \in \mathbb{C}^{1 \times L} \quad (13)$$

$$\mathbf{C}(n) = \begin{bmatrix} \mathbf{C}_1^T(n) \\ \mathbf{C}_2^T(n) \\ \vdots \\ \mathbf{C}_K^T(n) \end{bmatrix} \in \mathbb{C}^{KL \times 1} \quad (14)$$

$$\mathbf{Z}(n) = [z_1(n) \ z_2(n) \ \dots \ z_K(n)]^T \in \mathbb{C}^{K \times 1} \quad (15)$$

It is important to note that  $\mathbf{Z}(n)$  is a Gaussian vector and  $\mathbf{Z}(n) \sim \mathcal{N}(0, \mathbf{R})$ . In the sequel, we derive the block DFE and the new algorithm to improve block DFE based on equation (10) in iterative multiuser detection.

#### A. Conventional Block DFE

In this section, we focus on the iterative multiuser detection using block DFE algorithm in the synchronous CDMA system. Denote

$$\mathcal{C}_k^+(n) = \left\{ [c_1(n), \dots, c_{k-1}(n), +1, c_{k+1}(n), \dots, c_K(n)] : c_j(n) \in \{+1, -1\}, j \neq k \right\}. \quad (16)$$

Similarly define  $\mathcal{C}_k^-(n)$ . We now define  $\hat{\mathbf{C}}_k^{(m)}(n) \in \mathcal{C}_k^+(n)$  if  $\hat{\mathbf{C}}_k^{(m)}(n)$  is given by

$$\hat{\mathbf{C}}_k^{(m)}(n) = \begin{bmatrix} [c_1(n) \ \hat{c}_1^{(m)}(n-1) \ \dots \ \hat{c}_1^{(m)}(n-L+1)]^T \\ \vdots \\ [c_{k-1}(n) \ \hat{c}_{k-1}^{(m)}(n-1) \ \dots \ \hat{c}_{k-1}^{(m)}(n-L+1)]^T \\ [1 \ \hat{c}_k^{(m)}(n-1) \ \dots \ \hat{c}_k^{(m)}(n-L+1)]^T \\ [c_{k+1}(n) \ \hat{c}_{k+1}^{(m)}(n-1) \ \dots \ \hat{c}_{k+1}^{(m)}(n-L+1)]^T \\ \vdots \\ [c_K(n) \ \hat{c}_K^{(m)}(n-1) \ \dots \ \hat{c}_K^{(m)}(n-L+1)]^T \end{bmatrix}. \quad (17)$$

where  $\hat{c}_k^{(m)}(n-1) \in \{-1, +1\}$  is the estimated  $(n-1)$ th symbol of user  $k$  fed back by the equalizer in  $m$ th iteration. Based on the input-output relationship given in (10), we can write the *a posteriori* probability (APP) of the  $n$ th symbol of  $k$ th user being 1 as follows,

$$P[c_k(n) = +1 | \mathbf{Y}(n)] = \frac{\sum_{\hat{\mathbf{C}}_k^{(m)}(n) \in \mathcal{C}_k^+(n)} P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)] \cdot P[c_k(n) = +1] \cdot \prod_{j \neq k} P[c_j(n)]}{\sum_{\hat{\mathbf{C}}_k^{(m)}(n)} P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)] \cdot \prod_{j \neq k} P[c_j(n)]}, \quad (18)$$

$\{j, k \in \{1, 2, \dots, K\}\}$

where the conditional probability  $P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)]$  is defined as

$$P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)] = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left[ -\frac{1}{2\sigma^2} \Delta_n^* R^{-1} \Delta_n \right] \quad (19)$$

$$\Delta_n = \mathbf{Y}(n) - \mathbf{R}\mathbf{A}\mathbf{H}\hat{\mathbf{C}}_k^{(m)}(n) \quad (20)$$

and the operator  $\star$  is the complex conjugate transpose. Similarly, define  $P[c_k(n) = -1 | \mathbf{Y}(n)]$ . Given that the *a posteriori* LLR of  $n$ th symbol  $\Lambda(c_k^{(m)}(n)) = \log \frac{P[c_k^{(m)}(n) = +1 | \mathbf{Y}(n)]}{P[c_k^{(m)}(n) = -1 | \mathbf{Y}(n)]}$ , substitute (18) into it with some mathematical manipulation, we have

$$\Lambda(c_k^{(m)}(n)) = \log \frac{\sum_{\hat{\mathbf{C}}_k^{(m)}(n) \in \mathcal{C}_k^+(n)} P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)] \cdot \prod_{j \neq k} P[c_j(n)]}{\sum_{\hat{\mathbf{C}}_k^{(m)}(n) \in \mathcal{C}_k^-(n)} P[\mathbf{Y}(n) | \hat{\mathbf{C}}_k^{(m)}(n)] \cdot \prod_{j \neq k} P[c_j(n)]} + \log \frac{P[c_k^{(m)}(n) = +1]}{P[c_k^{(m)}(n) = -1]}. \quad (21)$$

$L_E(c_k^{(m)}(n))$   $L_D(c_k^{(m)}(n))$

It is seen from (21) that the *a posteriori* LLR consists of two terms, the first term  $L_E(c_k^{(m)}(n))$  is the extrinsic LLR computed by the block DFE, and the second term  $L_D(c_k^{(m)}(n))$  is the *a priori* LLR from the channel decoder. In the process of symbol detection, the block DFE, which relies on the soft information from (21), initially hard decides the  $k$ th user's  $n$ th symbol  $\hat{c}_k^{(m)}(n) \in \{-1, +1\}$ . Then, it is fed back to the equalizer for ISI and MAI cancellation. After the symbol detection, the block of *extrinsic* LLR is interleaved and delivered to the channel decoder as a set of *a priori* information.

Unfortunately, the analysis and simulation results from [6], [7] indicate that DFE is not an effective equalizer in turbo equalization. It has only small improvement throughout the iterations when compared with a linear equalizer. The loss of performance is mainly due to the residual interference in the presence of the severely multipath channels and the incorrect symbols being fed back during the equalization. Therefore, we propose a novel block DFE which is shown to be a low-complexity and efficient equalizer for turbo equalization.

#### B. Improved Block DFE

It is obvious that the feedback of incorrectly estimated symbols during equalization lead to loss of performance. As a consequence, the key idea of the proposed equalizer is increasing the reliability of the *extrinsic* information (LLR) by computing the extra metric. We further define the vector  $\hat{\mathbf{C}}_k^{(m,m-1)}(n) \in \mathcal{C}_k^+(n)$  if

$$\hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1) = \begin{bmatrix} [\hat{c}_1^{(m-1)}(n+1) & c_1(n) & \cdots & \hat{c}_1^{(m)}(n+1-L)]^T \\ \vdots & \vdots & \vdots & \vdots^T \\ [\hat{c}_{k-1}^{(m-1)}(n+1) & c_{k-1}(n) & \cdots & \hat{c}_{k-1}^{(m)}(n+1-L)]^T \\ [\hat{c}_k^{(m-1)}(n+1) & +1 & \cdots & \hat{c}_k^{(m)}(n+1-L)]^T \\ [\hat{c}_{k+1}^{(m-1)}(n+1) & c_{k+1}(n) & \cdots & \hat{c}_{k+1}^{(m)}(n+1-L)]^T \\ \vdots & \vdots & \vdots & \vdots^T \\ [\hat{c}_K^{(m-1)}(n+1) & c_K(n) & \cdots & \hat{c}_K^{(m)}(n+1-L)]^T \end{bmatrix} \quad (22)$$

In the first iteration of turbo equalization, there is no *a priori* LLR delivered from the channel decoder and estimated symbols from previous iteration by the equalizer. The equalizer thus detects the user symbols based on the *a posteriori* LLR calculated in the first term of (21). Starting from the second iteration, we define a new *a posteriori* probability of the  $n$ th symbol of  $k$ th user given by,

$$P[c_k(n) = +1 | \mathbf{Y}(n), \mathbf{Y}(n+1)] = \frac{\sum_{\mathbf{c}^+} P_Y^{(n,n+1)} \cdot P[c_k(n) = +1] \cdot \prod_{j \neq k} P[c_j(n)]}{\sum_{\mathbf{c}} P_Y^{(n,n+1)} \cdot \prod_{j \neq k} P[c_j(n)]}, \quad \{j, k \in \{1, 2, \dots, K\}\} \quad (23)$$

$$P_Y^{(n,n+1)} = P[\mathbf{Y}(n), \mathbf{Y}(n+1) | \hat{\underline{\mathbf{C}}}_k^{(m)}(n), \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1)] \quad (24)$$

where  $\mathbf{c} = \{\hat{\underline{\mathbf{C}}}_k^{(m)}(n), \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1)\}$  and  $\mathbf{c}^+ = \{\hat{\underline{\mathbf{C}}}_k^{(m)}(n), \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1)\} \in \mathcal{C}_k^+(n)$ . Similarly define  $P[c_k(n) = -1 | \mathbf{Y}(n), \mathbf{Y}(n+1)]$ . Given that the received samples are independent, the probability of received samples  $\mathbf{Y}(n)$  and  $\mathbf{Y}(n+1)$  at  $m$ th iteration is obtained using (19) and given by

$$P[\mathbf{Y}(n), \mathbf{Y}(n+1) | \hat{\underline{\mathbf{C}}}_k^{(m)}(n), \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1)] = \frac{1}{2\pi\sigma^2} \exp \left\{ -\frac{1}{2\sigma^2} \left[ g_0 \left( \hat{\underline{\mathbf{C}}}_k^{(m)}(n) \right) + g_1 \left( \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1) \right) \right] \right\} \quad (25)$$

where

$$g_0 \left( \hat{\underline{\mathbf{C}}}_k^{(m)}(n) \right) = \Delta_n^* R^{-1} \Delta_n$$

$$g_1 \left( \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1) \right) = \Delta_{n+1}^* R^{-1} \Delta_{n+1}$$

$$\Delta_{n+1} = \mathbf{Y}(n+1) - \mathbf{R} \mathbf{A} \mathbf{H} \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1)$$

Finally, let us define a new *a posteriori* LLR  $\Lambda(c_k^{(m)}(n)) = \ln \frac{P[c_k(n)=+1 | \mathbf{Y}(n), \mathbf{Y}(n+1)]}{P[c_k(n)=-1 | \mathbf{Y}(n), \mathbf{Y}(n+1)]}$ , and substituting (25) into (23). After some mathematical manipulation, the new *a posteriori* LLR of  $n$ th symbol at  $m$ th iteration is given by

$$\Lambda(c_k^{(m)}(n)) = \underbrace{\log \frac{\sum_{\mathbf{c}^+} \exp \left\{ -\frac{1}{2\sigma^2} \left[ g_0 \left( \hat{\underline{\mathbf{C}}}_k^{(m)}(n) \right) + g_1 \left( \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1) \right) \right] \right\}}{\sum_{\mathbf{c}^-} \exp \left\{ -\frac{1}{2\sigma^2} \left[ g_0 \left( \hat{\underline{\mathbf{C}}}_k^{(m)}(n) \right) + g_1 \left( \hat{\underline{\mathbf{C}}}_k^{(m,m-1)}(n+1) \right) \right] \right\}}}_{L_E(c_k^{(m)}(n))} + \log \frac{P(c_k^{(m)}(n) = +1)}{P(c_k^{(m)}(n) = -1)} \quad (26)$$

$$\underbrace{\hspace{10em}}_{L_D(c_k^{(m)}(n))}$$

It is obvious that the new algorithm considers the extra sample  $\mathbf{Y}(n+1)$  in the process of computing  $L_E(c_k^{(m)}(n))$  while compared with the conventional block DFE given in (21). However, it is important to note that in the first iteration, when computing  $L_E(c_k^{(1)}(n))$ , neither the *a priori* LLR  $L((c_k^{(1)}(n)))$  nor symbol  $\hat{c}_k^{(0)}(n)$  is available. As a consequence, the computation of metric  $\mathbf{Y}(n+1)$  is discarded and the new algorithm (26) becomes the conventional block DFE algorithm given in the first term of (21) in the first iteration. From the second iteration onward, the improved block DFE algorithm feedback the estimated set of  $k$ th user symbols  $\hat{\mathbf{C}}_k^{(m)} = [\hat{c}_k^{(m)}(0) \cdots \hat{c}_k^{(m)}(n) \cdots \hat{c}_k^{(m)}(K_c - 1)]$  to the detector for interference cancellation. It also keeps the data in memory for the computation of  $\mathbf{Y}(n+1)$  in the next iteration ( $m+1$ ). In short, the new algorithm treated the detected symbols  $\hat{\mathbf{C}}_k^{(m)}$  as another set of *a priori* information besides  $L(c_k^{(m)}(n))$  that is delivered from the channel decoder starting from the second iteration onward.

It is noted here that the new *a posteriori* probability can be calculated using  $J$  number of received samples, i.e.  $P[c_k(n) = +1 | \mathbf{Y}(n), \mathbf{Y}(n+1), \dots, \mathbf{Y}(n+j)]$ , where  $j = 0, 1, \dots, J-1$ . Given that  $j$  equals to 0 and 1, the APPs are simplified to (18) and (23), respectively. Due to the tradeoff between the computational complexity and system performance gain, we only compute the new *a posteriori* probability using  $j = 0, 1$  throughout this paper.

#### IV. SIMULATION RESULTS

In this section, we present several simulation results obtained using the block DFE and improved block DFE presented in Section III. The entire scenario of iterative multiuser detection is depicted in Fig. 1. In the 3 users CDMA system, the binary data is first encoded through rate  $r = 1/2$  and constraint length  $M = 5$  convolutional encoder. The generator code given in octal notation is  $G = [23, 35]$ . For the multipath channel, we consider a static ISI channel (slow fading) with  $L = 3$  and the CIRs of the 3 users are given in Table I, where  $i$  is  $\sqrt{-1}$  and the com-

TABLE I  
CHANNEL IMPULSE RESPONSE IN CDMA SYSTEM

	$\delta(n)$	$\delta(n-1)$	$\delta(n-2)$
User 1	0.7095 + 0.3752 <i>i</i>	0.4404 + 0.3507 <i>i</i>	0.1713 + 0.0979 <i>i</i>
User 2	0.6887 + 0.2175 <i>i</i>	0.3625 + 0.5075 <i>i</i>	0.2900 + 0.0725 <i>i</i>
User 3	0.7054 + 0.2748 <i>i</i>	0.5955 + 0.1924 <i>i</i>	0.1283 + 0.1374 <i>i</i>

plex path gains are normalized such that  $\sum_{l=0}^{L-1} |h_{kl}|^2 = 1$ . The additive white Gaussian noise added to the received symbols is determined by the desired  $E_b/N_o$ .

Based on the derivation given by the improved block DFE algorithm, it has the same performance as the conventional block DFE algorithm in the first iteration due to the lack of *a priori* information from the decoder and the equalizer. From the second iteration onward, the conventional block DFE algorithm has only a small improvement throughout the iterations shown

in all the figures. In Fig. 2, we assume that there is no correlation between the users;  $\rho_{kj} = 0, j \neq k$ . 'DFE 5' denotes the BER performance computed by the conventional DFE at 5th iteration. The conventional block DFE has BER of 0.0047 at 6 dB  $E_b/N_o$  after 5 iterations. We obtain only an 1dB gain if compared to the BER after the first iteration. Nevertheless, after 5 iterations, the improved block DFE algorithm has BER of 0.0006 at 6 dB  $E_b/N_o$ , and achieve extra 1.3dB gain than the conventional block DFE algorithm. Fig. 3 depicts the simulation results that the correlation of the users is  $\rho_{kj} = 0.5$ . After one iteration, the receiver achieve a BER of 0.0222 at 7dB  $E_b/N_o$ . Obviously, the gain of the new method is thus 2.4dB while the conventional DFE only produces the gain of 1.2dB after 5 iterations. It is apparent in Fig. 2 and Fig. 3 that the improved DFE algorithm requires only 2 iterations to achieve overall better performance when compared to that computed by the conventional DFE algorithm.

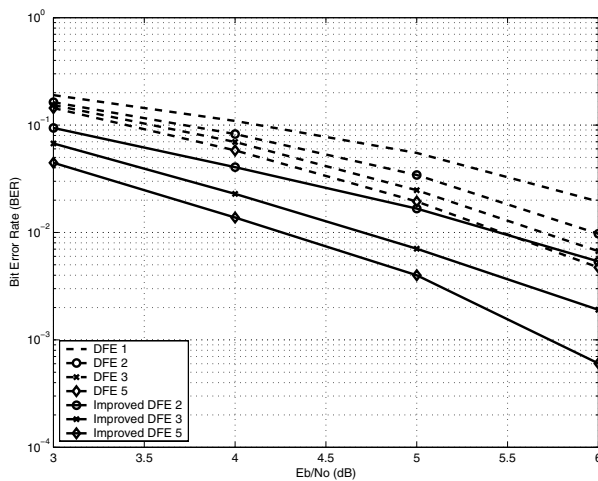


Fig. 2. BER performance of the Turbo multiuser detector while  $\rho_{kj} = 0$ .

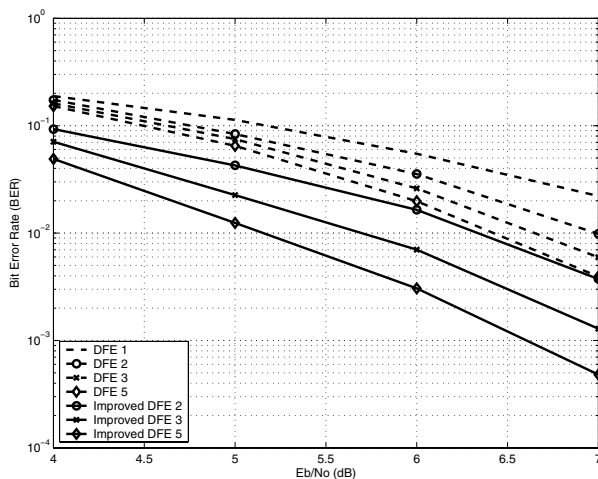


Fig. 3. BER performance of the Turbo multiuser detector while  $\rho_{kj} = 0.5$ .

## V. CONCLUSION

In this paper, the block DFE algorithms are introduced and analyzed for iterative multiuser receiver in synchronous CDMA system. We first address the computational complexity of the MAP equalizer and the ineffectiveness of the conventional block DFE algorithm to mitigate the MAI and ISI in the multipath CDMA channels. A novel low-complexity improved block DFE algorithm is proposed for iterative multiuser detection. The new method improves the BER performance by computing the *extrinsic* LLR using two consecutive received samples. At each iteration, the hard detected symbols are fed back to cancel MAI and ISI in equalization, they are also saved as another set of *a priori* data for next iteration. We verified the proposed algorithm through the simulation results in the 3 users synchronous CDMA system. The simulation results indicate that the proposed low complexity block DFE algorithm improves the BER performance throughout the iterations when compared to the conventional block DFE algorithm.

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