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FDTD Modeling of Skin Effect

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Abstract

The data rates and clock speeds of current high-speed signals are increasing rapidly, consequently, not only the lossy nature of FR-4 but also the lossy nature of good conductors, such as copper, need to be taken into account in high-speed signal designs. In order to well predict the loss caused by both dielectric loss and skin effect loss, a suitable simulation tool is needed. A surface impedance boundary condition (SIBC) algorithm was implemented in FDTD modeling herein to accommodate the skin effect loss due to finite conductivity of good conductors. Good agreement between the FDTD result and the measurements as well as SPICE result was obtained for a 14 mils wide strip line.

Introduction

With the increase of speed in digital systems, not only dielectric loss but also skin effect loss needs to be taken into account in signal designs. This work focuses on implementing surface impedance boundary condition SIBC [1] algorithm in FDTD modeling in order to include skin effect loss.

Implementing a SIBC in the FDTD method

If the cell size in the FDTD method is greater than the thickness of copper traces, the skin effect loss due to the finite conductivity of copper can be implemented using a SIBC. In particular,

$$\vec{E}_t(\omega) = Z_c(\omega)\left[\hat{n}\times\vec{H}_t(\omega)\right] \tag{1}$$

$$\begin{aligned} Z_c(\omega) &= \frac{\sqrt{j\omega\mu}}{\sqrt{\sigma + j\omega\varepsilon}} = \eta\frac{\sqrt{j\omega\varepsilon/\sigma}}{\sqrt{1+j\omega\varepsilon/\sigma}} \\ \Rightarrow Z_c(s) &= \eta\sqrt{\frac{s/a}{1+s/a}} \end{aligned} \tag{2}$$

where

$$a \equiv \sigma/\varepsilon\,. \tag{3}$$

An efficient implementation of (1) for the FDTD method was reported in [2]. The time-domain expression of (1) is the convolution of Z_c and the tangential magnetic field H_t. A normalized impedance Z_N, in the Laplace domain was defined in [2] in order to remove the dependency of medium parameters from (2).

$$Z_N(s') = \frac{1}{\eta}Z_c(as) = \sqrt{\frac{s'}{1+s'}} \tag{4}$$

To obtain an efficient way to evaluate this convolution, Z_N was approximated using a series of first order rational functions.

$$Z_N(s') \approx k_0 - \sum_i^L \frac{C_i}{\omega_i + s'} \qquad (5)$$

A set of coefficients corresponding to k_0, ω_i and $C_i (i = 1 \cdots L)$ was published in [2] for relatively poor conductors. However, those coefficients were not applicable to good conductors because the value of s' for good conductors is very low and the coefficients given in [2] did not approximate well in the range of the very low end. Therefore, recalculation of the coefficients applicable to good conductors is needed. In paper [3], the coefficients were recalculated for good conductors for Z_c over $s = j\omega$ in the range of f = *100kHz - 20 GHz*. However, the coefficients given in [3] were material-dependent.

To keep the merit of material-independence, here Z_N was approximated over the interval $s'=[0, 1.918\times10^{-8}]$ as a rational function using polynomial fit function. Similar to [3], a partial fraction expansion on the resultant polynomial was accomplished using residue function. Though this interval was chosen based on the conductivity of copper and frequency range of interest from *0 Hz* to *20 GHz*, because Z_N is material independent, the resultant coefficients can be applied to other good conductors as long as s' is within $[0, 1.918\times10^{-8}]$. The computed coefficients are given in Table 1 and the constant $k_0=1.532698004830299e\text{-}003$. A comparison of the approximate rational function (5) using coefficients published in [2] to that using coefficients computed in this report is shown in Figure 1. It is clear that the coefficients calculated in this paper fit the curve much better.

Table 1. Coefficients for rational approximation

C_i	ω_i
1.177932463687e-9	9.442612037863e-7
1.404050989993e-11	9.812840071306e-8
1.678974189768e-12	3.073562546828e-8
3.776480354506e-13	1.252531366479e-8
1.080843138507e-13	5.404256812475e-9
3.215680970141e-14	2.168192687294e-9
7.917791843695e-15	6.634831968668e-10
7.251180041003e-16	7.028860313400e-11

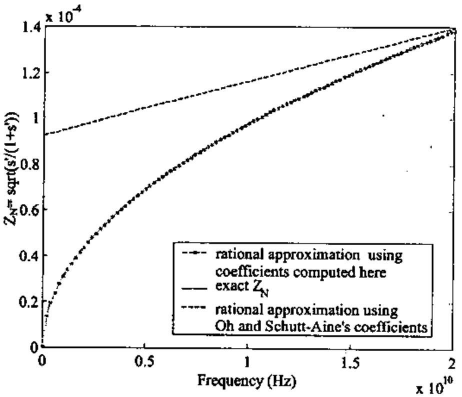

Figure 1. Comparison of rational approximation using different coefficients.

Numerical Results

A 3-layer board with dimensions of 8.273 mils × 161.2 mils × 30 mils was modeled in the FDTD, as shown in Figure 2. The top and bottom layers of the board were metal planes, whereas the inner layer located at 1/3 of the thickness of the board contained an 8" long and 14 mils wide trace. The characteristic impedance of the offset strip line is 50 Ω for a dielectric constant of approximately 3.46. The strip line was excited by a voltage source with source impedance 50 Ω, and was terminated by a lumped 50 Ω resistor. Thin wires were used to connect the strip line to the source/load, then to the closer metal plane. This geometry was employed to model a 14mils wide and 8" long strip line in a test board with N4000-13SI as the substrate material.

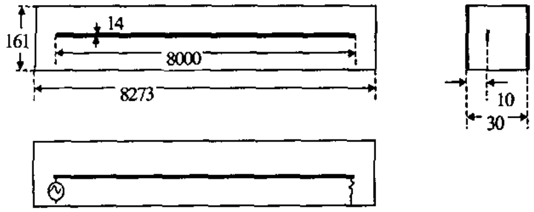

Figure 2. Geometry of the FDTD modeling for a 14 mils wide strip line. Units: mils.

A uniform cell size of 14 mils × 2.92mils × 3.33mils was employed throughout the modeling. The cell size was chosen such that the width of the trace was discretized into 5 cells and the thickness of the board was discretized into 9 cells. Since the cell size is very small, the time step has also to be small to meet the Corant stability criterion in FDTD.

Two simulations of FDTD modeling were performed, one with all the metal planes and trace modeled as PEC and the other with all the metal planes and trace modeled as copper. The latter accounted for the skin effect whereas the former did not. All the remaining parameters, such as dielectric constant and loss tangent, were the same for both simulations. A typical value of dielectric constant of N4000-13SI is 3.46 and the loss tangent given by the manufacturer is 0.007 @ 1 GHz [4]. A Debye model was employed to model the dielectric parameters in the FDTD modeling, as shown in Figure 3. Though physically the loss tangent is almost constant in the frequency range of interest, it was approximated to the Debye model because Debye model is easier to be handled with in our current FDTD code. Also, the dip of the modeled loss tangent in lower frequency is not very big (10% less than the maximum value), the effect should be minor in the final result of the FDTD modeling. The $|S_{21}|$ results are shown in Figure 4. This strip line geometry was also modeled with the W-element using Star-Hspice 2000.2 in order for comparison. The RLGC parameters of the W-element were extracted using XFX. The $|S_{21}|$ results of the SPICE model are also shown in Figure 4. The dark curve in Figure 4 is the measurement result.

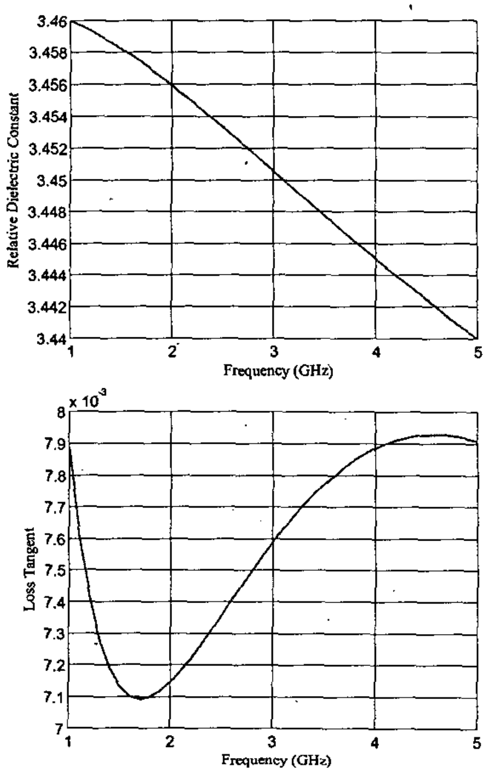

Figure 3. Loss tangent and dielectric constant in the FDTD modeling.

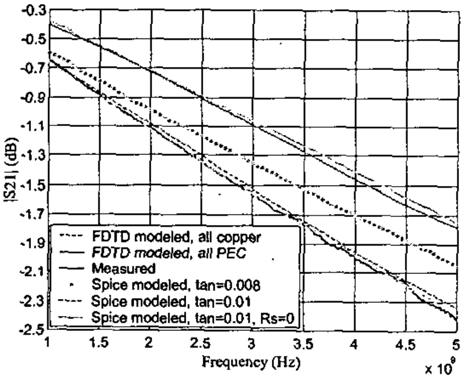

Figure 4. Measured and modeled results for an 8" long and 14 mils wide single ended trace on the N4000-13SI test board.

From the results, several observations can be made. While the curve of the FDTD modeling with all the metals modeled as PEC, is approximately 0.4 dB higher than the measurements, the curve of the FDTD modeling with skin effect taken into account, is very close to the measurement results. This indicates that FDTD modeling is suitable for modeling the skin effect loss. The curve of the W-element modeling is also very close to the measurement result. However, the loss tangent used was 0.01, which was slightly higher than that used in the FDTD modeling and the given value. The given loss tangent is 0.007 @ 1GHz and 0.006 @ 10.4GHz [4]. The dotted curve, which slightly underestimates the loss, is the W-element modeling result when the loss tangent was chosen to be 0.008. Finally, the difference between the FDTD modeling with and without skin effect is consistent to the difference between the W-element modeling with and without skin effect. This further indicates that both modeling tools employed the surface impedance boundary condition algorithm in order to take into account the skin effect loss at high frequency.

Conclusion

A set of material-independent coefficients for good conductors has been calculated for the rational function approximation that is used to implement SIBC in the FDTD modeling. Good agreement between the FDTD modeling and the measurement demonstrated that FDTD is suitable for model skin effect loss with this set of coefficients.

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