

01 Jan 1987

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Recommended Citation

A. Omar et al., "Estimation Approach to Locating Buried Geological Interfaces," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, 1987*, Institute of Electrical and Electronics Engineers (IEEE), Jan 1987.

The definitive version is available at <https://doi.org/10.1109/ICASSP.1987.1169853>

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ESTIMATION APPROACH TO LOCATING BURIED GEOLOGICAL INTERFACES

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ABSTRACT

The objective of the estimation work described in this paper is to improve algorithms for reconstructing the geometric configuration of acoustic reflecting surfaces. The conventional migration method of locating these surfaces is reviewed. A recent imaging condition which is closely related to the boundary condition at the geological interface is then described. This later procedure results in two outputs. One can be used to reconstruct the shape of the unknown acoustic reflector, while the second output estimates the acoustic wavespeed on the transmission side of the boundary. When noise is present in the reflected signal, the performance of the method is degraded. To improve the performance an empirical model for the noisy reflected signal is proposed, and its parameters are estimated. Computer-generated synthetic data are used to test the performance of the algorithms for a SNR range of -10dB to 10dB and for both normal incidence and non-normal incidence.

INTRODUCTION

Of considerable interest in many fields of applied physics is the problem of determining the internal structure and composition of an object based upon a set of measurements made on the external surface of the object. In the area of geophysical exploration, for example, the conventional approach [1], which is referred to as conventional migration, to this problem assumes a large number of seismic source/receiver pairs. Each source/receiver pair operates by generating a seismic pulse which propagates downward, strikes a single reflecting geological boundary and then propagates upward until it is detected by the seismic receiver, which is located at the same point as the seismic source. If the incident seismic wave and its returning echo can be regarded as impulsive waves, the presence of the reflecting boundary can be detected at some particular point $\underline{r}_0 = (x_0, y_0, z_0)$ in the subsurface by the following procedure:

- (1) Solve the wave equation to evaluate the function $P_R(\underline{r}_0, t)$, which is the pressure of reflected seismic wave as a function of time at the specified point \underline{r}_0 .

- (2) Evaluate this function of time at the instant t_s , which corresponds to the arrival time of the incident seismic wave at \underline{r}_0 .
- (3) Since $P_R(\underline{r}_0)$ is pulse-like, $P_R(\underline{r}_0, t)$ will be essentially 0 unless $P_R(\underline{r}_0, t)$ is a point at which the incident and reflected waves are nearly time-coincident, that is, (x_0, y_0, z_0) is a point on the reflector. Thus, the points (x_r, y_r, z_r) at which $P_R(x_r, y_r, z_r, t_s)$ is not essentially zero will indicate the location of the seismic reflector.

This procedure is referred to as seismic wave equation migration.

In 1983, DuBroff [2] proposed a modification to the conventional approach of wave-equation migration in order to replace the existing imaging condition with a new imaging condition that would be more closely related to the boundary conditions at the reflecting surface. The simplest case requires that the pressure fluctuations associated with the passage of a seismic wave must be continuous at the boundary. A second boundary condition requires that the normal derivative of the pressure must also be continuous at the boundary.

As a consequence of the boundary conditions, one can show that [2]:

$$\left[\frac{\nabla P_A(x_r, y_r, z_r, \omega)}{\omega P_A(x_r, y_r, z_r, \omega)} \right]^2 = - \frac{1}{C_B^2} \quad (1)$$

where P represents the acoustic pressure wave, and C represents the acoustic wavespeed. The A and B subscripts are used to distinguish the unequal acoustic waves and acoustic wave speeds above and below, respectively, the acoustic reflector. The A wavefield is the sum of the incident and the reflected wavefields while the B wavefield is the same as the transmitted wavefield. The ω is the angular frequency, and ∇ is the gradient operator. See Fig. 1.

The geometrical interpretation of this equation is that the normalized gradient vector on the left-hand side of the above equation must lie

where N_z is the number of observations. Substituting from (5) and (6) into (4)

$$\hat{P}_R(x, z, \omega) = \hat{A}(x, z) e^{-i\omega\hat{\tau}(x, z)} P_I(x, z, \omega) \quad (7)$$

and from (7), (2) and (3)

$$\hat{R}^2(x, z, \omega) = \frac{\{(\sqrt{N_x} P_{I_x} + \sqrt{N_x} \hat{P}_R)^2 + (\sqrt{N_z} P_{I_z} + \sqrt{N_z} \hat{P}_R)^2\}}{\omega^2 \hat{P}_A^2} \quad (8)$$

$$= \frac{-1}{C_B^2}$$

The mean (over frequency) of (8) gives an estimate of the acoustic wavespeed on the transmission side while the minimum variance of (8) produces an estimate of the location of the reflector surface.

RESULTS AND COMPARISON

In order to test the performance of the estimation approach, computer-generated synthetic data is used to simulate waves reflected from a flat reflector surface. A white Gaussian noise simulating the seismic noise is added to the reflected signal such that SNR ranging from -10dB to 10dB is considered. Both normal and non-normal incidence are tested.

As a measure of the performance of the different approaches (conventional migration, DuBroff's method, and estimation approach) a discrimination ratio is computed. Let the number of geophones $N_x=16$, the number of grid levels in the Z-direction $N_z=10$, and let the reflector be located at $z=5$, then the Discrimination Ratio (DR) equals

$$\frac{\frac{1}{N_x \cdot N_z} \cdot \left[\sum_{x=1}^{N_x} \sum_{z=1}^{N_z} [S(x, z)]^2 \right]}{\frac{1}{N_x} \cdot \left[\sum_{x=1}^{N_x} [S(x, 5)]^2 \right]} \cdot N_z \quad (9)$$

where $S(x, z)$ is the variance of the characteristic radius squared as expressed in (8). A DR of 1.0 indicates perfect detection. In addition to the DR measure, the number of times (out of N_x) the algorithm fails to detect the reflector is considered. Table 1 as well as Fig.'s 2 through 6 show the performance of the three approaches and the improvement obtained by the estimation approach.

CONCLUSIONS

We have described here an estimation approach to the seismic inversion problem. Both normal incidence and sloping incidence are considered. The analysis assumes the presence of one reflector of general shape as well as general noise characteristics. Two measures of performance, the

number of errors, NE, and the discrimination ratio, DR are considered. The results demonstrate that the estimation approach has better performance over its counterparts. Other performance measures are to be investigated.

For the special case of a flat reflector and incoherent noise, the ensemble average of the reflected signal over all traces (though not discussed here) was found by the authors to give better estimates for the time delay parameter and the location of the reflector subsurface.

The approach used in estimating τ needs to be generalized to include the estimation of multiple layers of reflection, where the reflected waves may overlap and cause the peaks to be adjacent. The estimate of the amplitude can be further improved by using recursive estimation algorithms, which will decrease the computational cost and enhance the accuracy of the results. Furthermore, time domain techniques can be investigated and compared with frequency domain techniques. Other techniques such as Kalman filtering may possibly be applied as well.

Table 1 Performance Measure

SNR	$\Theta = 0$				$\Theta = 30$			
	DR		NE		DR		NE	
	D	E	D	E	D	E	D	E
.1	.5191	1.0	4	0	.5536	.9909	7	2
.15	.5423	1.0	3	0	.5481	.9858	7	1
.7	.6584	1.0	2	0	.5478	.9785	3	1
.85	.6835	1.0	1	0	.5516	.9778	3	1
1	.7070	1.0	0	0	.5562	.9774	3	1
1.2	.7355	1.0	0	0	.5631	.9777	3	1

Θ =angle of incidence, DR: discrimination ratio exact reflector surface at $z=5$

D: DuBroff's method E: estimation method

NE: number of errors out of 16 points

SNR: signal to noise ratio

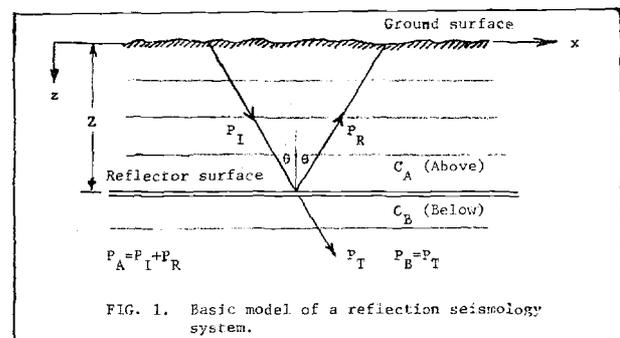


FIG. 1. Basic model of a reflection seismology system.

on the surface of a sphere having an imaginary radius of $1/C_B$, where \underline{r} lies on the surface of the reflector. Furthermore, although the vector will generally vary with ω for any fixed point \underline{r} in space, the square of the vector will be frequency-independent when \underline{r} is on a reflecting boundary.

This result has the effect of allowing the original problem of determining the reflector geometry to be expressed as a problem in estimation. Specifically, we wish to estimate the mean and variance of the square of the normalized gradient vector for each point, \underline{r} , in the subsurface. The points \underline{r} corresponding to the minimum variance provide an estimate of the location and configuration of the unknown reflector. The mean value may be used to deduce the phase-propagation velocity below the reflector.

Estimation approaches are needed to improve the computation of the mean and variance within the environment of notoriously noisy seismic data. These approaches are discussed in the following sections, which include analytical descriptions of the estimation problem and its results. A summary of recent results in addition to a discussion of future work needed are presented in the conclusion.

ESTIMATION APPROACH

The estimation approach applies estimation theory to DuBroff's method for the 1-D inversion problem ($y=0$). The observation data is obtained from the left hand side of (1). This quantity is called the characteristic radius squared (R^2), and will have the form:

$$\begin{aligned}
 R^2(x,z,\omega) &= \underline{R}(x,z,\omega) \cdot \underline{R}(x,z,\omega) \\
 &= \left[\frac{\nabla_x P_A(x,z,\omega) + \nabla_z P_A(x,z,\omega)}{\omega P_A(x,z,\omega)} \right]^2 \\
 &= \frac{[\nabla_x P_I(x,z,\omega) + \nabla_x P_R(x,z,\omega)]^2}{\omega^2 (P_I(x,z,\omega) + P_R(x,z,\omega))^2} \\
 &+ \frac{[\nabla_z P_I(x,z,\omega) + \nabla_z P_R(x,z,\omega)]^2}{\omega^2 (P_I(x,z,\omega) + P_R(x,z,\omega))^2} \\
 &= \frac{-1}{C_B^2} \tag{2}
 \end{aligned}$$

A white Gaussian noise (coherent or incoherent [3]) is added to the reflected signal to account for the seismic noise. Accordingly, all terms in (2) with subscript R are contaminated with the seismic noise, and hence their true values need to be estimated. $P_R(x,z,\omega)$ and its derivatives have the following relationship.

$$\nabla_x P_R(x,z,\omega) = ik_x P_R(x,z,\omega) \tag{3a}$$

and

$$\nabla_z P_R(x,z,\omega) = -ik_z P_R(x,z,\omega) \tag{3b}$$

where k_x and k_z are the wave numbers in the x and z directions, respectively.

Thus, estimating $P_R(x,z,\omega)$ enables us to directly find $\nabla_x P_R(x,z,\omega)$ and $\nabla_z P_R(x,z,\omega)$. To estimate $P_R(x,z,\omega)$ an empirical model which models $P_R(x,z,\omega)$ as a delayed and attenuated version of $P_I(x,z,\omega)$ is used such that

$$P_R(x,z,\omega) = A(x,z) e^{-i\omega\tau(x,z)} P_I(x,z,\omega) \tag{4}$$

where $A(x,z)$ is the attenuation factor, and $\tau(x,z)$ is the delay time between the incident and reflected waves. Both $A(x,z)$ and $\tau(x,z)$ need to be estimated.

For the case under investigation (assumption of one reflector) estimation of $\tau(x,z)$ is based on finding the time between the peaks of the incident and reflected waves. Since the peak of the reflected wave is distorted with noise, filtering is used to improve detecting the peak. Since the incident wave and the reflected wave have similar Fourier spectrum, we proceed as follows:

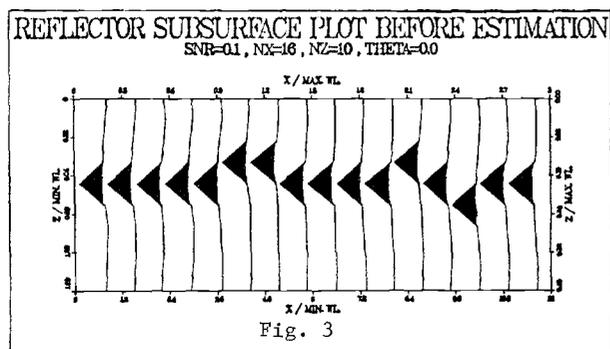
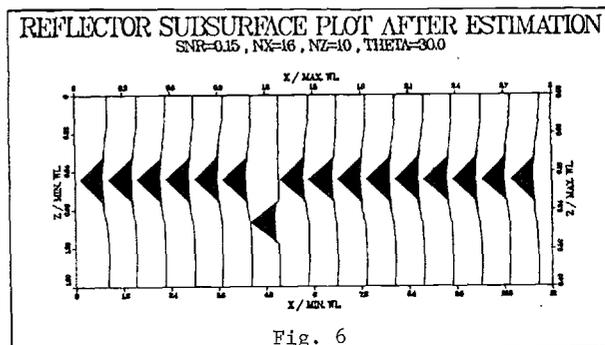
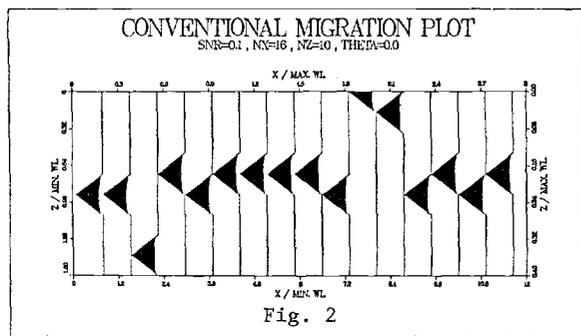
- Define the frequency band (f_1, f_2) of $P_I(x,z,\omega)$ where the signal is dominant
- Use a BPF (f_1, f_2) to exclude some of the noise from $P_R(x,z,\omega)$
- Detect the peak of $P_R(x,z,t)$ and find its corresponding time (t_2)
- Detect the peak of $P_I(x,z,t)$ and find its corresponding time (t_1)
- Calculate Time Delay $\hat{\tau}(x,y) = t_2 - t_1$ \tag{5}

Now, to estimate the attenuation factor $A(x,z)$ of (4), least square estimation is used as follows. Define the global error as

$$\begin{aligned}
 E &= \frac{1}{N} \sum_{k=1}^N e^2(\omega_k) \\
 &= \frac{1}{N} \sum_{k=1}^N \{ |P_R(x,z,\omega_k)| - |A(x,z)| |P_I(x,z,\omega_k)| \}^2
 \end{aligned}$$

Then from $\partial E / \partial A = 0$ we obtain

$$|\hat{A}| = \frac{\sum_{k=1}^N |P_R(x,z,\omega_k)| |P_I(x,z,\omega_k)|}{\sum_{k=1}^N |P_I(x,z,\omega_k)|^2} \tag{6}$$



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