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## Erratum: Gravitationally Coupled Dirac Equation for Antimatter (Physical Review a (2013) 87 (032101) Doi:10.1103/ physreva.87.032101)

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**Erratum: Gravitationally coupled Dirac equation for antimatter [Phys. Rev. A **87**, 032101 (2013)]**

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The central result of our paper [Phys. Rev. A **87**, 032101 (2013)] is given by the particle-antiparticle symmetry relation [see Eq. (47) of our paper],

$$E \leftrightarrow -E, \quad f(r) \leftrightarrow g(r), \quad \varkappa \leftrightarrow -\varkappa, \quad (1)$$

valid for eigensolutions of the Dirac equation in a generalized Schwarzschild metric. This result is not affected by the current Erratum, but a number of typographical errors in intermediate steps of the derivation need to be corrected. First, we have to point out that Eq. (40) of our paper should have read

$$\bar{\gamma}^0 \bar{\gamma}^\mu \Gamma_\mu = -\frac{\tilde{\gamma}^0 \tilde{\gamma} \cdot \hat{r}}{v(r)w(r)} G(r), \quad (2a)$$

$$G(r) = \frac{2v'(r)w(r) + v(r)w'(r)}{2v(r)w(r)}, \quad (2b)$$

where  $\hat{r} = \vec{r}/|\vec{r}|$  is the position unit vector. Furthermore,  $\tilde{\gamma}^0$  is the Dirac  $\beta$  matrix,

$$\tilde{\gamma}^0 = \begin{pmatrix} \mathbb{1}_{2 \times 2} & 0 \\ 0 & -\mathbb{1}_{2 \times 2} \end{pmatrix}, \quad \tilde{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \quad (3)$$

and  $\tilde{\gamma}$  is the vector of flat-space Dirac  $\gamma$  matrices. For the record, we would like to clarify that Eq. (40) of our paper remains valid if the curved-space  $\bar{\gamma}^\mu$  matrices on the left-hand side of Eq. (40) of our paper are replaced by flat-space  $\tilde{\gamma}^\mu$  matrices. The correction from Eq. (2) entails the following change for Eq. (46) of our paper,

$$\left( \frac{\partial}{\partial r} + \frac{1 - \varkappa}{r} + G(r) \right) g(r) = v(r) \left( m - \frac{E}{w(r)} \right) f(r), \quad (4a)$$

$$\left( \frac{\partial}{\partial r} + \frac{1 + \varkappa}{r} + G(r) \right) f(r) = v(r) \left( m + \frac{E}{w(r)} \right) g(r), \quad (4b)$$

where the invariant line element is  $ds^2 = w(r)^2 dt^2 - v(r)^2 d\vec{r}^2$ . Furthermore, Eq. (45) of our paper also needs to be modified (we suppress one intermediate step),

$$\begin{aligned} \frac{i \partial_t \psi(\vec{r})}{w^2(r)} &= \begin{pmatrix} \frac{m}{w(r)} & \frac{\vec{\sigma} \cdot \hat{r}}{v(r)w(r)} \left( -i \frac{\partial}{\partial r} + i \frac{\vec{\sigma} \cdot \vec{L}}{r} - i G(r) \right) \\ \frac{\vec{\sigma} \cdot \hat{r}}{v(r)w(r)} \left( -i \frac{\partial}{\partial r} + i \frac{\vec{\sigma} \cdot \vec{L}}{r} - i G(r) \right) & -\frac{m}{w(r)} \end{pmatrix} \begin{pmatrix} f(r) \chi_{\varkappa\mu}(\hat{r}) \\ i g(r) \chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix} \\ &= \begin{pmatrix} \left[ \frac{1}{v(r)w(r)} \left( -\frac{\partial}{\partial r} + \frac{1}{r}(\varkappa - 1) - G(r) \right) g(r) + \frac{m}{w(r)} f(r) \right] \chi_{\varkappa\mu}(\hat{r}) \\ i \left[ \frac{1}{v(r)w(r)} \left( \frac{\partial}{\partial r} + \frac{1}{r}(\varkappa + 1) + G(r) \right) f(r) - \frac{m}{w(r)} g(r) \right] \chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix} = \frac{E}{w^2(r)} \begin{pmatrix} f(r) \chi_{\varkappa\mu}(\hat{r}) \\ i g(r) \chi_{-\varkappa\mu}(\hat{r}) \end{pmatrix}. \end{aligned} \quad (5)$$

For completeness, we also indicate that the vector of  $\vec{\alpha}$  matrices in Eq. (41) should be understood as  $\alpha^i = \bar{\gamma}^0 \bar{\gamma}^i$ , where  $\bar{\gamma}^0 = \tilde{\gamma}^0/w(r)$  and  $\bar{\gamma}^i = \tilde{\gamma}^i/v(r)$ . We reemphasize that the symmetry relation (47) of our paper, on which all further considerations in the paper and all conclusions are based, remains valid.