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Echo State Networks for Determining Harmonic Contributions from Nonlinear Loads

Joy Mazumdar, *Student Member, IEEE*, Ganesh K. Venayagamoorthy, *Senior Member, IEEE*, Ronald G. Harley, *Fellow, IEEE*, and Franklin C. Lambert, *Senior Member, IEEE*

Abstract—This paper investigates the application of a new kind of recurrent neural network called Echo State Networks (ESNs) for the problem of measuring the actual amount of harmonic current injected into a power network by a nonlinear load. The interaction between loads connected to a point of common coupling (PCC) is a highly dynamic process. The determination of true harmonic current injection by individual loads is further complicated by the fact that the supply voltage waveform at the PCC is distorted by other loads at the PCC or further upstream and is therefore rarely a pure sinusoid. Harmonics in a power system are classified as either load harmonics or as supply harmonics. The principles of ESN are based on the use of a Recurrent Neural Network (RNN) as a dynamic reservoir. In order to compute the desired output dynamics, only the weights of connections from the reservoir to the output units are calculated. This is simply a linear regression problem. Experimental results presented in this paper confirm that attempting to predict the Total Harmonic Distortion (THD) of a load by simply measuring the load's current may not be accurate. The main advantage of this new method is that only waveforms of voltages and currents at the PCC have to be measured. This method is applicable for both single and three phase loads.

I. INTRODUCTION

POWER system harmonics have been known to exist on the power system for a long time. With the widespread proliferation of power electronic loads and other nonlinear loads, significant amounts of harmonic currents are being injected into the network. Identification of harmonic sources in a power system has been a challenging task for many years. Harmonic distortions have become an important concern for all utility companies. This concern has led to the evolution of various instruments like harmonic analyzers, disturbance monitors etc. The most common approach

adopted to tackle this problem was the establishment of limits on the amount of harmonic currents and voltages generated by customers and utilities. The IEEE standard 519[1, 2] and the IEC-1000-3[3] are the perfect examples. Customers are required to comply with the regulations and when any customer exceeds the limits, the only enforcement power the utility has is to disconnect the customer. This is not a desirable action. In any case before this could happen, an accurate measurement is needed.

Figure 1 shows a simple network structure. When the nonlinear load is supplied from a sinusoidal voltage source, its injected harmonic current $i_s(t)$ is referred to as contributions from the load. The harmonic currents cause harmonic volt drops in the supply network. Any other loads, even linear loads, connected to the point of common coupling (PCC), will have harmonic currents injected into them by the distorted PCC voltage. Such currents are referred to as contributions from the power system, or supply harmonics.

If several loads are connected to a PCC, it is not possible to accurately determine the amount of harmonic current injected by each load, in order to tell which load(s) is

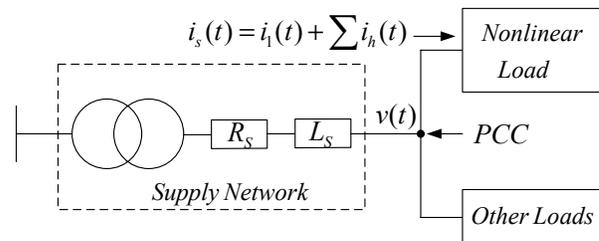


Fig. 1. Typical power distribution network with loads connected at the point of common coupling (pcc).

injecting the excessively high harmonic currents. If individual harmonic current injections were known, then a utility could penalize the offending consumer in some appropriate way, including say a special tariff or insist on corrective action by the consumer. Simply measuring the harmonic currents at each individual load is not sufficiently accurate since these harmonic currents may be caused by not only the nonlinear load, but also by a non-sinusoidal PCC voltage.

This is not a new issue and researchers have proposed tools based on traditional power system analysis methods to solve this problem. The harmonic active power method [4] and critical impedance measurement method [5] yield results

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to a certain degree of accuracy; however they are based on some fundamental assumptions like prior knowledge of the source impedance. The authors of [6] presented a novel modeling approach based on neural networks to identify the admittance of a nonlinear load. The rationale behind using neural networks was that, neural networks provide the flexibility of identifying dynamic systems online without the need to make assumptions. The most important criteria for the choice of a particular neural network structure for this problem are its ability to learn the admittance of the nonlinear load online and in the shortest possible time. This paper extends the concept proposed in [6] by using Echo State Networks to determine the true harmonic current of a nonlinear load in a three phase power system.

II. ECHO STATE NETWORKS

Artificial Neural Networks have provided an alternative modelling approach for power system applications [7-9]. The multilayer perceptron network (MLPN) is one of the most popular topologies in use today [10]. This network consists of a set of input neurons, output neurons and one or more hidden layers of intermediate neurons. Data flows into the network through the input layer, passes through the hidden layers and finally flows out of the network through the output layer. The network thus has a simple interpretation as a form of input-output model, with network weights as free parameters. The use and training of MLPNs is well understood.

Recurrent neural networks (RNN) are feedback networks in which the present activation state is a function of the previous activation state as well as the present inputs. The recurrent connections contain memory states. Thus, RNNs are better suited for identifying dynamic processes and systems with transients. However they can be difficult to train.

Echo State Networks provide a novel and easier-to-manage approach to supervised training of RNNs. Echo State Networks (ESN) [11 - 14] are a special form of recurrent neural networks (RNNs) recently proposed for modelling complex dynamic systems. A large (order of 100s of units) RNN is used as a “reservoir” of dynamics which can be excited by suitably presented input and/or feedback output. The connection weights of the reservoir network are not changed by training. In order to compute the desired output dynamics, only the weights of connections from the reservoir to the output units are calculated. This is simply a linear regression problem.

The ESN shown in Fig. 2 is a sparsely connected RNN with most of its weights fixed a priori to randomly chosen values. In contrast to normal RNNs, where the input and output weights are adapted depending on the minimization of the output error, ESNs only adjust the set of output weights leading from the hidden layer to the output layer. The hidden layer is known as the “Dynamic Reservoir”.

The key to understanding ESN training is the concept of echo states. Having echo states (or not having them) is a property of the network prior to training, that is, a property of the weight matrices W^{in} , W , and W^{fb} . Intuitively, the echo state property says, “if the network has been run for a very long time [from minus infinity time in the definition], the current network state is uniquely determined by the history of the input and the teacher forced output”.

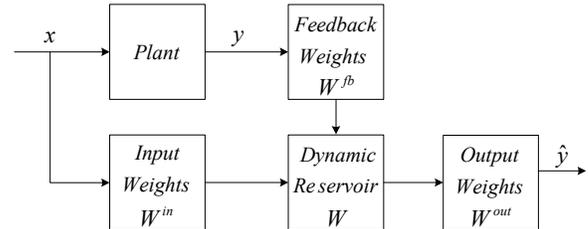


Fig. 2. Structure of Echo State Network

The echo state property is related to algebraic properties of the weight matrix W . Unfortunately, there is no known necessary and sufficient algebraic condition which allows one to decide, given $(W^{in}, W, \text{ and } W^{fb})$, whether the network has the echo state property.

A typical ESN consists of the following: discrete-time input/output (I/O) sequence, K input neurons, i.e., $\underline{x} = [x(1), \dots, x(K)]^T$, N neurons in the hidden layer, the decision vector is $\underline{d} = [d(1), \dots, d(N)]^T$, M neurons in the output i.e., $\underline{\hat{y}} = [\hat{y}(1), \dots, \hat{y}(M)]^T$, Input weight matrix W^{in} of size $N \times K$, Dynamic Reservoir weight matrix W of size $N \times N$, Output weight matrix W^{out} of size $M \times (K + N + M)$, Feedback weight matrix W^{fb} of size $N \times M$ and sigmoidal activation function for the dynamic reservoir.

There are certain conditions for determination of the weights for the dynamic reservoir.

- Generate a sparse random matrix in the range $[-1, 1]$
- Scale the matrix by its highest eigenvalue
- Multiply the matrix by α , known as the spectral radius. $[\alpha \in 0, 1]$

$$W = \frac{\alpha \cdot W_0}{|\lambda_{\max}|} \quad (1)$$

W^{in} and W^{fb} are drawn from a uniform distribution over $[-1, 1]$. The target is to calculate the value of W^{out} .

Given the ESN and the I/O sequences of the system, the network is trained to learn the system characteristics. The available I/O sequences are divided into three parts:

- An initial part, which is not used for training but serves the purpose of getting rid of initial transients in the network’s internal states.
- A training part, which is used in the actual learning procedure of adjusting the output weights.
- A testing part, which is used to test the newly

trained network on additional data.

The training of the network is done as follows: First, the network's internal state vector \underline{d} is initialized to random values. Then, the system is run on the initial part and the training part of the I/O sequence, i.e. input samples are written into the input nodes, output samples are written into the output nodes, and the internal states of the next time step are computed according to

$$d(n+1) = \text{sig}(W^{in}x(n+1) + Wd(n) + W^{fb}y(n)) \quad (2)$$

The output at time $n+1$ is computed as;

$$\hat{y}(n+1) = \text{lin}(W^{out}(x(n+1), d(n+1), y(n))) \quad (3)$$

Here the commas mean vector concatenation.

It is important to remember that the output update equation is not used during training, since the output weights are not yet set to their final values. Instead, the output nodes are just overwritten by the output part of the I/O sequence.

During the training period, the internal states are collected into the rows of a state-collecting matrix P of size $N \times K$. At the same time, the actual system outputs $y(n)$ are collected into the rows of a matrix T of size $N \times M$.

Once the training is completed, multiply the pseudoinverse of P with T , to obtain a $(K+N+M) \times M$ sized matrix $(W^{out})^T$ whose i^{th} column contains the output weights from all network units to the i^{th} output unit.

$$(W^{out})^T = P^{-1}T \quad (4)$$

Now the output $\hat{y}(n)$ of the ESN approximates the actual system output $y(n)$ by the equation,

$$\hat{y}(n) \approx y(n) = \sum_{i=1}^L W_i^{out} \cdot d(n) \quad (5)$$

More specifically, the output weights are computed such that the mean squared training error MSE is minimized.

$$MSE = \frac{1}{r} \sum_{n=1}^r (y(n) - \hat{y}(n))^2 = \frac{1}{r} \sum_{n=1}^r (y(n) - \sum_{i=1}^L W_i^{out} \cdot d(n))^2 \quad (6)$$

where r is the length of the I/O sequence used for testing.

III. ESTIMATION OF HARMONIC CURRENT

The method originally proposed in [6], predicts the true harmonic current distortion that can be attributed to a load. Figure 3 is a one-line diagram of a three-phase supply network having a sinusoidal voltage source v_s , network impedance L_s, R_s and several loads (one of which is nonlinear) connected to a PCC.

The nonlinear load injects distorted line three-phase line current i_{abc} into the network. The Identification Neural Network (ESN1) is trained to identify the nonlinear characteristics of the load. The Estimation Neural Network (ESN2) predicts the true harmonic current that would be injected by the load into the network, if it were possible to

isolate the load and supply it from a pure sinusoidal source. ESN2 is an exact replica of the trained ESN1 structurally.

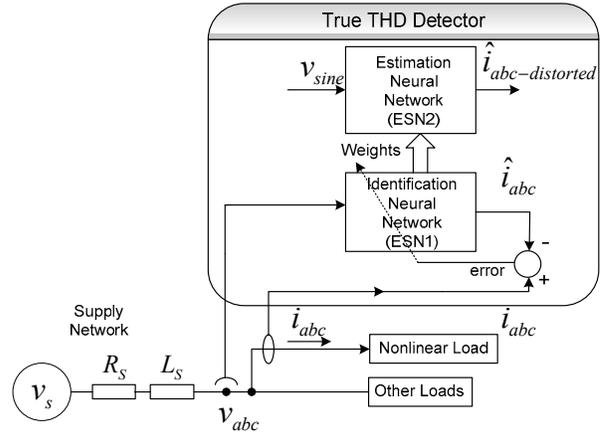


Fig. 3. Proposed scheme (v_{abc} is the voltage at the PCC)

The function of ESN2 can very well be carried out by ESN1; however that would disrupt the continual online training of ESN1 during the brief moments of estimating.

A. Identification Neural Network

The proposed method measures the instantaneous values of the three voltages v_{abc} at the PCC, as well as the three line currents i_{abc} at the k^{th} moment in time. The voltages v_{abc} could be line-to-line or line-to-neutral measurements. The neural network is designed to predict one step ahead line current \hat{i}_{abc} as a function of the present voltage vector value.

The length of the training samples is predetermined. Once all the samples are processed by the neural network, the output weights are computed. Now the actual instantaneous values of \hat{i}_{abc} are compared with the previously predicted values of \hat{i}_{abc} , and the difference (or error e) is used to fine tune the ESN1 output weights.

After several runs, the training converges and the value of the error e diminishes to an acceptably small value. Proof of this is illustrated by the fact that the waveforms for i_{abc} and \hat{i}_{abc} should practically lie on top of each other. At this point the ESN1 therefore represents the admittance of the nonlinear load. This process is called identifying the load admittance.

B. Estimation Neural Network

ESN2 is supplied with a mathematically generated sine wave voltage to estimate its output. The output of ESN2 called $\hat{i}_{abc-distorted}$ therefore represents the current which the nonlinear load would have drawn had it been supplied by a sinusoidal voltage source. In other words, this gives the same information that could have been obtained by quickly removing the distorted PCC voltage (if this were possible)

and connecting a pure sinusoidal voltage to supply the nonlinear load, except that it is not necessary to actually do this interruption. Any distortion present in the $\hat{i}_{abc-distorted}$ waveform can now be attributed to the nonlinearity of the load admittance.

C. Data Scaling

Due to the nature of the sigmoidal transfer function, the outputs of the neurons in the hidden layer are limited to values between 0 and 1. The inputs to the neural networks are therefore limited to values between -1 and 1. The scaling of the acquired data is done using software and hence that removes any limitations whatsoever on the data acquisition system and the transducers.

IV. EXPERIMENTAL RESULTS

For illustrative purposes, the scheme has been applied on a variable speed drive, ABB make ACS 500. The load is supplied from the utility source as well as a clean power source. The clean power source used is a California Instruments 5001 iX harmonic generator which is capable of outputting voltages with programmable distortion levels and zero internal impedance. Figure 4 shows the experimental setup.

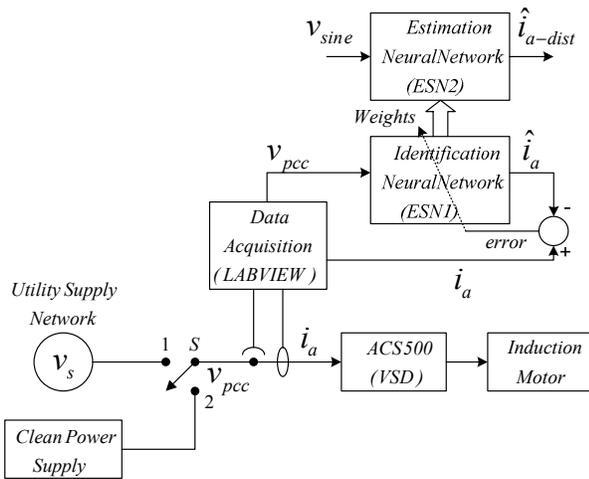


Fig. 4. Experimental Setup with a Variable Speed Drive

The scheme has to be applied to each phase individually. The method of using online trained ESNs to identify the load admittance and utilizing the trained neural network to estimate the harmonic current of the VSD, is now demonstrated for phase A.

With switch S in position 1, the VSD is supplied from the utility source. The three phase line to neural voltages and the phase currents are recorded. Now with switch S in position 2, the drive is supplied from the clean power source. The measured phase A voltage and current waveforms with switch in position 1 are shown in Fig. 5.

Figure 6 shows the measured phase A voltage and current waveforms with the switch in position 2. The total harmonic distortion (THD) of the utility voltage is 4.5% and the THD

of the CI 5001 iX voltage is 0.2% (near sinusoid).

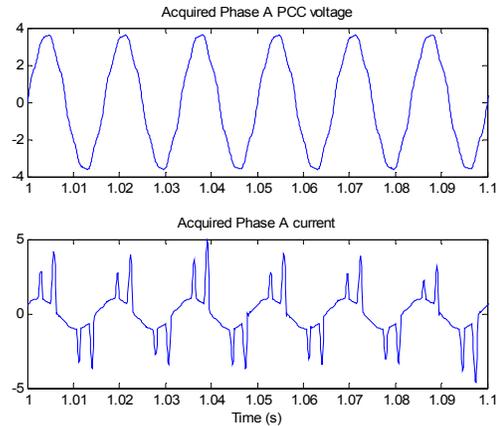


Fig. 5. Measured voltage and current with S in position 1

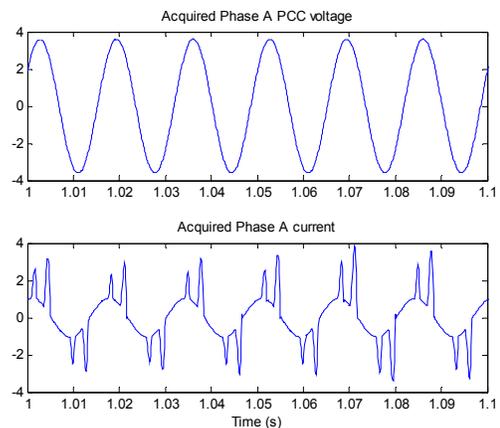


Fig. 6. Measured voltage and current with S in position 2

In a real life application, the use of the clean power source is not required for the implementation of this scheme, nor will such a power source be available.

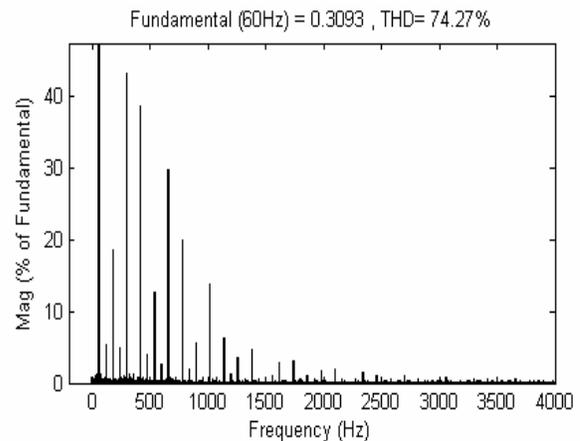


Fig. 7. FFT spectrum of current with S in position 1

The THD of the current with S in position 1 is 74.27% and the THD of the current with S in position 2 is 68.5%. The FFT spectrums are shown in Fig. 7 and Fig. 8 respectively.

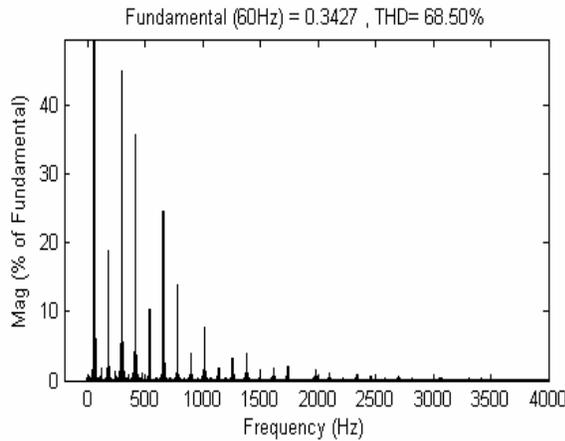


Fig. 8. FFT spectrum of current with S in position 2

The data obtained with switch S in position 1 is used to train the neural network ESN1 until the training error converges to near zero, and the output of ESN1 correctly tracks the actual current i_a . Figure 9 indicates how well the training of ESN1 has converged since its output \hat{i}_a coincides with the actual i_a waveform.

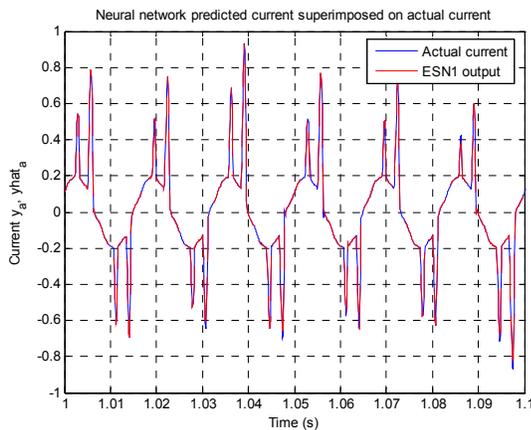


Fig. 9. ESN1 Convergence Result for Phase A current

The convergence of the training can also be verified by considering the MSE of ESN1 in Fig. 10.

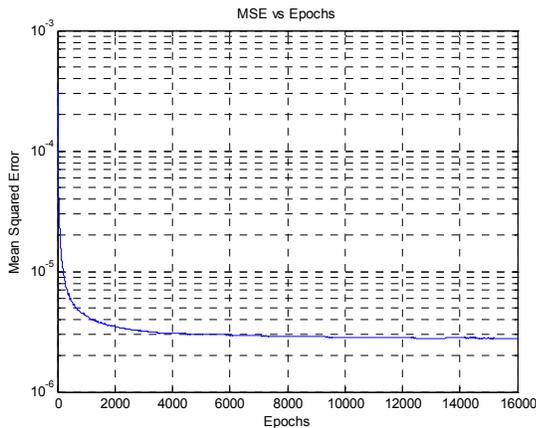


Fig. 10. MSE in Phase A current training

The sampling rate for data acquisition is set at 128 samples per cycle. The number of neurons used in the hidden layer of ESN1 is 20. Data acquisition is carried out by a National Instruments data acquisition system. The voltage transducers used are LEM LV 25-P and the current transducers used are LEM LAH 25-NP. The FFT of the acquired waveforms are computed using the *powergui* block of SIMULINK.

Once ESN1 has learned the admittance of the phase A of the VSD, the weights of ESN1 are transferred to ESN2. The output of ESN2 is \hat{i}_{a-dist} and is obtained by using a mathematically generated sine wave voltage with zero distortion as its input.

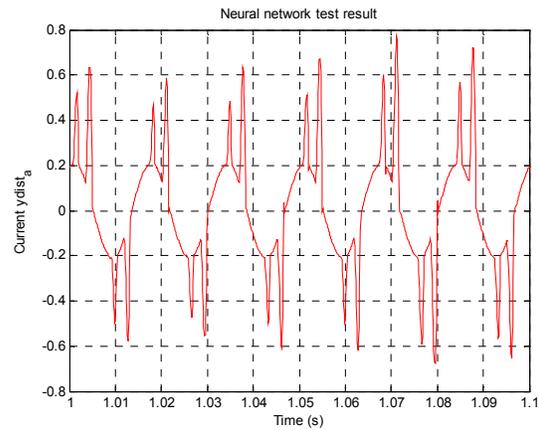


Fig. 11. Output of ESN2, estimated Phase A current

Fig. 11 shows what Fig. 5 would have looked like if it were possible to isolate the VSD and supply it from a pure sine wave. Fig. 12 shows the frequency spectrum of Fig. 11.

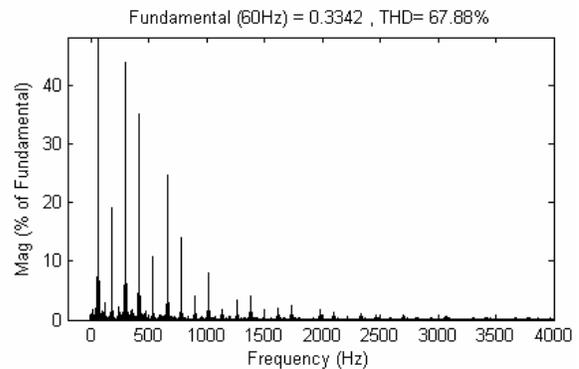


Fig. 12. FFT spectrum of the ESN2 output

The true current distortion of \hat{i}_{a-dist} turns out to be 67.88% (instead of the 74.27% of Fig. 7). This result agrees well with the measured value of 68.5% of Fig. 8 where the VSD was supplied by a 0.2% distorted voltage.

Similar to phase A, the scheme has also been applied to phase B and phase C of the VSD. Figure 13 shows the training result for Phase B.

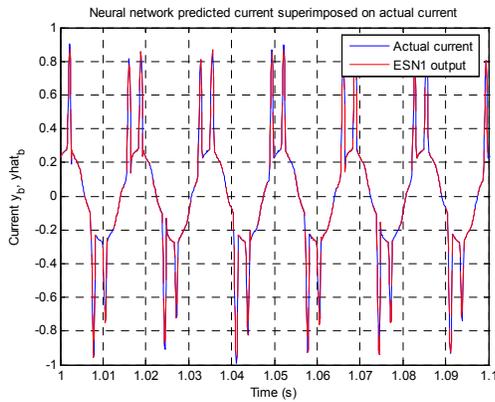


Fig. 13. ESN1 Convergence Result for phase B current

Figure 14 shows the training result for Phase C.

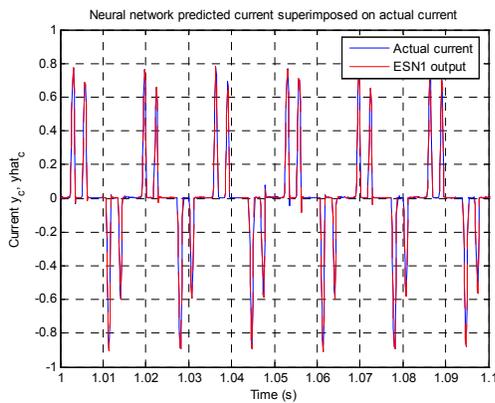


Fig. 14. ESN1 Convergence Result for phase C current

The above experiment has shown that there is a difference in the current distortion of a load depending on whether the loads are served by a clean supply or a distorted supply. Any load serviced by a utility is designed and optimized to operate at 60 Hz. For the purpose of quantification of this difference, a new parameter e_m , known as the resultant error in measurement, is introduced and is defined as:

$$e_m = \left(\frac{THD_s - THD_d}{THD_s} \right) \% \quad (7)$$

where THD_d is i_{THD} from a distorted v_{pcc} , and THD_s is i_{THD} from a mathematical sine wave, i.e. the output of ESN2. e_m can be \pm depending on the type of load and the condition of the network. The results for the three phases are summarised in Table I.

TABLE I
SUMMARY OF RESULTS

Phase	THD_d	THD_s	THD_{CI}	e_m
A	74.27%	67.88%	68.5%	-9.41%
B	59.67%	49.02%	47.72%	-21.73%
C	152%	132.27%	132.47%	-14.92%

THD_{CI} is the distortion in current with the switch S in position 2. This value is used for validation of the results obtained using the proposed scheme. In an actual implementation of the proposed scheme, the value of THD_{CI} will not be required since it is not used in the training algorithm and nor will such a value be available in any real power system application.

Some of the other experimental details of the neural network implementation are given below:

- Echo state network implemented in MATLAB.
- FFT computation : *powergui* block of SIMULINK
- Number of Neurons in the hidden layer: 25
- Sampling frequency for data acquisition: 8 kHz. Power quality instrumentations require approximately 128 samples/cycle.
- Computation time for the MATLAB code to compute the output weights (with 2 sec of acquired data) run on a 1.8 GHz PC: 10.2 sec

The above experiments show an important aspect of ESN training. The trained ESN closely approximates the actual output after the initial transient dynamics have washed out, which are caused by the initial untrained and random network starting state. Hence the training performance of the ESN is judged after the initial transients have passed. Depending on the network size and the sampling rate of the input data, typical range for the initial transients is about 10 cycles of the input data.

Sometimes with switching power electronic loads, the initial transients may be a problem. However with the present experiment, this problem did not arise.

The well known Least-Mean-Square (LMS) algorithm is difficult to use with ESNs. The performance of this algorithm depends critically on the eigenvalue spread of the cross-correlation matrix R .

$$R = E[d(n)d(n)^T] \quad (8)$$

where $d(n)$ is the networks internal state.

The eigenvalue spread s of the cross-correlation matrix R is defined as;

$$s = \frac{\lambda_{\max}(R)}{\lambda_{\min}(R)} \quad (9)$$

When the eigenvalue spread is large, the LMS algorithm converges very slowly and becomes inefficient. The adaptation of the spectrum of the cross-correlation matrix R to permit the use of LMS algorithm, without compromising the training performance, is still an important area of research [15].

There are issues with ESNs which need to be resolved; however preliminary results do indicate that ESNs could be used in the proposed scheme. ESNs demonstrated fast convergence times compared to other neural network topologies in identifying nonlinear load admittance.

V. CONCLUSION

This paper conceptually demonstrated the ability of Echo state neural networks to learn the load admittance and utilize the trained neural network for estimating the true harmonic distortion caused by that load. The proposed method has been successfully applied to a specific three phase load. The advantages of the proposed method are that it can be implemented online without disrupting the operation of any load, only voltages and currents need to be measured, it does not require any special instruments and it does not need to make any assumptions about any quantities, e.g. the impedance of the source.

Standards like IEEE 519 provide guidelines for controlling harmonic distortion levels that divide the responsibility between the utility and the customer. The utility has to maintain voltage distortion at the PCC below the specified limits and the customer has to limit the amount of harmonic current injection onto the utility system. However, disputes may arise between utilities and customers regarding who is responsible for the harmonic distortions due to the lack of a reliable single index which can precisely point out the source of the harmonic pollution.

Experimental results presented in this paper confirm that an error in the measurement is made if the calculation of current THD is done by simply measuring the input current of the nonlinear load. The information provided by the proposed method regarding the true current distortion of a load could be used to persuade an offending load to take steps to mitigate an unacceptably high level of distortion.

On a practical system the neural network computations could be carried out on a DSP, together with a suitable A/D interface. Utilities stand to benefit from this work, since it provides a tool to model a load under distorted supply conditions and may help the utility to check the accuracy of the load model provided by a customer during commissioning of a new service.

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