

01 Jun 2004

Electron Self-Energy for Higher Excited S Levels

Ulrich D. Jentschura

Missouri University of Science and Technology, ulj@mst.edu

Peter J. Mohr

Follow this and additional works at: https://scholarsmine.mst.edu/phys_facwork

 Part of the [Physics Commons](#)

Recommended Citation

U. D. Jentschura and P. J. Mohr, "Electron Self-Energy for Higher Excited S Levels," *Physical Review A - Atomic, Molecular, and Optical Physics*, vol. 69, no. 6, pp. 064103-1-064103-2, American Physical Society (APS), Jun 2004.

The definitive version is available at <https://doi.org/10.1103/PhysRevA.69.064103>

This Article - Journal is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in Physics Faculty Research & Creative Works by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.

Electron self-energy for higher excited S levels

 Ulrich D. Jentschura^{1,2} and Peter J. Mohr²
¹*Physikalisches Institut der Universität Freiburg, Hermann-Herder-Straße 3, 79104 Freiburg im Breisgau, Germany*
²*National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8401, USA*

(Received 2 March 2004; published 7 June 2004)

A nonperturbative numerical evaluation of the one-photon electron self-energy for the $3S$ and $4S$ states with charge numbers $Z=1$ to 5 is described. The numerical results are in agreement with known terms in the expansion of the self-energy in powers of $Z\alpha$.

DOI: 10.1103/PhysRevA.69.064103

PACS number(s): 12.20.Ds, 31.30.Jv, 06.20.Jr, 31.15.-p

In this Brief Report, we consider the one-loop self-energy shift which is the dominant radiative correction to the energy of hydrogenic bound states. For high-precision spectroscopy, S states are rather important because they can be excited from the ground state via Doppler-free two-photon spectroscopy. We calculate the self-energy numerically to high accuracy for $3S$ and $4S$ states (nuclear charge number $Z=1, \dots, 5$). We follow the approach previously outlined for $1S$ (Ref. [1]) and $2S$ and $2P$ states (Ref. [2]).

The natural unit system with $\hbar=c=m_e=1$ and $e^2=4\pi\alpha$ is employed, as is customary in bound-state quantum electrodynamics. The (real part of the) energy shift ΔE_{SE} due to the electron self-energy radiative correction is usually written as [3]

$$\Delta E_{SE} = \frac{\alpha (Z\alpha)^4 m_e}{\pi n^3} F(nl_j, Z\alpha), \quad (1)$$

where F is a dimensionless quantity. In writing the expression $F(nl_j, Z\alpha)$, we follow the usual spectroscopic notation for an atomic state with principal quantum number n , orbital angular momentum l and total electron angular momentum j .

The leading terms in the semianalytic expansion of $F(nS_{1/2}, Z\alpha)$ about $Z\alpha=0$ read

$$\begin{aligned} F(nS_{1/2}, Z\alpha) &= A_{41}(nS_{1/2}) \ln(Z\alpha)^{-2} + A_{40}(nS_{1/2}) \\ &+ (Z\alpha) A_{50}(nS_{1/2}) + (Z\alpha)^2 [A_{62}(nS_{1/2}) \ln^2(Z\alpha)^{-2} \\ &+ A_{61}(nS_{1/2}) \ln(Z\alpha)^{-2} + G_{SE}(nS_{1/2}, Z\alpha)]. \end{aligned} \quad (2)$$

The A coefficients have two indices, the first of which denotes the power of $Z\alpha$ [including those powers explicitly contained in Eq. (1)], while the second index denotes the power of the logarithm $\ln(Z\alpha)^{-2}$.

We now list the analytic coefficients and the Bethe logarithms relevant to the atomic states under investigation [4–23],

$$A_{41}(nS_{1/2}) = \frac{4}{3}, \quad (3a)$$

$$A_{40}(nS_{1/2}) = \frac{10}{9} - \frac{4}{3} \ln k_0(nS), \quad (3b)$$

$$A_{50}(nS_{1/2}) = 4\pi \left[\frac{139}{128} - \frac{1}{2} \ln 2 \right], \quad (3c)$$

$$A_{62}(nS_{1/2}) = -1. \quad (3d)$$

A_{61} coefficients read

$$A_{61}(1S_{1/2}) = \frac{21}{20} + \frac{28}{3} \ln 2, \quad (4a)$$

$$A_{61}(2S_{1/2}) = \frac{67}{30} + \frac{16}{3} \ln 2, \quad (4b)$$

$$A_{61}(3S_{1/2}) = \frac{6163}{1620} + \frac{28}{3} \ln 2 - 4 \ln 3, \quad (4c)$$

$$A_{61}(4S_{1/2}) = \frac{4}{3} \ln 2 + \frac{391}{80}. \quad (4d)$$

The Bethe logarithms $\ln k_0(nS)$ are known [24–29], and we here present a reevaluation,

$$\ln k_0(1S) = 2.984\,128\,555\,765\,497\,611(1), \quad (5a)$$

$$\ln k_0(2S) = 2.811\,769\,893\,120\,563\,520(1), \quad (5b)$$

$$\ln k_0(3S) = 2.767\,663\,612\,491\,821\,190(1), \quad (5c)$$

$$\ln k_0(4S) = 2.749\,811\,840\,454\,057\,422(1). \quad (5d)$$

The evaluation of the coefficient

$$A_{60}(nS_{1/2}) \equiv \lim_{Z\alpha \rightarrow 0} G_{SE}(nS_{1/2}, Z\alpha) \quad (6)$$

has been historically problematic [19–23], and it has therefore been a considerable challenge to reliably estimate the self-energy remainder function G_{SE} , especially in the range of low nuclear charge number Z . Our calculation of the nonperturbative (in $Z\alpha$) electron self-energy for the $3S_{1/2}$ state (see Table I) has a numerical uncertainty of 2 Hz in atomic hydrogen. For the $4S_{1/2}$ state, the numerical uncertainty is $Z^4 \times 3$ Hz (see Table II). The value of the fine-structure constant α employed in the calculation is $\alpha^{-1} = 137.036$; this is close to the 1998 and 2002 CODATA recommended values

TABLE I. Numerical results for the scaled self-energy function F ($3S$ state) and the self-energy remainder function G_{SE} , in the regime of low nuclear charge numbers Z .

Z	$F(3S_{1/2}, Z\alpha)$	$G_{SE}(3S_{1/2}, Z\alpha)$
1	10.605 614 22(5)	−31.047 7(9)
2	8.817 615 14(5)	−30.512 6(2)
3	7.794 461 17(5)	−30.022 7(1)
4	7.083 612 42(5)	−29.564 53(6)
5	6.543 385 98(5)	−29.130 61(4)

[30,31]. The entries for the self-energy remainder function G_{SE} in Tables I and II are in agreement with those used in the latest adjustment of the fundamental physical constants [31] (the G_{SE} values used in [31] are based on an extrapolation of numerical data previously obtained [32] for higher nuclear charge numbers). Our all-order evaluation eliminates any uncertainty due to the unknown higher-order analytic terms that contribute to the bound electron self-energy of $3S$ and $4S$

TABLE II. Numerical results for the scaled self-energy function $F(4S_{1/2}, Z\alpha)$ and the self-energy remainder function G_{SE} .

Z	$F(4S_{1/2}, Z\alpha)$	$G_{SE}(4S_{1/2}, Z\alpha)$
1	10.629 388 4(2)	−30.912(4)
2	8.841 324 1(2)	−30.380 0(9)
3	7.818 078 5(2)	−29.892 4(4)
4	7.107 116 6(2)	−29.437 1(2)
5	6.566 758 8(2)	−29.006 0(2)

states [see Eq. (2)]. This improves our knowledge of the spectrum of hydrogenlike atoms (e.g., atomic hydrogen, He^+).

U.D.J. thanks the National Institute of Standards and Technology for kind hospitality during a number of extended research appointments. The authors acknowledge E.-O. LeBigot for help in obtaining numerical results for selected partial contributions to the electron self-energy, for the hydrogenic energy levels discussed in this work.

-
- [1] U. D. Jentschura, P. J. Mohr, and G. Soff, Phys. Rev. Lett. **82**, 53 (1999).
 - [2] U. D. Jentschura, P. J. Mohr, and G. Soff, Phys. Rev. A **64**, 042512 (2001).
 - [3] J. Sapirstein and D. R. Yennie, in *Quantum Electrodynamics*, edited by T. Kinoshita (World Scientific, Singapore, 1990), pp. 560–672.
 - [4] H. A. Bethe, Phys. Rev. **72**, 339 (1947).
 - [5] R. P. Feynman, Phys. Rev. **74**, 1430 (1948).
 - [6] R. P. Feynman, Phys. Rev. **76**, 769 (1949).
 - [7] J. B. French and V. F. Weisskopf, Phys. Rev. **75**, 1240 (1949).
 - [8] N. M. Kroll and W. E. Lamb, Phys. Rev. **75**, 388 (1949).
 - [9] J. Schwinger, Phys. Rev. **75**, 898 (1949).
 - [10] H. Fukuda, Y. Miyamoto, and S. Tomonaga, Prog. Theor. Phys. **4**, 47 (1949).
 - [11] M. Baranger, Phys. Rev. **84**, 866 (1951).
 - [12] R. Karplus, A. Klein, and J. Schwinger, Phys. Rev. **86**, 288 (1952).
 - [13] M. Baranger, H. A. Bethe, and R. P. Feynman, Phys. Rev. **92**, 482 (1953).
 - [14] H. M. Fried and D. R. Yennie, Phys. Rev. **112**, 1391 (1958).
 - [15] H. M. Fried and D. R. Yennie, Phys. Rev. Lett. **4**, 583 (1960).
 - [16] A. J. Layzer, Phys. Rev. Lett. **4**, 580 (1960).
 - [17] A. J. Layzer, J. Math. Phys. **2**, 292 (1961).
 - [18] A. J. Layzer, J. Math. Phys. **2**, 308 (1961).
 - [19] G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) **35**, 271 (1965).
 - [20] G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) **35**, 447 (1965).
 - [21] G. W. Erickson, Phys. Rev. Lett. **27**, 780 (1971).
 - [22] J. Sapirstein, Phys. Rev. Lett. **47**, 1723 (1981).
 - [23] K. Pachucki, Ann. Phys. (N.Y.) **226**, 1 (1993).
 - [24] S. Klarsfeld and A. Maquet, Phys. Lett. **43**, 201 (1973).
 - [25] H. A. Bethe, L. M. Brown, and J. R. Stehn, Phys. Rev. **77**, 370 (1950).
 - [26] J. M. Harriman, Phys. Rev. **101**, 594 (1956).
 - [27] C. Schwartz and J. J. Tieman, Ann. Phys. (N.Y.) **6**, 178 (1959).
 - [28] M. Lieber, Phys. Rev. **174**, 2037 (1968).
 - [29] R. W. Huff, Phys. Rev. **186**, 1367 (1969).
 - [30] P. J. Mohr and B. N. Taylor, Rev. Mod. Phys. **72**, 351 (2000).
 - [31] P. J. Mohr and B. N. Taylor, CODATA recommended values of the fundamental physical constants: 2002, Rev. Mod. Phys. (to be published).
 - [32] P. J. Mohr and Y. K. Kim, Phys. Rev. A **45**, 2727 (1992).