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Decentralized Neural Network Control of a Class of Large-Scale Systems with Unknown Interconnections¹

Wenxin Liu, S. Jagannathan, Donald C. Wunsch II, and Mariesa L. Crow

Abstract—A novel decentralized neural network (DNN) controller is proposed for a class of large-scale nonlinear systems with unknown interconnections. The objective is to design a DNN for a class of large-scale systems which do not satisfy the matching condition requirement. The NNs are used to approximate the unknown subsystem dynamics and the interconnections. The DNN is designed using the back stepping methodology with only local signals for feedback. All of the signals in the closed loop (system states and weights estimation errors) are guaranteed to be uniformly ultimately bounded and eventually converge to a compact set.

Index Terms—Decentralized control, neural networks, adaptive neural network control, backstepping.

I. INTRODUCTION

MANY physical systems, such as power grid, computer and communication network are complex large-scale interconnected systems. To coordinate the control activities of the overall system, centralized control schemes are proposed by assuming that global information of the overall system is available. But centralized controllers are very difficult to implement for complex large scale systems due to technical and economic reasons. Furthermore, centralized controller designs are dependent upon the system structure and cannot handle the structural changes.

To overcome the problems of centralized control, decentralized schemes are currently being addressed in the literature [1~4]. Instead of designing a global controller, decentralized control design aims at designing separate controllers for each subsystem. The subsystem controllers require only local signals or a minimum amount of information from other subsystems.

Earlier works on decentralized control of nonlinear systems assumed that the interconnection dynamics are linear in the unknown parameters (LIP) and bounded with first order terms. Moreover, a major structural restriction is imposed on the system, for instance, uncertainties and interconnections are in the range space of the input matrix, which is basically the strict matching condition. Therefore, global stabilization of systems with mismatched uncertainties is not possible using these schemes.

Moreover, to further relax the LIP assumption on the nonlinear system, recently, neural networks have been applied to design decentralized NN controllers [1~3] by assuming that the interconnections can also be approximated by using the NNs. Further, the direct controller designs require neither the knowledge of nor the direct estimation of the unknown input gain matrix. Thus the singularity problems that are typically observed with the class of nonlinear systems are overcome. However, controller design in [1, 2] is only applicable to nonlinear systems in Brunovsky Canonical Form (BCF).

This paper proposes a novel decentralized NN controller (DNN) design for the control of a class of more general large-scale unknown nonlinear systems. The NNs are used to approximate the unknown nonlinear dynamics of the subsystems and to compensate the unknown nonlinear interactions. The first or higher order polynomial bound assumption of earlier works [1, 2] on the unknown interconnection terms can be treated here as special cases. All the signals in the closed loop are guaranteed to be uniformly ultimately bounded.

II. BACKGROUND

The following mathematical notions are required for the development of adaptive decentralized NN controller.

A. Approximation Property of NN

The commonly used universal functional approximation property of NN is used in this work. Let $f(x)$ be a smooth function from $R^n \rightarrow R^m$, then it was demonstrated that, as long as x is restricted to a compact set $S \in R^n$, for some sufficiently large number of hidden-layer neurons, there exist weights and thresholds such that

$$f(x) = W^T \varphi(x) + \varepsilon(x) \quad (1)$$

where x is the input vector, $\varphi(\cdot)$ is the activation function, W is the weight matrix of the output layer and $\varepsilon(x)$ is the approximation error. Equation (1) implies that a NN can approximate any continuous function in a compact set. In fact, for any choice of a positive number ε_N , one can find a NN such that $\varepsilon(x) \leq \varepsilon_N$ for all $x \in S$. For suitable function approximation, $\varphi(x)$ must form a basis. It has been shown in [5] that $\varphi(x)$ can form a basis if V is chosen randomly. The larger the number of the hidden layer neurons, the smaller is the approximation error $\varepsilon(x)$.

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B. Stability of Systems

To formulate the controller, the following stability notion is needed. Consider the nonlinear system given by

$$\begin{aligned}\dot{x} &= f(x, u) \\ y &= h(x)\end{aligned}\quad (2)$$

where $x(t)$ is a state vector, $u(t)$ is the input vector and $y(t)$ is the output vector. The solution to (2) is uniformly ultimately bounded (UUB) if for any U , a compact subset of R^n , and all $x(t_0) = x_0 \in U$ there exists an $\varepsilon > 0$ and a number $T(\varepsilon, x_0)$ such that $\|x(t)\| < \varepsilon$ for all $t \geq t_0 + T$.

III. THE CLASS OF LARGE-SCALE NONLINEAR SYSTEMS

The large-scale nonlinear system considered in this paper is comprised of N interconnected subsystems. The i -th subsystem is expressed as follow.

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = x_{i3} \\ \dots \\ \dot{x}_{ik} = f_{i0}(x_i) + g_{i0}(x_i)\xi_{i1} + \Delta_{i0}(X) + d_{i0} \\ \dot{\xi}_{i1} = f_{i1}(x_i, \xi_{i1}) + g_{i1}(x_i, \xi_{i1})\xi_{i2} + \Delta_{i1}(X, \xi_{i1}) + d_{i1} \\ \dots \\ \dot{\xi}_{ij} = f_{ij}(x_i, \xi_{i,1-j}) + g_{ij}(x_i, \xi_{i,1-j})\xi_{i,j+1} + \Delta_{ij}(X, \xi_{i,1-j}) + d_{ij} \\ \dots \\ \dot{\xi}_{i,n-k} = f_{i,n-k}(x_i, \xi_i) + g_{i,n-k}(x_i, \xi_i)u_i + \Delta_{i,n-k}(X, \xi_i) + d_{i,n-k} \\ y_i = x_{i1} \end{cases}\quad (3)$$

where $j = 1, \dots, n-k$; n is the dimension of each subsystem; $x_i = [x_{i1}, x_{i2}, \dots, x_{ik}]^T$ and $\xi_i = [\xi_{i1}, \dots, \xi_{i,n-k}]^T$ are the subsystem's states; $\xi_{i,1-j}$ stands for $[\xi_{i1}, \dots, \xi_{ij}]^T$; X and Ξ are defined as $X = [x_1^T, \dots, x_N^T]^T$ and $\Xi = [\xi_1^T, \dots, \xi_N^T]^T$ respectively; $f_{ij}(x_i, \xi_{i,1-j})$ and $g_{ij}(x_i, \xi_{i,1-j})$ are unknown smooth functions; $\Delta_{ij}(X, \xi_{i,1-j})$ stands for the interconnection terms; d_{ij} is the disturbances bounded by $|d_{ij}| \leq d_{ij}^M$; u_i is the input and y_i is the output of the i -th subsystem.

Assumption 1: The interconnection term of the i -th subsystem $\Delta_{ij}(X, \xi_{i,1-j})$ is bounded whose bound is given by

$$|\Delta_{ij}(X, \xi_{i,1-j})| \leq \delta_{ij0}(\xi_{i,1-j}) + \sum_{p=1}^N \delta_{ijp}(|z_{p0}|)\quad (4)$$

where $\delta_{ij0}(\xi_{i,1-j})$ and $\delta_{ijp}(|z_{p0}|)$ are unknown smooth functions; z_{p0} is the filtered tracking error defined as (6).

Assumption 2: The input gain function $g_{ij}(x_i, \xi_{i,1-j})$ is bounded, continuous, differentiable, and the sign of which is known. That is, they are either positive or negative. Without

losing generality, $g_{ij}(x_i, \xi_{i,1-j})$ is assumed to be positive, and $0 < g_{ij}^m \leq g_{ij}(x_i, \xi_{i,1-j})$, where g_{ij}^m is a positive constant. Moreover, $\dot{g}_{ij}(x_i, \xi_{i,1-j})$, is considered to be continuous.

Assumption 3: The desired trajectory vector, $x_{id} = [y_d, \dot{y}_d, \ddot{y}_d, \dots, y_d^{(n)}]$, is continuous and available.

Remark 1: In [1], the interconnection terms are a function of all the system states. But the interconnection terms only appear in the same equations as the control signals. By contrast, the interconnection terms are a function of local signals and they appear in all the equations. Future work includes relaxing the dependability of interconnection terms on local signals.

Remark 2: Unlike in [4], in this paper, $g_{ij}(x_i, \xi_{i,1-j})$ is considered to be unknown. Moreover, the controller designed here does not require the assumption on the bounded of $\dot{g}_{ij}(x_i, \xi_{i,1-j})$, such as $|\dot{g}_{ij}(x_i, \xi_{i,1-j})| \leq g_d^M$, in contrast with [2].

IV. DECENTRALIZED CONTROLLER DESIGN

In this section, the DNN design is combined with the backstepping method and it is given next.

Step 0:

First consider the i -th isolated subsystem. Define the error between the actual and desired system output as

$$e_i = y_i - y_{id}\quad (5)$$

Furthermore, define $x_{id} = [y_{id}, \dot{y}_{id}, \dots, y_{id}^{(k-1)}]^T$ and $\bar{e}_i = x_i - x_{id} = [e_i, \dot{e}_i, \dots, e_i^{(k-1)}]^T$. Thus, the filtered error z_{i0} can be defined as

$$z_{i0} = [\Lambda_i^T \quad 1]^T \bar{e}_i\quad (6)$$

where $\Lambda_i = [\lambda_{i1}, \lambda_{i2}, \dots, \lambda_{ik}]^T$ is an appropriately chosen coefficient vector such that $e_i \rightarrow 0$ as $z_{i0} \rightarrow 0$ (i.e. $s^k + \lambda_{ik}s^{k-1} + \dots + \lambda_{i2}s + \lambda_{i1} = 0$ is Hurwitz).

Taking the derivative of (6) with respect to time and using (3) to get

$$\dot{z}_{i0} = f_{i0}(x_i) + [0 \quad \Lambda_i^T] \bar{e}_i - y_{id}^{(k)} + g_{i0}(x_i)\xi_{i1} + \Delta_{i0}(X) + d_{i0}\quad (7)$$

By viewing ξ_{i1} as the virtual control signal, the ideal value of which can be chosen according to

$$\begin{aligned} r_{i0}^* &= -K_{i0}z_{i0} - \frac{f_{i0}(x_i) + [0 \quad \Lambda_i^T] \bar{e}_i - y_{id}^{(k)}}{g_{i0}(x_i)} \\ &+ \frac{\dot{g}_{i0}(x_i)}{2g_{i0}^2(x_i)} z_{i0} - \text{sign}(z_{i0}) \frac{1}{g_{i0}^m} \sum_{j=1}^N [\delta_{j00} + \delta_{j0i}(|z_{i0}|)] \end{aligned}\quad (8)$$

where K_{i0} is a positive design constant.

Based on NN approximation theory [5] and applying the Assumptions, the latter part of the above equation can be approximated by using two NNs. That is one NN satisfying

$$W_{i01}^T \Phi_{i01}(X_{i01}) + \varepsilon_{i01} = \frac{f_{i0}(x_i) + [0 \quad \Lambda_i^T] \bar{e}_i - y_{id}^{(k)}}{g_{i0}(x_i)} - \frac{\dot{g}_{i0}(x_i)}{2g_{i0}^2(x_i)} z_{i0} \quad (9)$$

where $X_{i01} = [x_i, y_{id}^{(0)\sim(k)}, 1]^T$, $|\varepsilon_{i01}| \leq \varepsilon_{i01}^M$, $y_{id}^{(0)\sim(k)}$ is defined as $y_{id}^{(0)\sim(k)} = [y_{id}, \dot{y}_{id}, \dots, y_{id}^{(k)}]$.

And another NN satisfying

$$W_{i02}^T \Phi_{i02}(X_{i02}) + \varepsilon_{i02} = \frac{1}{g_{i0}^m} [\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|)] \quad (10)$$

where $X_{i02} = [|z_{i0}|, 1]^T$ and $|\varepsilon_{i02}| \leq \varepsilon_{i02}^M$.

Thus, if ξ_{i1} is the actual control signal, the virtual control signal can be chosen as

$$\hat{r}_{i0} = -K_{i0} z_{i0} - \hat{W}_{i01}^T \Phi_{i01}(X_{i01}) - \text{sign}(z_{i0}) \hat{W}_{i02}^T \Phi_{i02}(X_{i02}) \quad (11)$$

Remark 3: During the following controller design, we need to calculate the derivative of the desired virtual control signal. The procedure cannot proceed if \hat{r}_{i0} is chosen as the desired virtual control signal because of the discontinuities of $|z_{i0}|$ and $\text{sign}(z_{i0})$. This problem is confronted by employing a smooth function instead of the sign function. A choice of the function is $f_1(x) = (1 - e^{-Ex}) / (1 + e^{-Ex})$ with $E > 0$. Thus $|x|$ can be approximated by $f_2(x) = x f_1(x)$. It is easy to verify that the error function is bounded by $||x| - f_2(x)| \leq 1/E$.

According to the NN approximation theory in [5], the first layer weights can be randomly chosen between zero and one and kept fixed thereafter. Thus, the above disturbance will result a disturbance bounded by I/E to the inputs of the hidden neurons, where I is the number of input neurons. In this paper, only bounded and smooth NN transfer functions are considered. For simplification, the output of the transfer function is assumed to be bounded by one. Thus, the disturbance to the output of the hidden neuron is bounded by IM/E , where M is the maximum slope of the transfer function. For example, for logarithmic sigmoid transfer function, defined as $y = 1/(1 + e^{-x})$, $M_s = 0.25$. Finally, the disturbance to the NN output is bounded by $(HIM/E)|W|$, where H is the number of hidden neurons and $|W|$ is bound of the weights of the output layer. For simplification, define

$$C = HIM/E \quad (12)$$

where C is a constant value once E and the NN parameters (number of input neurons, number of hidden neurons, and type of transfer functions) are decided.

Thus, instead of using $X_{i02} = [|z_{i0}|, 1]^T$ as the NN input, we can use $X'_{i02} = [f_2(z_{i0}), 1]^T$ as the NN input. Then, the desired realizable virtual control signal will become

$$r_{i0} = -K_{i0} z_{i0} - \hat{W}_{i01}^T \Phi_{i01}(X_{i01}) - f_1(z_{i0}) \hat{W}_{i02}^T \Phi_{i02}(X'_{i02}) \quad (13)$$

Now, the calculation of \hat{r}_{i0} is made possible because r_{i0} is a continuously differentiable function. Define

$$z_{i1} = \xi_{i1} - r_{i0} \quad (14)$$

Expressing ξ_{i1} as a function of z_{i1} and substituting it into (7) to get

$$\dot{z}_{i0} = g_{i0}(x_i) [z_{i1} - K_{i0} z_{i0} + \frac{\dot{g}_{i0}(x_i)}{2g_{i0}^2(x_i)} z_{i0} - \tilde{W}_{i01}^T \Phi_{i01}(X_{i01})] \quad (15)$$

$$- \varepsilon_{i01} - f_1(z_{i0}) \hat{W}_{i02}^T \Phi_{i02}(X'_{i02}) + \Delta_{i0}(X) + d_{i0}$$

where $\tilde{W}_{i01} = \hat{W}_{i01} - W_{i01}$ and $\tilde{W}_{i02} = \hat{W}_{i02} - W_{i02}$ are the weights estimation errors.

Choose the Lyapunov function for this step as

$$V_0 = \sum_{i=1}^N \left[\frac{z_{i0}^2}{2g_{i0}(x_i)} + \frac{1}{2} \tilde{W}_{i01}^T \Gamma_{i01}^{-1} \tilde{W}_{i01} + \frac{1}{2} \tilde{W}_{i02}^T \Gamma_{i02}^{-1} \tilde{W}_{i02} \right] \quad (16)$$

where $\Gamma_{i01}, \Gamma_{i02} > 0$ are the adaptation gain matrix.

Evaluating the derivative of V_0 to get

$$\dot{V}_0 = \sum_{i=1}^N \left\{ \left[\frac{\dot{z}_{i0}}{g_{i0}(x_i)} - \frac{\dot{g}_{i0}(x_i)}{2g_{i0}^2(x_i)} z_{i0} \right] z_{i0} \right. \\ \left. + \frac{1}{\tilde{W}_{i01}^T \Gamma_{i01}^{-1} \tilde{W}_{i01} + \tilde{W}_{i02}^T \Gamma_{i02}^{-1} \tilde{W}_{i02}} \left\{ \begin{aligned} & -K_{i0} z_{i0}^2 + z_{i0} z_{i1} - z_{i0} \tilde{W}_{i01}^T \Phi_{i01}(X_{i01}) + \tilde{W}_{i01}^T \Gamma_{i01}^{-1} \tilde{W}_{i01} \\ & - [\varepsilon_{i01} - \frac{d_{i0}}{g_{i0}(x_i)}] z_{i0} - z_{i0} f_1(z_{i0}) \hat{W}_{i02}^T \Phi_{i02}(X'_{i02}) \\ & + \frac{1}{g_{i0}(x_i)} z_{i0} \Delta_{i0}(X) + \tilde{W}_{i02}^T \Gamma_{i02}^{-1} \tilde{W}_{i02} \end{aligned} \right\} \right\} \quad (17)$$

It is important to note that the following equality hold.

$$\sum_{i=1}^N \sum_{j=1}^N \delta_{i0j}(|z_{j0}|) = \sum_{i=1}^N \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) \quad (18)$$

Considering the bounds of $\Delta_{i0}(X)$ and $g_{i0}(x_i)$, we have

$$\sum_{i=1}^N \left| \frac{1}{g_{i0}(x_i)} z_{i0} \Delta_{i0}(X) \right| \leq \sum_{i=1}^N |z_{i0}| \frac{1}{g_{i0}^m} [\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|)] \quad (19)$$

$$\leq \sum_{i=1}^N |z_{i0}| [W_{i02}^T \Phi_{i02}(X_{i02}) + \varepsilon_{i02}^M]$$

According to Remark 3 and (12), we have

$$W_{i02}^T \Phi_{i02}(X_{i02}) \leq W_{i02}^T \Phi_{i02}(X'_{i02}) + C_{i02} |W_{i02}| \quad (20)$$

Thus,

$$\sum_{i=1}^N \left| \frac{1}{g_{i0}(x_i)} z_{i0} \Delta_{i0}(X) \right| \leq \sum_{i=1}^N |z_{i0}| [W_{i02}^T \Phi_{i02}(X'_{i02}) + \varepsilon'_{i02}] \quad (21)$$

where $|\varepsilon'_{i02}| \leq C_{i02} |W_{i02}| + \varepsilon_{i02}^M$. Therefore (17) becomes

$$\dot{V}_0 \leq \sum_{i=1}^N \left\{ \begin{aligned} & -K_{i0} z_{i0}^2 + z_{i0} z_{i1} - z_{i0} \tilde{W}_{i01}^T \Phi_{i01}(X_{i01}) + \tilde{W}_{i01}^T \Gamma_{i01}^{-1} \tilde{W}_{i01} \\ & + (\varepsilon_{i01}^M + \frac{d_{i0}^M}{g_{i0}^m}) |z_{i0}| + \tilde{W}_{i02}^T \Gamma_{i02}^{-1} \tilde{W}_{i02} - |z_{i0}| \tilde{W}_{i02}^T \Phi_{i02}(X'_{i02}) \\ & + |z_{i0}| |\varepsilon'_{i02}| + |z_{i0}| - f_2(z_{i0}) \tilde{W}_{i02}^T \Phi_{i02}(X'_{i02}) \end{aligned} \right\} \quad (22)$$

Choose the weights updating rules for \hat{W}_{i01} and \hat{W}_{i02} as

$$\dot{\hat{W}}_{i01} = \Gamma_{i01}[z_{i0}\Phi_{i01}(X_{i01}) - \alpha_{i01}\hat{W}_{i01}] \quad (23)$$

$$\dot{\hat{W}}_{i02} = \Gamma_{i02}[z_{i0}|\Phi_{i02}(X'_{i02}) - \alpha_{i02}\hat{W}_{i02}]$$

where α_{i01} , and α_{i02} are positive design constants.

Since the outputs of transfer functions are selected to be bounded by one, we have $|\Phi_{i02}(X'_{i02})| \leq H_{i02}$. Thus,

$$| [|z_{i0}| - f_2(z_{i0})] \hat{W}_{i02}^T \Phi_{i02}(X'_{i02}) | \leq (H_{i02}/E) |\hat{W}_{i02}| \quad (24)$$

$$\leq T_{i02} |\hat{W}_{i02}| \leq T_{i02} (|W_{i02}| + |\tilde{W}_{i02}|) \leq \frac{1}{2} |\tilde{W}_{i02}|^2 + \frac{1}{2} T_{i02}^2 + T_{i02} |W_{i02}|$$

where $T_{i02} = H_{i02}/E$. Furthermore, we have

$$|z_{i0}' \varepsilon_{i02}'| \leq (C_{i02} |W_{i02}| + \varepsilon_{i02}^M) |z_{i0}| \leq z_{i0}^2 + \frac{C_{i02}^2}{2} |W_{i02}|^2 + \frac{1}{2} \varepsilon_{i02}^M{}^2 \quad (25)$$

Since

$$\begin{aligned} -\alpha_{i01} \tilde{W}_{i01}^T \hat{W}_{i01} &\leq -\alpha_{i01} \tilde{W}_{i01}^T (\tilde{W}_{i01} + W_{i01}) \\ &\leq -\alpha_{i01} |\tilde{W}_{i01}|^2 + \alpha_{i01} |\tilde{W}_{i01}| |W_{i01}| \leq -\frac{\alpha_{i01}}{2} |\tilde{W}_{i01}|^2 + \frac{\alpha_{i01}}{2} |W_{i01}|^2 \end{aligned} \quad (26)$$

Similarly

$$-\alpha_{i02} \tilde{W}_{i02}^T \hat{W}_{i02} \leq -\frac{\alpha_{i02}}{2} |\tilde{W}_{i02}|^2 + \frac{\alpha_{i02}}{2} |W_{i02}|^2 \quad (27)$$

and

$$\left(\varepsilon_{i01}^M + \frac{d_{i0}^M}{g_{i0}^m} \right) |z_{i0}| \leq \frac{z_{i0}^2}{2} + \frac{1}{2} \left(\varepsilon_{i01}^M + \frac{d_{i0}^M}{g_{i0}^m} \right)^2 \quad (28)$$

Substituting (23) into (22) and using the bounds analysis of (24-28), we get

$$\dot{V}_0 \leq \sum_{i=1}^N \left[-k_{i01} z_{i0}^2 + z_{i0} z_{i1} - k_{i02} |\tilde{W}_{i01}|^2 - k_{i03} |\tilde{W}_{i02}|^2 + k_{i04} \right] \quad (29)$$

where

$$k_{i01} = K_{i0} - \frac{3}{2} > 0, k_{i02} = \frac{\alpha_{i01}}{2} > 0, k_{i03} = \frac{\alpha_{i02} - 1}{2} > 0, \text{ and}$$

$$\begin{aligned} k_{i04} &= \frac{\alpha_{i01}}{2} |W_{i01}|^2 + \frac{\alpha_{i02} + C_{i02}^2}{2} |W_{i02}|^2 + T_{i02} |W_{i02}| + \frac{1}{2} T_{i02}^2 \\ &+ \frac{1}{2} \left(\varepsilon_{i01}^M + \frac{d_{i0}^M}{g_{i0}^m} \right)^2 + \frac{1}{2} \varepsilon_{i02}^M{}^2 \end{aligned}$$

Step 1:

Taking the derivative of (14) and using (3) to get

$$\dot{z}_{i1} = f_{i1}(x_i, \xi_{i1}) + g_{i1}(x_i, \xi_{i1}) \xi_{i2} + \Delta_{i1}(X, \xi_{i1}) + d_{i1} - \dot{r}_{i0} \quad (30)$$

where

$$\begin{aligned} \dot{r}_{i0} &= \frac{\partial r_{i0}}{\partial x_i} \dot{x}_i + \sum_{p=0}^k \frac{\partial r_{i0}}{\partial y_{id}^{(p)}} y_{id}^{(p+1)} + \frac{\partial r_{i0}}{\partial \hat{W}_{i01}} \dot{\hat{W}}_{i01} + \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \dot{\hat{W}}_{i02} \\ &= \phi_{i0} + \frac{\partial r_{i0}}{\partial x_{ik}} [\Delta_{i0}(X) + d_{i0}] + \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \Gamma_{i02} |z_{i0}| \Phi_{i02}(X'_{i02}) \end{aligned} \quad (31)$$

with

$$\phi_{i0} = \sum_{p=0}^k \frac{\partial r_{i0}}{\partial y_{id}^{(p)}} y_{id}^{(p+1)} + \frac{\partial r_{i0}}{\partial \hat{W}_{i01}} \dot{\hat{W}}_{i01} - \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \Gamma_{i02} \alpha_{i02} \hat{W}_{i02} \quad (32)$$

$$+ \sum_{q=1}^{k-1} \frac{\partial r_{i0}}{\partial x_{iq}} x_{i,q+1} + \frac{\partial r_{i0}}{\partial x_{ik}} [f_{i0}(x_i) + g_{i0}(x_i) \xi_{i1}]$$

Define

$$z_{i2} = \xi_{i2} - r_{i1} \quad (33)$$

By viewing ξ_{i2} as the virtual control signal, the ideal virtual control signal r_{i1}^* can be chosen according to

$$\begin{aligned} r_{i1}^* &= -z_{i0} - K_{i1} z_{i1} - \frac{f_{i1}(x_i, \xi_{i1}) - \phi_{i0}}{g_{i1}(x_i)} + \frac{\dot{g}_{i1}(x_i, \xi_{i1})}{2g_{i1}^2(x_i, \xi_{i1})} z_{i1} \\ &- \text{sign}(z_{i1}) \frac{1}{g_{i1}^m} \left\{ \begin{aligned} &\delta_{i10}(\xi_{i1}) + \sum_{j=1}^N \delta_{j1i}(|z_{i0}|) + d_{i1}^M + H_{i02}^2 \Gamma_{i02} |z_{i0}| \\ &+ \left| \frac{\partial r_{i0}}{\partial x_{ik}} \left[\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) + d_{i0}^M \right] \right| \end{aligned} \right\} \end{aligned} \quad (34)$$

Assuming there exist one NN such that

$$W_{i11}^T \Phi_{i11}(X_{i11}) + \varepsilon_{i11} = \frac{f_{i1}(x_i, \xi_{i1}) - \phi_{i0}}{g_{i1}(x_i, \xi_{i1})} - \frac{\dot{g}_{i1}(x_i, \xi_{i1})}{2g_{i1}^2(x_i, \xi_{i1})} z_{i1} \quad (35)$$

with $X_{i11} = [x_i, \xi_{i1}, y_{id}^{(0) \sim (k+1)}, \hat{W}_{i01}, \hat{W}_{i02}, 1]^T$ and $|\varepsilon_{i11}| \leq \varepsilon_{i11}^M$.

Define another NN such that

$$\begin{aligned} &W_{i12}^T \Phi_{i12}(X_{i12}) + \varepsilon_{i12} \\ &= \frac{1}{g_{i1}^m} \left\{ \begin{aligned} &\delta_{i10}(\xi_{i1}) + \sum_{j=1}^N \delta_{j1i}(|z_{i0}|) + d_{i1}^M + H_{i02}^2 \Gamma_{i02} |z_{i0}| \\ &+ \left| \frac{\partial r_{i0}}{\partial x_{ik}} \left[\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) + d_{i0}^M \right] \right| \end{aligned} \right\} \end{aligned} \quad (36)$$

with $X_{i12} = [z_{i0}, \xi_{i1}, \left| \frac{\partial r_{i0}}{\partial x_{ik}} \right|, 1]^T$ and $|\varepsilon_{i12}| \leq \varepsilon_{i12}^M$

Similar to Step 0, change the input vector to the NN to

$$X'_{i12} = \left[f_2(z_{i0}), \xi_{i1}, f_2 \left(\frac{\partial r_{i0}}{\partial x_{ik}}, 1 \right) \right]^T \quad (37)$$

Then, the desired realizable virtual control signal is

$$r_{i1} = -z_{i0} - K_{i1} z_{i1} - \hat{W}_{i11}^T \Phi_{i11}(X_{i11}) - f_{i1}(z_{i1}) \hat{W}_{i12}^T \Phi_{i12}(X'_{i12}) \quad (38)$$

Choose the Lyapunov function for this step as

$$V_1 = V_0 + \sum_{i=1}^N \left[\frac{z_{i1}^2}{2g_{i1}(x_i, \xi_{i1})} + \frac{1}{2} \tilde{W}_{i11}^T \Gamma_{i11}^{-1} \tilde{W}_{i11} + \frac{1}{2} \tilde{W}_{i12}^T \Gamma_{i12}^{-1} \tilde{W}_{i12} \right] \quad (39)$$

where $\Gamma_{i11}, \Gamma_{i12} > 0$ are the adaptation gain matrix \tilde{W}_{i11} and \tilde{W}_{i12} are the weights estimation errors. Taking the derivative (39) and using (30) to get

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + \sum_{i=1}^N \left\{ \begin{aligned} &\left[\frac{\dot{z}_{i1}}{g_{i1}(x_i, \xi_{i1})} - \frac{\dot{g}_{i1}(x_i, \xi_{i1})}{2g_{i1}^2(x_i, \xi_{i1})} z_{i1} \right] z_{i1} \\ &+ \tilde{W}_{i11}^T \Gamma_{i11}^{-1} \dot{\tilde{W}}_{i11} + \tilde{W}_{i12}^T \Gamma_{i12}^{-1} \dot{\tilde{W}}_{i12} \end{aligned} \right\} \\ &\leq \sum_{i=1}^N \left\{ \begin{aligned} &-k_{i01} z_{i0}^2 + z_{i1} z_{i2} - k_{i02} |\tilde{W}_{i01}|^2 - k_{i03} |\tilde{W}_{i02}|^2 + k_{i04} \\ &-K_{i1} z_{i1}^2 - z_{i1} \tilde{W}_{i11}^T \Phi_{i11}(X_{i11}) + \tilde{W}_{i11}^T \Gamma_{i11}^{-1} \dot{\tilde{W}}_{i11} + (\varepsilon_{i11}^M + \frac{d_{i1}^M}{g_{i1}^m}) |z_{i1}| \\ &-f_{i1}(z_{i1}) z_{i1} \hat{W}_{i12}^T \Phi_{i12}(X'_{i12}) + \tilde{W}_{i12}^T \Gamma_{i12}^{-1} \dot{\tilde{W}}_{i12} \\ &+ \frac{1}{g_{i1}^m} |z_{i1}| \left\{ \begin{aligned} &[|\Delta_{i1}(X, \xi_{i1})| + d_{i1}^M] + \left| \frac{\partial r_{i0}}{\partial x_{ik}} \right| [|\Delta_{i0}(X)| + d_{i0}^M] \\ &+ \left| \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \right| \Gamma_{i02} |z_{i0}| \Phi_{i02}(X'_{i02}) \end{aligned} \right\} \end{aligned} \right\} \end{aligned} \quad (40)$$

Considering the expression of (13), we know that $\frac{\partial r_{i0}}{\partial \hat{W}_{i02}}$ is bounded by H_{i02} . Thus, we have

$$\left| \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \Gamma_{i02} |z_{i0}| \Phi_{i02}(X'_{i02}) \right| \leq H_{i02}^2 \Gamma_{i02} |z_{i0}| \quad (41)$$

Similar to Step 0, we have

$$\begin{aligned} & \sum_{i=1}^N \frac{1}{g_{i1}^m} \left\{ \left[|\Delta_{i1}(X, \xi_{i1})| + d_{i1}^M \right] + \left| \frac{\partial r_{i0}}{\partial x_{ik}} \right| \left[|\Delta_i(X)| + d_{i0}^M \right] \right. \\ & \left. + \left| \frac{\partial r_{i0}}{\partial \hat{W}_{i02}} \right| \Gamma_{i02} |z_{i0}| \Phi_{i02}(X'_{i02}) \right\} \\ & \leq \sum_{i=1}^N \frac{1}{g_{i1}^m} \left\{ \delta_{i10}(\xi_{i1}) + \sum_{j=1}^N \delta_{j1i}(|z_{i0}|) + d_{i1}^M + H_{i02}^2 \Gamma_{i02} |z_{i0}| \right. \\ & \left. + \left| \frac{\partial r_{i0}}{\partial x_{ik}} \right| \left[\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) + d_{i0}^M \right] \right\} \\ & \leq \sum_{i=1}^N [W_{i12}^T \Phi_{i12}(X_{i12}) + \varepsilon_{i12}^M] \leq \sum_{i=1}^N [W_{i12}^T \Phi_{i12}(X'_{i12}) + \varepsilon'_{i12}] \end{aligned} \quad (42)$$

where $|\varepsilon'_{i12}| \leq C_{i12} |W_{i12}| + \varepsilon_{i12}^M$. The weights updating rules are chosen as

$$\begin{aligned} \dot{\hat{W}}_{i11} &= \Gamma_{i11} [z_{i1} \Phi_{i11}(X_{i11}) - \alpha_{i11} \hat{W}_{i11}] \\ \dot{\hat{W}}_{i12} &= \Gamma_{i12} [z_{i1} \Phi_{i12}(X'_{i12}) - \alpha_{i12} \hat{W}_{i12}] \end{aligned} \quad (43)$$

where α_{i11} , and α_{i12} are positive design constants.

Similar procedure as Step 0 can be taken to get the following result

$$\dot{V}_1 \leq \sum_{i=1}^N \left[z_{i1} z_{i2} - k_{i01} z_{i0}^2 - k_{i02} |\tilde{W}_{i01}|^2 - k_{i03} |\tilde{W}_{i02}|^2 + k_{i04} \right. \\ \left. - k_{i11} z_{i1}^2 - k_{i12} |\tilde{W}_{i11}|^2 - k_{i13} |\tilde{W}_{i12}|^2 + k_{i14} \right] \quad (44)$$

where

$$k_{i11} = K_{i1} - \frac{3}{2} > 0, \quad k_{i12} = \frac{\alpha_{i11}}{2} > 0, \quad k_{i13} = \frac{\alpha_{i12} - 1}{2} > 0$$

$$k_{i14} = \frac{\alpha_{i11}}{2} |W_{i11}|^2 + \frac{\alpha_{i12} + C_{i12}^2}{2} |W_{i12}|^2 + T_{i12} |W_{i12}| + \frac{1}{2} T_{i12}^2$$

and

$$+ \frac{1}{2} \left(\varepsilon_{i11}^M + \frac{d_{i1}^M}{g_{i1}^m} \right)^2 + \frac{1}{2} \varepsilon_{i12}^M$$

Step $n-k$ (final step):

Taking the derivative of $z_{i,n-k}$ and using (3) to get

$$\begin{aligned} \dot{z}_{i,n-k} &= f_{i,n-k}(x_i, \xi_i) + g_{i,n-k}(x_i, \xi_i) u_i \\ &+ \Delta_{i,n-k}(X, \xi_i) + d_{i,n-k} - \dot{r}_{i,n-k-1} \end{aligned} \quad (45)$$

where

$$\begin{aligned} \dot{r}_{i,n-k-1} &= \frac{\partial r_{i,n-k-1}}{\partial x_{ik}} [\Delta_{i0}(X) + d_{i0}] + \sum_{s=0}^{n-k-1} \frac{\partial r_{i,l-1}}{\partial \hat{W}_{is2}} \Gamma_{is2} |z_{is}| \Phi_{is2}(X'_{is2}) \\ &+ \sum_{t=1}^{n-k-1} \frac{\partial r_{i,n-k-1}}{\partial \xi_{it}} [\Delta_{it}(X, \xi_{i,1-t}) + d_{it}] + \phi_{i,n-k} \end{aligned} \quad (46)$$

with

$$\begin{aligned} \phi_{i,n-k} &= \sum_{p=1}^{k-1} \frac{\partial r_{i,n-k-1}}{\partial x_{ip}} x_{i,p+1} + \frac{\partial r_{i,n-k-1}}{\partial x_{ik}} [f_{i0}(x_i) + g_{i0}(x_i) \xi_{i1}] \\ &+ \sum_{q=0}^{n-1} \frac{\partial r_{i0}}{\partial y_{id}^{(q)}} y_{id}^{(q+1)} + \sum_{s=0}^{n-k-1} \left[\frac{\partial r_{i,n-k-1}}{\partial \hat{W}_{is1}} \hat{W}_{is1} - \frac{\partial r_{i,n-k-1}}{\partial \hat{W}_{is2}} \Gamma_{is2} \alpha_{is2} \hat{W}_{is2} \right] \\ &+ \sum_{t=1}^{n-k-1} \frac{\partial r_{i,n-k-1}}{\partial \xi_{it}} [f_{it}(x_i, \xi_{i,1-t}) + g_{it}(x_i, \xi_{i,1-t}) \xi_{i,t+1}] \end{aligned} \quad (47)$$

The desired control signal is

$$\begin{aligned} u_i^* &= -z_{i,n-k-1} - K_{i,n-k} z_{i,n-k} - \frac{f_{i,n-k}(x_i, \xi_i) - \phi_{i,n-k}}{g_{i,n-k}(x_i, \xi_i)} \\ &+ \frac{\dot{g}_{i,n-k}(x_i, \xi_i)}{2g_{i,n-k}^2(x_i, \xi_i)} z_{i,n-k} - \text{sign}(z_{i,n-k}) \frac{1}{g_{i,n-k}^m} \\ &\left\{ \delta_{i,n-k,0}(\xi_i) + \sum_{j=1}^N \delta_{j,n-k,i}(|z_{i0}|) + d_{i,n-k}^M + \sum_{s=0}^{n-k-1} H_{is2}^2 \Gamma_{is2} |z_{is}| \right. \\ &\times \left. + \sum_{t=1}^{n-k-1} \left| \frac{\partial r_{i,n-k-1}}{\partial \xi_{it}} \right| \left[\delta_{it0}(\xi_i) + \sum_{j=1}^N \delta_{jit}(|z_{i0}|) + d_{it}^M \right] \right. \\ &\left. + \left| \frac{\partial r_{i,n-k-1}}{\partial x_{ik}} \right| \left[\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) + d_{i0}^M \right] \right\} \end{aligned} \quad (48)$$

Similarly, there exist two NNs such that

$$\begin{aligned} W_{i,n-k,1}^T \Phi_{i,n-k,1}(X_{i,n-k,1}) + \varepsilon_{i,n-k,1} \\ = \frac{f_{i,n-k}(x_i, \xi_i) - \phi_{i,n-k}}{g_{i,n-k}(x_i, \xi_i)} - \frac{\dot{g}_{i,n-k}(x_i, \xi_i)}{2g_{i,n-k}^2(x_i, \xi_i)} z_{i,n-k} \end{aligned} \quad (49)$$

where $|\varepsilon_{i,n-k,1}| \leq \varepsilon_{i,n-k,1}^M$ and the NN input is defined as

$$X_{i,n-k,1} = [x_i, \xi_i, y_{id}^{(0)-(n)}, \hat{W}_{i01}, \dots, \hat{W}_{i,n-k-1,1}, \hat{W}_{i02}, \dots, \hat{W}_{i,n-k-1,2}, 1]^T \quad (50)$$

and another neural network satisfying

$$\begin{aligned} W_{i,n-k,2}^T \Phi_{i,n-k,2}(X_{i,n-k,2}) + \varepsilon_{i,n-k,2} \\ = \frac{1}{g_{i,n-k}^m} \left\{ \delta_{i,n-k,0}(\xi_i) + \sum_{j=1}^N \delta_{j,n-k,i}(|z_{i0}|) + d_{i,n-k}^M + \sum_{s=0}^{n-k-1} H_{is2}^2 \Gamma_{is2} |z_{is}| \right. \\ \left. + \sum_{t=1}^{n-k-1} \left| \frac{\partial r_{i,n-k-1}}{\partial \xi_{it}} \right| \left[\delta_{it0}(\xi_i) + \sum_{j=1}^N \delta_{jit}(|z_{i0}|) + d_{it}^M \right] \right. \\ \left. + \left| \frac{\partial r_{i,n-k-1}}{\partial x_{ik}} \right| \left[\delta_{i00} + \sum_{j=1}^N \delta_{j0i}(|z_{i0}|) + d_{i0}^M \right] \right\} \end{aligned} \quad (51)$$

where $|\varepsilon_{i,n-k,2}| \leq \varepsilon_{i,n-k,2}^M$ and the NN input is defined as

$$X_{i,n-k,2} = \left[|z_{i0}|, \dots, |z_{i,n-k-1}|, \xi_i, \left| \frac{\partial r_{i,n-k-1}}{\partial x_{ik}} \right|, \left| \frac{\partial r_{i,n-k-1}}{\partial \xi_{i1}} \right|, \dots, \left| \frac{\partial r_{i,n-k-1}}{\partial \xi_{i,n-k-1}} \right|, 1 \right]^T \quad (52)$$

Since this is the last step, there is no need to approximate $\text{sign}(\cdot)$ using $f_1(\cdot)$. Thus the actual control signal can be chosen is

$$u_i = -z_{i,n-k-1} - K_{i,n-k} z_{i,n-k} - \hat{W}_{i,n-k,1}^T \Phi_{i,n-k,1}(X_{i,n-k,1}) - \text{sign}(z_{i,n-k}) \hat{W}_{i,n-k,2}^T \Phi_{i,n-k,2}(X_{i,n-k,2}) \quad (53)$$

The Lyapunov function for the whole system is chosen as

$$V = \sum_{i=1}^N \left[\sum_{j=0}^{n-k} \frac{z_{ij}^2}{2g_{ij}} + \sum_{j=0}^{n-k} \frac{\tilde{W}_{ij1}^T \Gamma_{ij1}^{-1} \tilde{W}_{ij1}}{2} - \sum_{j=0}^{n-k} \frac{\tilde{W}_{ij2}^T \Gamma_{ij2}^{-1} \tilde{W}_{ij2}}{2} \right] \quad (54)$$

where $\Gamma_{i,n-k,1}, \Gamma_{i,n-k,2} > 0$ are the adaptation gain matrix $\tilde{W}_{i,n-k,1}$ and $\tilde{W}_{i,n-k,2}$ are the weights estimation errors.

By choosing the weight updating rules as

$$\dot{\hat{W}}_{i,n-k,1}^T = \Gamma_{i,n-k,1} [z_{i,n-k} \Phi_{i,n-k,1}(X_{i,n-k,1}) - \alpha_{i,n-k,1} \hat{W}_{i,n-k,1}^T] \quad (55)$$

$$\dot{\hat{W}}_{i,n-k,2}^T = \Gamma_{i,n-k,2} [z_{i,n-k} \Phi_{i,n-k,2}(X'_{i,n-k,2}) - \alpha_{i,n-k,2} \hat{W}_{i,n-k,2}^T]$$

where $\alpha_{i,n-k,1}$, and $\alpha_{i,n-k,2}$ are positive design constants.

We get

$$\dot{V} \leq \sum_{i=1}^N \left[-\sum_{j=0}^{n-k} k_{ij1} z_{ij}^2 - \sum_{j=0}^{n-k} k_{ij2} |\tilde{W}_{ij1}|^2 - \sum_{j=0}^{n-k} k_{ij3} |\tilde{W}_{ij2}|^2 + \sum_{j=0}^{n-k} k_{ij4} \right] \quad (56)$$

where $k_{i,n-k,1} = K_{i,n-k} - 1 > 0$, $k_{i,n-k,2} = \frac{\alpha_{i,n-k,1}}{2}$, $k_{i,n-k,3} = \frac{\alpha_{i,n-k,2}}{2}$,

$$k_{i,n-k,4} = \frac{\mathcal{E}_{i,n-k,1}^M + \mathcal{E}_{i,n-k,2}^M + \alpha_{i,n-k,1} |W_{i,n-k,1}|^2 + \alpha_{i,n-k,2} |W_{i,n-k,2}|^2}{2}.$$

For simplification, define $\delta = \sum_{i=1}^N \sum_{j=0}^{n-k} k_{ij4}$. If the selecting of

design parameters $K_{i0}, \dots, K_{i,n-k}, \alpha_{i01}, \dots, \alpha_{i,n-k-1}, \alpha_{i02}, \dots, \alpha_{i,n-k-2}$ and NN properly, such that $k_{ij1} > \gamma/2g_{ij}^m$, $k_{ij2} \geq \gamma\lambda_{\max}(\Gamma_{ij1}^{-1})$, and $c_{ij3} \geq \gamma\lambda_{\max}(\Gamma_{ij2}^{-1})$, then we get

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^N \left[-\sum_{j=0}^{n-k} k_{ij1} z_{ij}^2 - \sum_{j=0}^{n-k} k_{ij2} |\tilde{W}_{ij1}|^2 - \sum_{j=0}^{n-k} k_{ij3} |\tilde{W}_{ij2}|^2 - \sum_{j=0}^{n-k} k_{ij4} \right] \\ &\leq \sum_{i=1}^N \left[-\sum_{j=0}^{n-k} \frac{\gamma}{2g_{ij}^m} z_{ij}^2 - \sum_{j=0}^{n-k} \frac{\gamma \tilde{W}_{ij1}^T \Gamma_{ij1}^{-1} \tilde{W}_{ij1}}{2} - \sum_{j=0}^{n-k} \frac{\gamma \tilde{W}_{ij2}^T \Gamma_{ij2}^{-1} \tilde{W}_{ij2}}{2} \right] + \delta \\ &\leq -\gamma \sum_{i=1}^N \left[\sum_{j=0}^{n-k} \frac{z_{ij}^2}{2g_{ij}(x_i, \xi_{i,1-j})} + \sum_{j=0}^{n-k} \frac{\tilde{W}_{ij1}^T \Gamma_{ij1}^{-1} \tilde{W}_{ij1}}{2} - \sum_{j=0}^{n-k} \frac{\tilde{W}_{ij2}^T \Gamma_{ij2}^{-1} \tilde{W}_{ij2}}{2} \right] + \delta \\ &= -\gamma V + \delta \end{aligned} \quad (57)$$

Theorem 1: Consider the closed-loop system consisting of system (3), the reference trajectory, x_d , the controller (53), and the NN weight updating laws (23), (43) and (55). If the NN transfer functions are selected to be smooth and bounded, and the NNs are large enough, such that they can approximate their objective functions accurately, then for bounded initial conditions, we have the following conclusion.

All signals in the closed loop system remain uniformly ultimately bounded, and the system states X and Ξ , and NN weight estimates $\hat{W}_{i01}^T, \dots, \hat{W}_{i,n-k,1}^T$ and $\hat{W}_{i02}^T, \dots, \hat{W}_{i,n-k,2}^T$ eventually converge to a compact set Ω .

$$\Omega = \left\{ X, \Xi, \hat{W}_{i01}^T, \dots, \hat{W}_{i,n-k,1}^T, \hat{W}_{i02}^T, \dots, \hat{W}_{i,n-k,2}^T \mid V < \frac{\delta}{\gamma} \right\} \quad (58)$$

Proof: From (57), we can see that if z_{ij} , \tilde{W}_{ij1} , and \tilde{W}_{ij2} are outside of the compact set defined as (58), then \dot{V} will remain negative definite until the systems state and the weight estimate errors enter the Ω . Thus, z_{ij} , \tilde{W}_{ij1} , and \tilde{W}_{ij2} are uniformly ultimately bounded [6]. Furthermore, since W_{ij1}^T and W_{ij2}^T exist and are bounded, then \hat{W}_{ij1}^T and \hat{W}_{ij2}^T are also bounded. Considering (6) and the

boundedness of z_{i0} and x_{id} , we can conclude that x_i is bounded. From $z_{ij} = \xi_{ij} - r_{i,j-1}$, $j=1, \dots, n-k$, and the definitions of the virtual control $r_{i,j-1}$, $j=1, \dots, n-k$, we have that ξ_{ij} , $j=1, \dots, n-k$ remain bounded. Using (53), we conclude that control u is also bounded.

Thus, all signals in the closed loop system remain bounded, and the system states X and Ξ , and NN weight estimates $\hat{W}_{i01}^T, \dots, \hat{W}_{i,n-k,1}^T$ and $\hat{W}_{i02}^T, \dots, \hat{W}_{i,n-k,2}^T$ eventually converge to a compact set Ω .

Remark 4: In NN literature, the unboundedness of parameter estimates when persistence of excitation (PE) fails to hold is known as "weight overtraining". The PE condition ensures that the parameter drift is avoided. Since it is difficult to verify or guarantee the PE condition, this theorem relaxes the PE condition by modifying the NN weight update rules.

Remark 5: The initial weights of the output layer are set to zero and then tuned online according to updating laws. There is no preliminary off-line training phase. This is a significant improvement over other NN control techniques where one must find some initial stabilizing weights, generally an impossible feat for complex nonlinear systems.

V. CONCLUSION

This paper proposed a NN based decentralized controller for a class of large-scale systems with unknown interconnections wherein the proposed work does not require the strict matching condition that is considered in other works. The controller design is based on back stepping methodology and NN approximation theory. All of the system states and weight estimation are guaranteed to be uniformly ultimately bounded and eventually converge to a compact set. Future work includes considering more general assumption on the interconnection terms and to use fewer number of NNs.

VI. REFERENCES

- [1] S. N. Huang, K. K. Tan, and T. H. Lee, "Decentralized control design for large-scale systems with strong interconnections using neural networks," IEEE Transactions on Automatic Control, vol. 48, no. 5, pp. 805-810, May 2003.
- [2] J. T. Spooner and K. M. Passino, "Decentralized adaptive control of nonlinear systems using radial basis neural networks," IEEE Transaction on Automatic Control, vol. 44, pp. 2050-2057, Nov. 1999.
- [3] N. Hovakimyan, E. Lavretsky, A. J. Calise, and R. Sattigeri, "Decentralized adaptive output feedback control via input/output inversion", Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, Hawaii USA, December 2003.
- [4] S. Jain and F. Khorrami, "Decentralized adaptive control of a class of large-scale interconnected nonlinear systems," IEEE Transactions on Automatic Control, vol. 42, no. 2, pp. 136-154, February, 1997.
- [5] B. Igel'nik and Y-H. Pao, "Stochastic choice of basis functions in adaptive function approximation and the functional-link net", IEEE Transaction on Neural Networks, vol. 6, no. 6, pp. 1320-1329, November 1995.
- [6] F. L. Lewis, S. Jagannathan, and A. Yesildirek, "Neural network control of robot manipulators and nonlinear systems", Taylor and Francis, 1999.