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Dominance of virtual Delbrück scattering for the photon emission by nuclei in relativistic electron–nucleus and nucleus–nucleus collisions

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Abstract

We consider the contribution of virtual Delbrück scattering to the photon emission by nuclei in the collision of two ultra-relativistic nuclei $Z_1 Z_2 \to Z_1 Z_2 \gamma$ and electron–nucleus collisions $eZ \to eZ\gamma$, in the photon energy range $m \ll E_\gamma \ll m\gamma$, where m is the electron mass and γ is the Lorentz factor of the colliding nucleus. The discussed process has no infrared divergence. The total cross section is 50 barn for the Pb–Pb collisions at the LHC collider. The spectral distribution obtained is considerably larger than for ordinary nuclear bremsstrahlung in the same photon energy range.

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1. Introduction

In this Letter we consider the emission of photons in nucleus–nucleus and electron–nucleus collisions. For definiteness, we start with the Au–Au collisions at the RHIC collider with the Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma = 108$, charge numbers of nuclei $Z_1 = Z_2 \equiv Z = 79$ and Pb–Pb collisions at the LHC collider with $\gamma = 3000$, $Z = 82$. As is well known, a freely moving, relativistic nucleus is accompanied by an electromagnetic field that can be described classically by Liénard–Wiechert potentials. These potentials can be expanded in terms of plane electromagnetic waves which are known as equivalent photons (so-called Weizsäcker–Williams approximation [1,2]). Equivalent photons are not real photons, but they can be used in order to describe the interaction of the relativistic nucleus with other charged particles, and in the course of these interactions, real photons can be emitted.

In principle, for the process of photon emission one would expect that the tree-level diagram in Fig. 1(a) should give the by far dominant contribution especially in the region of not too energetic real photons. It means that the equivalent photon is converted to a real one by virtual nuclear Compton scattering off the incoming nucleus. The scale for this process is given by the comparatively huge nuclear mass: $d\sigma_{\text{brems}} \sim Z^6\alpha^3/M^2$ [here and below we use units with $\hbar = c = 1$, denote the electron (nucleus) mass by m (M), and $\alpha = 1/137$].

The diagram in Fig. 1(b) represents a quantum electrodynamic (QED) electron–positron loop correction to the diagram in Fig. 1(a). It corresponds to Delbrück scattering of the equivalent photon off the second nucleus, with the emission of a real photon. This process has been mentioned for the first time in the paper [3], but its cross section has never been calculated to the best of our knowledge. This is done in the present Letter. The

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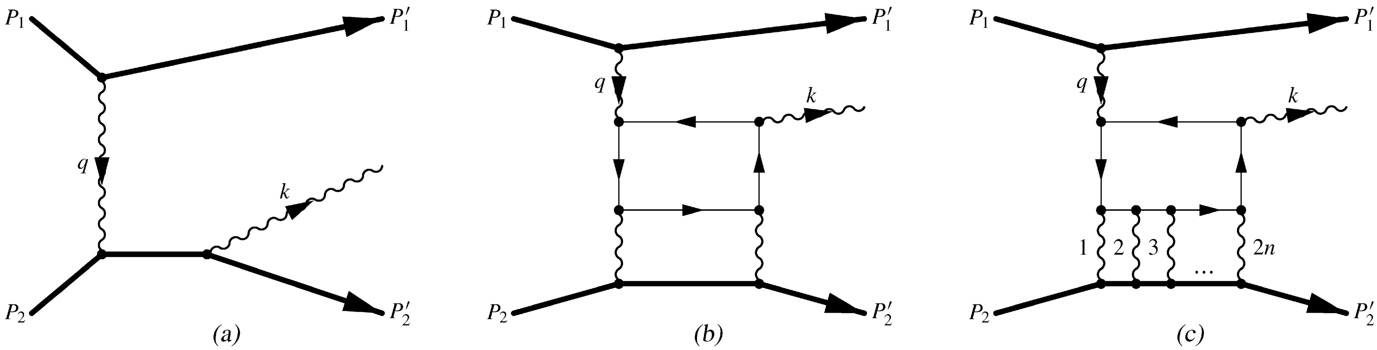

Fig. 1. (a) Representative diagram for ordinary nuclear bremsstrahlung which is the emission of a photon in a nuclear collision via a virtual Compton subprocess. Bold lines denote nuclei. (b) Representative diagram for the emission of a photon in a nuclear collision via virtual Delbrück scattering in the leading QED order. Thin lines denote the electron propagators. (c) Amplitude M^1_{2n} for the emission of a photon in the nuclear collision with the exchange of $2n$ virtual photons between electron loop and the second nucleus.

perhaps surprising conclusion of this Letter is the following: the processes Fig. 1(b) gives a contribution which is by one order of magnitude lager than that of Fig. 1(a) in the photon energy region

$$m \ll E_\gamma \ll m\gamma. \tag{1}$$

The reason for the dominance of the virtual Delbrück scattering is that the scale for the cross section of the process Fig. 1(b) is given by the small electron mass: $\mathrm{d}\sigma \sim Z^6\alpha^7/m^2$. Specifically, the ratio of the cross sections of the processes in Fig. 1(b) and (a) is roughly $(\alpha^2 M/m)^2 \gg 1$. In the following, we explain this conclusion in more detail.

2. Delbrück scattering

Properties of Delbrück scattering with real photons are well known (see, e.g., the review [4], recent experiments [5] and numerical results for the Delbrück scattering amplitudes in Ref. [6]). The total cross section of this process $\sigma_\mathrm{D}(\omega_L, Z)$ depends on the invariant (see Fig. 1 for the identification of q and P)

$$\omega_L = (q \cdot P_2)/M_2, \tag{2}$$

which is equal to the initial photon energy in the laboratory system (lab-system, denoted by the subscript L). Here, the lab-system means the rest frame of the scattering nucleus, in which the 4-momentum of the initial photon takes the form $q = (\omega_L, 0, 0, \omega_L)$, and the 4-momentum of the initial nucleus is $P_2 = (M_2, 0, 0, 0)$.

The Delbrück scattering cross section vanishes at small energies, $\sigma_\mathrm{D}(\omega_L \ll m, Z) \propto \omega_L^4$, and tends to a constant, independent of ω_L, in the limit $\omega_L \gg m$. In the lowest order of the QED perturbation theory, this constant is

$$\sigma_\mathrm{D}(\omega_L \gg m, Z) = \sigma_\mathrm{D}^{(0)}(Z) = 1.07(Z\alpha)^4\frac{\alpha^2}{m^2}. \tag{3}$$

For heavy nuclei, the strong-field effects [so-called Coulomb corrections $\sim (Z\alpha)^{2n}$ corresponding to the exchange of $2n$ virtual photons between the electron loop and the nucleus—see Fig. 1(c)] decreases significantly this constant,

$$\sigma_\mathrm{D}^{(0)}(Z) \to \sigma_\mathrm{D}(Z) \equiv \frac{\sigma_\mathrm{D}^{(0)}(Z)}{r_Z}, \tag{4}$$

where the reduction factor $r_Z > 1$. The factor $1/r_Z$ takes care of the Coulomb correction reduction. For example, for Delbrück scattering off the Au ($Z = 79$) and Pb ($Z = 82$) nuclei, the cross section $\sigma_\mathrm{D}(Z)$ is 5.5×10^{-3} barn for Au and 6.2×10^{-3} barn for Pb. These are, respectively, $r_{79} = 1.7$ and $r_{82} = 1.8$ times smaller than $\sigma_\mathrm{D}^{(0)}(Z)$.

The main contribution to the total cross section given in Eqs. (3) and (4) for $\omega_L \gg m$ comes from a region where the transverse momenta of the final photon $k_\perp \sim m$. For transverse momenta larger than the electron mass, but still smaller than the photon energy, the differential cross section has the form

$$\mathrm{d}\sigma_\mathrm{D} = \alpha^2(Z\alpha)^4 f_Z(k_\perp/m)\frac{\mathrm{d}k_\perp^2}{m_\perp^4} \quad (m \lesssim k_\perp \ll \omega_L). \tag{5}$$

Here $m_\perp = \sqrt{m^2 + k_\perp^2}$, and $f_Z(k_\perp/m)$ is a slowly varying function of the ratio $k_\perp/m$. Numerical values of this function can be found from plots and numbers given in Refs. [4,6]. In particular, for $Z = 82$, this function is $f_{82}(k_\perp/m) \approx 1.2$ at $k_\perp \gg m$ and $f_{82}(k_\perp/m) \approx 0.48$ at $k_\perp = m$.

It should be noted that the applicability of this distribution is limited not only by the condition $k_\perp \ll \omega_L$, but also by the additional condition $k_\perp \lesssim 1/R$, where $R \approx 1.2A^{1/3}$ fm is the radius of the nucleus with A the nucleon number. We have, roughly, $R \approx 7$ fm and $1/R \approx 28$ MeV for Au and Pb.

3. Delbrück scattering and the $Z_1Z_2 \to Z_1Z_2\gamma$ process

Let us consider now the process of a photon emission without any excitation of nuclei in the final state: $Z_1(P_1) + Z_2(P_2) \to Z_1(P_1') + Z_2(P_2') + \gamma(k)$. In this process, two nuclei with charges Z_1e and Z_2e and 4-momenta P_1 and P_2 collide with each other and produce a photon with the total four-momentum k. Let E_i ($\gamma_i = E_i/M_i$) and E_γ be the energy (the Lorentz factor) of the ith nucleus and the photon energy in the collider system, respectively. Here, the collider system means the rest frame of the particle collider, which is not necessarily equal to our "lab-system" (the latter we define to be the rest frame of the scattering nucleus). For identical colliding nuclei, the collider system coincides with the center-of-mass system, and in this case $\gamma_1 = \gamma_2 = \gamma$.

Because $Z\alpha \approx 0.6$ for the collisions considered, the whole series in $Z\alpha$ has to be summed to obtain the cross section with sufficient accuracy. Fortunately, there is another small parameter $1/L = 1/\ln(\gamma_1\gamma_2) = 0.1$–$0.06$, and it will be sufficient to calculate the cross section in the leading logarithmic approximation (LLA) only.

Let $\mathcal{M}$ be the sum of all amplitudes $M^n_{n'}$ which correspond to n exchanges with the first and n' exchanges with the second nucleus. This sum can be presented in the form

$$\mathcal{M} = \sum_{nn'} M^n_{n'} = M_1 + \tilde{M}_1 + M_2,$$
$$M_1 = \sum_{n\geqslant 1} M^1_{2n}, \qquad \tilde{M}_1 = \sum_{n\geqslant 1} M^{2n}_1. \tag{6}$$

The amplitude M_1 contains a one-photon exchange with the first nucleus [see also Fig. 1(c)], the amplitude $\tilde{M}_1$ describes a one-photon exchange with the second nucleus, and the amplitude M_2 has no one-photon exchange. According to this classification, we write the total cross section as

$$\sigma = \sigma_1 + \tilde{\sigma}_1 + \sigma_2, \tag{7}$$

where

$$d\sigma_1 \propto |M_1|^2, \qquad d\tilde{\sigma}_1 \propto |\tilde{M}_1|^2, \tag{8}$$

and $d\sigma_2$ corresponds to the rest of the terms. The integration over the transferred momentum squared q^2 results in the large Weizsäcker–Williams logarithm $\sim L$ for σ_1 and $\tilde{\sigma}_1$, but not for σ_2. Therefore, the relative contribution of the σ_2 term is

$$\sigma_2/\sigma_1 \sim (Z\alpha)^2/L < 0.04. \tag{9}$$

As a result, with an accuracy of the order of a few percent we can neglect σ_2 in the total cross section and use the equation $\sigma = \sigma_1 + \tilde{\sigma}_1$.

Let us consider the cross section σ_1. In the LLA, it can be calculated using the equivalent photon approximation, in which $d\sigma_1$ is expressed via the number of equivalent photons dn_1, emitted by the first nuclei, and the cross section for the Delbrück scattering off the second nuclei (see, e.g., Ref. [7]):

$$d\sigma_1 = dn_1\, \sigma_D(\omega_L, Z_2). \tag{10}$$

The virtual Delbrück scattering amplitude decreases when the virtuality of the initial photon $Q^2 = -q^2$ becomes larger than $m_\perp^2$ (here, $q = P_1 - P_1'$ is the 4-momentum of the equivalent photon). This means that the main contribution to $d\sigma_1$ is given by photons from the first nucleus with a small virtuality

$$Q^2 = -q^2 = \boldsymbol{q}_\perp^2 + (\omega/\gamma_1)^2 \ll m_\perp^2, \tag{11}$$

where $\omega = E_1 - E_1'$ is the energy of the equivalent photon in the collider system. Therefore, we can neglect the virtuality of this photon in the description of the cross section $\sigma_D(\omega_L, Z_2)$ for the subprocess. From (11), we learn that we can usually assume $\omega \ll m_\perp\gamma_1$. Because $\omega_L = (q \cdot P_2)/M_2 = 2\omega\gamma_2$, the most important region for this cross section is [in accordance with Eqs. (4) and (11)]

$$m \ll \omega_L = 2\omega\gamma_2 \ll m\gamma_1\gamma_2, \qquad k_\perp \sim m. \tag{12}$$

To calculate the spectrum of equivalent photons, we can use Eq. (D.4) from Ref. [7] neglecting terms proportional to ω/E_1, since in our case $\omega \lesssim m\gamma_1 \ll E_1$:

$$dn_1(\omega, Q^2) = \frac{Z_1^2\alpha}{\pi}\frac{d\omega}{\omega}\left(1 - \frac{Q^2_{\min}}{Q^2}\right)F^2(Q^2)\frac{dQ^2}{Q^2}, \tag{13}$$

where $Q^2_{\min} = (\omega/\gamma_1)^2$ and $F(Q^2)$ is the nuclear electromagnetic form factor. Here, we assume that $m_\perp^2$ is considerably smaller than $1/R^2$. This implies that $k_\perp \ll 1/R$, so that we can put $F(Q^2) = 1$ in our calculation. Integrating $dn_1(\omega, Q^2)$ over Q^2 in the region

$$Q^2_{\min} = (\omega/\gamma_1)^2 \leqslant Q^2 \lesssim m^2, \tag{14}$$

and then integrating the cross section (10) over ω in the region

$$m/\gamma_2 \lesssim \omega \lesssim m\gamma_1, \tag{15}$$

we obtain the total cross section σ_1 in the LLA

$$\sigma_1 = \frac{\alpha}{\pi} Z_1^2 \sigma_D(Z_2) L^2, \tag{16}$$

and the cross section $\tilde{\sigma}_1$ is obtained by the replacement $Z_1 \leftrightarrow Z_2$. As a result, the total contribution of the Delbrück scattering to the cross section of the discussed process is equal to

$$\sigma = \sigma_1 + \tilde{\sigma}_1 = \frac{\alpha}{\pi}\left[Z_1^2\sigma_D(Z_2) + Z_2^2\sigma_D(Z_1)\right]L^2, \tag{17}$$

where L is given by

$$L = \ln\left(\frac{P_1 \cdot P_2}{2M_1M_2}\right) = \ln(\gamma_1\gamma_2), \tag{18}$$

and $\sigma_D(Z)$ is given in Eq. (4).

4. Energy and angular distribution of photons

In the above calculations, we have considered only the total cross section in the LLA. In a similar way, the energy and angular distribution of the final photons can be obtained. We present here the final results only for the case of identical nuclei. The differential over the photon momentum cross section reads

$$d\sigma = \frac{2}{\pi^2}\alpha(Z\alpha)^6\frac{f_Z(k_\perp/m)}{(m^2+k_\perp^2)^2}L\frac{d^3k}{E_\gamma},$$
$$m_\perp \ll E_\gamma \ll m_\perp\gamma, \tag{19}$$

and the spectrum of photons is

$$d\sigma = \frac{4}{\pi}Z^2\alpha\sigma_D(Z)L\frac{dE_\gamma}{E_\gamma} \quad (m \ll E_\gamma \ll m\gamma). \tag{20}$$

Let us stress that this type of distribution is only valid for not too soft photons. The Delbrück cross section vanishes for soft photons, and, therefore, the discussed cross section in fact has no infrared divergence.

5. Comparison to nuclear bremsstrahlung

Tree-level photon emission by nuclear bremsstrahlung (virtual Compton scattering) is described by the Feynman diagram of Fig. 1(a), with the cross section $d\sigma_{\text{brems}}$.

Now we can repeat the previous calculations with minor changes, replacing virtual Delbrück scattering by virtual Compton scattering. In particular, the expression analogous to (10) has the form

$$d\sigma_{\text{brems}} = dn_1(\omega)\, d\sigma_C(\omega, E_2, E_\gamma, Z_2), \tag{21}$$

where

$$dn_1(\omega) = 2\frac{Z_1^2\alpha}{\pi} \ln\left(\frac{\gamma_1}{\omega R}\right)\frac{d\omega}{\omega} \tag{22}$$

is the number of the equivalent photons emitted by the first nucleus, and σ_C is the cross section for the Compton scattering of this photon off the second nucleus. For the Compton cross section, we can use well-known expressions valid for a nucleus approximated by a charged point particle, and this leads to

$$d\sigma_{\text{brems}} = \frac{2}{\pi} Z_1^2 \alpha \sigma_T(Z_2) \ln\left(\frac{4\gamma_1\gamma_2^2}{E_\gamma R}\right)\frac{dE_\gamma}{E_\gamma}, \tag{23}$$

where the nuclear Thomson cross section $\sigma_T(Z_2)$ is

$$\sigma_T(Z_2) = \frac{8\pi}{3}\frac{Z_2^4\alpha^2}{M_2^2}. \tag{24}$$

The cross section $d\tilde{\sigma}_{\text{brems}}$ can be obtained by replacing indices $1 \leftrightarrow 2$. This approximation is justified because the equivalent photon energy range relevant for virtual Delbrück scattering, as seen by the incoming nucleus, fulfills $\omega_L \sim E_\gamma/(2\gamma) \ll m/2$. In this energy range, the conceivable influence of nuclear resonances can be safely excluded (at higher photon energies, their influence has been studied in [8]).

Comparing these formulae with the corresponding ones for the Delbrück scattering, we find that the ratio

$$\frac{d\sigma_{\text{brems}}}{d\sigma_1} \sim \frac{\sigma_T(Z)}{\sigma_D(Z)} = 7.83 r_Z \left(\frac{m}{\alpha^2 M}\right)^2 \tag{25}$$

is small in the energy region given by Eq. (1), since $\sigma_T(Z)/\sigma_D(Z) \approx 1/30$ for the considered heavy nuclei.

6. eRHIC option

The obtained results can be easily modified for the process of a photon emission in electron–nucleus collisions without excitation of nucleus $eZ \to eZ\gamma$. Certainly, the emission of photons in the direction of the electron beam is absolutely dominated by ordinary bremsstrahlung. But for the emission in the nuclear beam direction, we find out that the process via the Delbrück scattering is dominant in a certain region of the photon energy.

This consideration is motivated by the recent project of the eRHIC collider which is now actively discussed as a promising extension of the existing RHIC machine (see [9]). It is proposed to built an additional electron ring with the energy $E_e = 10$ GeV and, thus to create an electron–nucleus collider with parameters:

$$Z_1 = -1, \qquad \gamma_1 = 2\times 10^4,$$
$$Z_2 \equiv Z = 79, \qquad \gamma_2 = 108. \tag{26}$$

The process $eZ \to eZ\gamma$ via virtual Compton scattering and via virtual Delbrück scattering is described by Feynman diagrams Fig. 1(a) and (b), respectively, but with the first nucleus being replaced by the electron. The corresponding calculations are basically the same as above with some minor changes. In particular, in Eq. (7) we can neglect not only σ_2, but $\tilde{\sigma}_1$ as well:

$$d\sigma_{eZ} = dn_e\, \sigma_D(Z), \tag{27}$$

with (see, e.g., Ref. [7])

$$dn_e = 2\frac{\alpha}{\pi}\left(1 - x + \frac{1}{2}x^2\right)\frac{d\omega}{\omega} \ln\left(\frac{m\gamma_1\sqrt{1-x}}{\omega}\right), \tag{28}$$

where $x = \omega/E_e$. Integrating this cross section in the region (15), we obtain the total cross section

$$\sigma_{eZ} = \frac{\alpha}{\pi}\sigma_D(Z) L^2, \tag{29}$$

which leads to the value $\sigma_{eZ} = 0.19 \times 10^{-3}$ barn for the parameters (26).

The region (15) can be split into two subregions: $m/\gamma_2 \ll \omega \ll m$ (subregion A) and $m \ll \omega \ll m\gamma_1$ (subregion B). The most interesting is the subregion A, in which the photons fly along the beam direction of the scattering nucleus; their spectrum is given by

$$d\sigma_{eZ}^{(A)} = \frac{2\alpha}{\pi}\sigma_D(Z) \ln\left(\frac{\gamma_1 E_\gamma}{m}\right)\frac{dE_\gamma}{E_\gamma} \tag{30}$$

for $m \ll E_\gamma \ll m\gamma_2$. The contribution of the ordinary nuclear bremsstrahlung in the same direction and the same region of energy is smaller for heavy nuclei and equals:

$$d\sigma_{\text{brems}} = \frac{2\alpha}{\pi}\sigma_T(Z) \ln\left(\frac{4\gamma_1\gamma_2^2}{E_\gamma R}\right)\frac{dE_\gamma}{E_\gamma}. \tag{31}$$

In the subregion B, the photons fly along the electron beam direction, and their spectrum is absolutely dominated by the ordinary bremsstrahlung, described by the well-known Bethe–Heitler formula (see, for example, the textbook [1]).

7. Conclusions

For both the $Z_1Z_2 \to Z_1Z_2\gamma$ as well as the $eZ \to eZ\gamma$ processes, the emitted photon energy region we consider is $m \ll E_\gamma \ll m\gamma$ in the collider reference system, and we establish the dominance of the virtual Delbrück scattering process over ordinary nuclear bremsstrahlung. In the LLA, the total photon emission cross section corresponding to this region is given by Eq. (17) for $Z_1Z_2 \to Z_1Z_2\gamma$. In particular, for Au–Au collisions at the RHIC collider, $\sigma = 14$ barn, and for Pb–Pb collisions at the LHC collider, the total cross section is $\sigma = 50$ barn. Note for comparison, that the last cross section is 6 times larger than for the total hadronic/nuclear cross section in Pb–Pb collisions, which is roughly 7.9 barn. Corrections to this result are of the order of $1/\ln(\gamma^2) = 0.06$ for the LHC Pb–Pb option. The energy and angular distribution of photons is given in Eq. (19). For $eZ \to eZ\gamma$, our main result is given in Eq. (29).

If our process can be detected experimentally, then one can effectively study Delbrück scattering in the range of the initial photon energy up to $\omega_L \sim 2m\gamma^2$ which is 10 GeV for RHIC, 8 TeV for LHC and 2 TeV for eRHIC (in the rest frame of the colliding nucleus).

Finally, we would like to point out that there is a numerically not large, but conceptually interesting so-called unitarity correction to the process $Z_1 Z_2 \to Z_1 Z_2 \gamma$. It is due to the unitarity requirement for the S matrix and corresponds to the exchange of light-by-light blocks between nuclei; this correction is analyzed in detail in [10].

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