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Comment on "A Convergent Series for the QED Effective Action"

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In a recent Letter, Cho and Pak claim to have found an additional contribution to the quantum electrodynamic one-loop effective action: a "logarithmic correction term" [see the final result in Eq. (2) and the remark in Ref. [9] of [1]]. However, the logarithmic correction term found by Cho and Pak vanishes when the final result is written in terms of the finite, renormalized, physical electron charge.

Because current determinations of fundamental constants [2] rely on renormalized QED perturbation theory—without logarithmic correction terms of the kind advocated by Cho and Pak—it is of prime general interest to point out that these terms do not appear if on-mass shell renormalization is used. In the on-mass shell scheme, the renormalized QED effective Lagrangian (see, e.g., [3, Eq. (3.43)]) reads

$$\Delta \mathcal{L} = -\frac{e^2}{8\pi^2} \lim_{\epsilon,\eta\to 0^+} \int_{\eta}^{i\infty+\eta} \frac{ds}{s}\, e^{-(m^2-i\epsilon)s} \times \left[ab \coth(eas)\cot(ebs) - \frac{a^2-b^2}{3} - \frac{1}{(es)^2} \right]. \quad (1)$$

The two latter terms in the integrand are counterterms. The last term simply removes a divergent constant from the Lagrangian, while the term $-(a^2 - b^2)/3$—if it were not removed—would lead to a logarithmic divergence at small eigentime s. This logarithmically divergent term, however, is proportional to the leading-order Maxwell Lagrangian $\mathcal{L}_{\rm cl} = (b^2 - a^2)/2$ and leads to a Z_3 renormalization [see Eq. (8-97) of [4]]. Specifically, the introduction of the cutoff parameter μ in [1] leads to a logarithmic term $[1 - (e^2/12\pi^2)\ln(m^2/\mu^2)]$ which multiplies $\mathcal{L}_{\rm cl}$. In order to ensure compliance with the renormalization conditions of on-mass shell renormalization [see Eqs. (8-96d) and (8-96e) of [4]], a further counterterm $+(e^2/12\pi^2)\ln(m^2/\mu^2)\mathcal{L}_{\rm cl}$ has to be added to the Lagrangian. As a consequence, the logarithmic correction term is absent [see Eqs. (2)–(6) of [5]]. For the particular problem at hand, the on-mass shell scheme is well motivated even from a purely mathematical point of view, as it is evident from the partial fraction theorem discussed in Sec. 3 of [6].

If Cho and Pak use different renormalization conditions, then the logarithmic correction term has to be reabsorbed into the physical charge of the electron, by considering the effect that the term has on matrix elements of transition currents [see the elucidating discussion on page 325 of [4]]. In this case, we are forced to interpret $e^2_{\rm ph}(\mu) = e^2[1 - (e^2/12\pi^2)\ln(m^2/\mu^2)] + \mathcal{O}(e^4)$ as the *physical* charge, in which case the logarithmic correction term [1] is reabsorbed in a renormalization of charge. When expressing $\Delta\mathcal{L}$ in terms of $e^2_{\rm ph}(\mu)$ instead of e^2, the resulting further modification of $\Delta\mathcal{L}$ is of the same order as the two-loop effective Lagrangian and therefore beyond the validity of the one-loop approximation inherent to Eq. (1). Finally, we would like to remark here that a renormalization-group (RG) improved running of the electron charge, based on the RG invariance of the effective action, has been discussed by Dittrich and Reuter (Chap. 8 of [7]) and Ritus [8], and that, in the latter case, two-loop effects are consistently taken into account in the analysis of the evolution of the electromagnetic charge.

Finally, we stress that the potentially important remark in Ref. [9] of [1] falsely suggests that the "usual" result for $\Delta\mathcal{L}$ given in Eq. (1) is incomplete without the logarithmic correction term. Helpful conversations with H. Gies, B. R. Holstein, D. G. C. McKeon, C. Schubert, V. M. Shabaev, and G. Soff are gratefully acknowledged.

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