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An Approach to Active Sensor Imaging

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Abstract

In this paper, an alternative Target Density Function (TDF) is proposed to image the radar targets in a dense target environment. It is obtained by considering a novel range and scanning angle plane different from the conventional methods. An alternative method is briefly proposed for smoothing the target density function by taking advantage of Walsh functions. Although the imaging is obtained via the phased array radars, the problem associated with beamforming in linear phased array radar system is bypassed in this new algorithm.

1. Introduction

Target density function(TDF) is the reflectivity of spatially, continuously distributed targets and it is an important characteristic of radar imaging. TDF is known by different names such as ambiguity function, density function, target density function, object(target), object reflectivity function, doubly-spread reflectivity function, and reflection coefficient [1–6].

If TDF is assumed to be a reflection coefficient, then it is defined as the ratio of the received signal to the transmitted signal. By this definition, the reflected signals from the object space are amplitudes relevant to the intensities of the points on the target or objects. If the object geometric plane is considered, since the integration of these amplitudes or the illuminated intensities reveal information related to the object shape, TDF will have an important role in obtaining the radar images.

There are two well known approaches on TDF. First one considers point scatterers reflected off the target scatterer centers. Integration of all point scatterers is able to obtain the whole object. This radar imaging technique is based on inverse Fourier transform(IFT) and used mostly in inverse synthetic aperture radar(SAR) studies [7–12].

Second method on TDF is a dense target environment approach by Fowle and Naparst [13, 14]. It is based on the ambiguity functions with two variables as range and velocity [15–17]. Especially, the advanced function in the dense target environment by Naparst is developed in a novel way. Rather than typical radar imaging, this is an approach to measure the closeness of the targets to each other in the dense target environment.

In this study, a new TDF is theoretically developed by a new approach on a range-scanning angle plane different from the early approaches. This is obtained via a phased array radar system, the problem associated with beamforming [18] is bypassed [18]. Addition to well known alternatives such as filtration or compressing, an unconventional approaches, which is a Walsh function, is proposed for the smoothing of the new TDF.

2. Walsh Functions

Walsh functions are orthogonal functions and composed of square waves with (0-1) amplitudes. Unlike the Rademacher functions, Walsh functions are complete. Mathematical theory of Walsh functions corresponds to Fourier analysis-based sine-cosine functions [19–22].

Walsh functions are defined in a limited time interval, T, known as the time-base. Like the sine-cosine functions, two entities are required for a complete definition. These are a time period, t, which is normalized to the time base as t/T, and an ordering number, n, which is related to frequency. A Walsh basis function is represented by $Wal(n,t)$. A general Walsh function with pulse basis functions can be written as [19–22]

$$Wal(n,t) = sign[(sin2\pi t)^{b_0} \prod_{k=1}^{m} (cos2^k \pi t)^{b_k}] \quad (1)$$

where n and m are related to each other. If u is a binary value of the decimal, n, and $g(u)$ is a number of digits, then m is represented as

$$m = g(u) - 1 \tag{2}$$

b_0 and b_k in Equation 1 are either 0 or 1.

A set of Walsh functions derived from Equation 1 is given in Figure 1 [19–22]. These sets of Walsh functions are in form of typical radar pulse train.

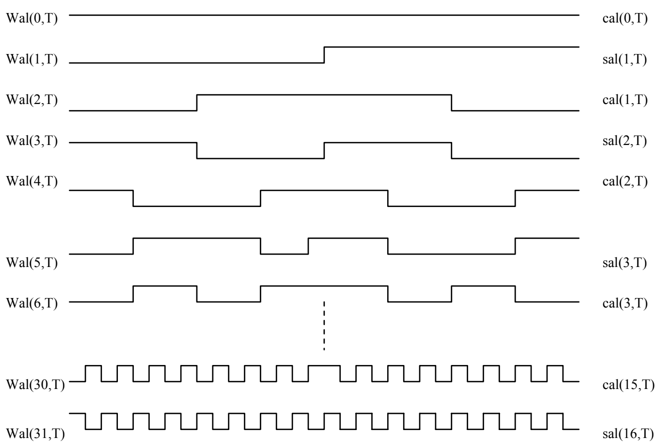

Figure 1. A set of Walsh functions.

While behavior of both Fourier and Walsh series are similar, basis functions have different forms. Walsh function can be expressed as a time series similar to the Fourier theory:

$$f(t) = \sum_{k=0}^{\infty} F_k Wal(k,t) \tag{3}$$

If this is compared with Fourier series, $p(t) = \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_0 t}$, their basis functions become important separators. The basis functions are infinite in Fourier series($-\infty \leq k \leq \infty$), while finite in Walsh series($0 \leq k \leq \infty$). The finite basis functions provide important advantage in signal processing in terms of dimension reduction.

Two new functions, **sal** and **cal**, which are analogues of **sine** and **cosine** functions in Fourier series, are defined by Walsh functions [19–22].

$$f(t) = a_0 Wal(0,t) + \sum_{k=1}^{\infty} [a_k cal(k,t) + b_k sal(k,t)] \tag{4}$$

where

$$a_0 = \int_{-1/2}^{1/2} f(t) Wal(0,t) dt,\ a_k = \int_{-1/2}^{1/2} f(t) cal(k,t) dt$$
$$b_k = \int_{-1/2}^{1/2} f(t) sal(k,t) dt \tag{5}$$

3. Preliminaries Of Target Density Functions

The background of the target density functions consists of the following main techniques;

- SAR-ISAR reflectivity functions
- Naparst's target density functions

3.1. SAR - ISAR Reflectivity Functions

Coherent SAR imaging is an alternative approach to remote sensing that provides contribution to the imaging over visible/infrared sensing technology [10, 11, 23–26]. If the target is composed of continuum point targets(scatterers), by the superposition principle, the echo(reflected signal) $e(t)$, from such a target at x, y points is:

$$e(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x,y) f(t - \frac{2R(x,y)}{c}) dxdy \tag{6}$$

Here, f is the transmitted signal function, ρ is reflectivity function, R is the range, and c is the speed of light. As stated in Equation 6, the returned signal $x(t)$ is a delayed and time-scaled version of the transmitted signal, $f(t)$.

If Inverse Fourier Transform is applied to Equation (6), the image $\rho(x,y)$ is obtained as a 2-D form of 3-D object [7, 10, 11].

$$\rho(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(f_x, f_y) e^{j2\pi(xf_x - yf_y)\frac{2R_p(t)}{c}} df_x df_y \tag{7}$$

where

$$f_x = \frac{2f_0}{c} cos\theta(t)\ f_y = \frac{2f_0}{c} sin\theta(t) \tag{8}$$

See Ref [27] for the details.

3.2. Target Density Functions

First *Density* term related to the target density function is called by Fowle et all [13]. Fowle is focused on the problem of the detection and resolution in two dimensions of a large number of targets in a fixed part of the target space and, he is inspired of ambiguity functions. Then, *Dense target environment* term is used by Naparst's paper [14] by taking advantage of Fowle work. His new approach is based on ambiguity and cross-ambiguity functions. In this work, the dense-target environment is defined the closeness of a lot of targets at a distance, which their velocities are so close to each other.

Definition by Naparst, density of targets at distance x and velocity y is $D(x,y)$. In this case, the echo or the reflected signal from targets is

$$e(t) = \int_{0}^{\infty}\int_{-\infty}^{\infty} D(x,y)\sqrt{y}s(y(t-x))dxdy \qquad (9)$$

In this approach, it is assumed that all targets are illuminated equally. As stated, the target density function is a function of the range and velocity variables similar to the ambiguity functions.

Reconstruction of the target density function in Naparst algorithm is finalized as fallows(see Ref [14] for the details);

$$D(x,y) = \sum_{n,m=0}^{\infty} < e_n, s_m > A_{nm}(x,y) \qquad (10)$$

where s_m are signals sent out and e_n are their echoes. The cross-ambiguity function of the signals sent out $(s_1, s_2, \ldots)$ is

$$A_{nm}(x,y) = \int_{-\infty}^{\infty} s_n(y(t-x))\bar{s_m}(t)dt \qquad (11)$$

4. Development Of An Alternative Target Density Function

In this paper, an alternative target density function(TDF) is obtained by a new algorithm. Rather than the measurement of a high dense target environment, this TDF is an effective tool for imaging of the radar targets in the dense target environment. The target density algorithm related to the radar imaging is theorized in a different way based on a linear phased array radar system and the range-scanning angle.

New target density function, $g(R,\beta)$ is composed of two variables, which are the range R, and the scanning angle β. If the target density function was the ratio of the received signal to the transmitted signal as stated early, by taking advantage of this definition, we give the following new definition utilized in our algorithm.

Definition: Target Density Function is the limit of the ratio of the amplitude of the signal reflected from an infinitesimally neighborhood about the point (R,β) to the amplitude of the incoming signal.

By this definition, the new target density function $g(R,\beta)$ is;

$$g(R,\beta) = \lim_{d(\Omega)\to 0} \frac{A_r}{A_t} \qquad (12)$$

where $d(\Omega)$ is the diameter of the disc about the point $(R,\beta) \in \Omega$, A_r and A_t are the amplitudes of the reflected and the transmitted signals, respectively.

In this definition, the target density function(TDF) is relevant to the the reflectivity of spatially, continuously distributed targets and emphasizes how much energy is reflected. This approach is different from the conventional target density function definitions stated early. Instead of ambiguity functions based on range-velocity variables, the imaging in a high dense target environment is taken by a new target density function with the range and scanning angle.

Let us consider the target plane shown in Figure 2, where β is $cos\theta$ and R is the range from the target to the radar, and the sensor elements in the linear phased array radar system are located equally.

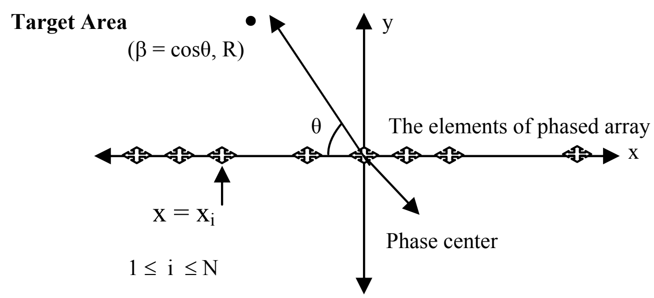

Figure 2. Phased array imaging.

As seen in Figure 2, the target density function is a function of the spatial coordinates (R,β) in the upper semi-plane.

Now, let us obtain the target density function. Let $P(t)$ be any periodic function of time, such as a train of pulses,where

$$p(t) = \sum_{k=-\infty}^{\infty} \alpha_k\, e^{jk\omega_0 t} \qquad (13)$$

$$\omega_0 = 2\pi \times \mathrm{PRF}, \qquad (14)$$

where PRF is the pulse repetition frequency.

$$s_c(t) = e^{j\omega_c t} \qquad (15)$$

Where $s_c(t)$ is the carrier signal.

$$s_m(t) = p(t)s_c(t) \qquad (16)$$

Where $s_m(t)$ is the modulated signal. The reflectivity of one point at $g(R,\beta)$

$$e(x,t) = s_m(t - 2R/c - \beta x/c)g(R,\beta) \qquad (17)$$

Let us generalize (17) for the whole radar-target semi upper plane by superpositioning principle considering all point scatterers related to the range-angle.

If $g(R,\beta)$ is the reflectivity of the point (R,β), and R_1 is the maximum range of interest target area; then

$$e(x,t) = \int_{-1}^{1}\int_{0}^{R_1} s_m(t-2R/c-\beta x/c)g(R,\beta)dRd\beta \tag{18}$$

Then

$$\begin{aligned} e(x,t) &= \int_{-1}^{1}\int_{0}^{R_1} p(t-2R/c-\beta x/c)e^{-j\omega_c(2R/c+\beta x/c)} \\ &\quad \times e^{j\omega_c t}g(R,\beta)dRd\beta \end{aligned} \tag{19}$$

where $e(x,t)$ is the output of the sensor located at center (the feature space), and c is the speed of light. The algorithm is as follows,

$$\begin{aligned} e(x,t) &= \sum_{k=-\infty}^{\infty}\alpha_k e^{j(\omega_c+k\omega_0)t}\int_{-1}^{1}\int_{0}^{R_1} e^{-j(\omega_c+k\omega_0)2R/c} \\ &\quad \times e^{-j(\omega_c+k\omega_0)\beta x/c}g(R,\beta)dRd\beta \end{aligned} \tag{20}$$

Then, demodulation of (20) via (21)

$$s_d(t) = e^{-j(\omega_c+k\omega_0)t} \tag{21}$$

yields,

$$\begin{aligned} E_k(x) &= \int_{-1}^{1}\int_{0}^{R_1} e^{-j(\omega_c+k\omega_0)2R/c} \\ &\quad \times e^{-j(\omega_c+k\omega_0)\beta x/c}g(R,\beta)dRd\beta \end{aligned} \tag{22}$$

Let apply a zero or first holder to $Y_k(x)$ to reach the continuous form of the function as $Y(k,x)$.

$$E(k,x) = \int_{-1}^{1}\int_{0}^{R_1} e^{-j(\omega_c+k\omega_0)(2R/c+\beta x/c)}g(R,\beta)dRd\beta \tag{23}$$

In case of $-\infty < k < \infty$ and for each β and R, let consider (23) as the trigonometric Fourier series of $g(R,\beta)$. Hence we estimate $g(R,\beta)$

$$g(R,\beta) = \sum_{k,x=-\infty}^{\infty} E(k,x)e^{j(\omega_c+k\omega_0)(2R/c+\beta x/c)} \tag{24}$$

as a desired target density function(TDF) in range-scanning angle plane. Thus, in a high dense target environment, radar targets can be imaged theoretically by TDF $g(R,\beta)$. As realized that although a phased array radar system is used during the producing of TDF, the problem associated with beamforming is bypassed.

Infinity of k can be optimized by some filtration, compressing or estimation methods. However, an alternative way may be proposed to reduce k dimension and smooth the new TDF;

- **Walsh Approach:** While the new TDF is developed, at the beginning, the basis functions of the modulating signal had infinite dimensions. In contrast to infinite basis functions($-\infty \leq k \leq \infty$) in the Equation 13, Walsh functions are expressed in finite pulse basis functions (3). They have an essential advantage to the radar imaging in terms of basis dimension reduction.

In case of using Walsh functions with pulse form, this function in (3) will replace the Equation 13 in the new algorithm as a modulating Walsh function with finite dimensions. The Walsh function in question is a modulating signal in the form of a pulse train. After Walsh function is chosen with respect to some parameters such as *PRF* in (13), the new algorithm can resume the remaining steps after (14) in a similar manner.

5. Summary and Conclusion

In this paper, radar imaging is studied as an active sensor. An alternative target density function(TDF) is obtained by a new algorithm. The proposed target density function is based on the range and angle information different from conventional approaches. The advantage of taking the angle-range plane is to provide both practical scanning and effective detection. Then, an unconventional alternative is briefly proposed for smoothing the TDF. Two main contributions of this study are as follows;

- *A proposed target density function algorithm:* Target density function (TDF) is represented by an algorithm that is capable of producing the radar images by desired scanning angle and range plane. While developing this algorithm, the high dense target environments with multiple targets are taken into account.

- *Bypassing the beam-forming problem:* Second contribution of this study is provided by the phased array radar system. Although the new TDF is produced via the phased array radar, the problem associated with beamforming is bypassed.

The present TDF is generated partly by analogy to Fowle-Naparst and SAR-ISAR approaches.

- *Comparing to Fowle-Naparst:* As an advanced work of Fowle, Naparst target density function is developed for a high dense target environment with multiple targets, whose velocities are close to each other. This TDF acts like a separator rather than an imaging function for the targets at the distance with a given velocity.

TDF proposed here is obtained by a scanning angle and range in a high dense target environment. The main difference is in the imaging approach, which is capable of sensor imaging the targets in a dense target environment via phased array radar system.

- *Comparing to ISAR:* While ISAR imaging is based on multi-aperture principle, the present imaging method is a multi-sensor image fusion technique based on the phased array radar system. TDF in this study is similar to the reflectivity function in conventional ISAR imaging. However, ISAR reflectivity function is obtained by the integration of the point scatterers on the target, while our target density function is produced by the integration of ranges and scanning angles.

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